Abstract : To be able to design state of art SAW filters, it is mandatory to precisely characterize the SAW propagation parameters and to quantify the effect of manufacturing variations. A classical approach consists in an experimental characterization using sets of test devices. One of the main difficulties of this experimental approach is the accuracy of both the geometrical and electrical measurements. It implies to manufacture new test devices each time a new configuration is used. Precise numerical methods advantageously replace experiments. For both infinite and finite grating modeling, we developed numerical mixed FEM/BEM (Finite Element Method - Boundary Element Method) models using an efficient interpolation function basis that takes into account the singularity at both edges of each electrode. An other formulation allowing to analyze more complex structures like passivated devices or interfacial waves is described. Simulations and comparisons with experiments are shown for each model.

1 INTRODUCTION

In the first times, SAW devices analysis methods were based mainly on physical approximated models, often considering the electrode thickness as a small perturbation. The last ten years have seen the development of several numerically intensive methods to describe the propagation of SAWs in a transducer. In addition to the computers speeds continuous improvement, this is due to the large demand in low loss filters for telecommunication market and in particular to the generalization of the use of larger metallization thicknesses in order to design unidirectional transducers or resonators. Further more, it was found that leaky waves or surface transverse waves, widely used in RF filters, are much more difficult to describe with simple approximation than Rayleigh wave, due to the very close slowness of other modes like SSBW. These waves are very sensitive to the surface conditions and in some cases, it may even not propagate without the presence of metal. It is now well known, to give an example, that it exists an optimum metal thickness-crystal cut operating point in term of propagation losses for leaky SAWs on lithium tantalate. It is also very important when designing filters for mass production to be able to predict the effect of the manufacturing dispersion on the filter transfer function. Further more, it is interesting to take into account the electrode shape in the simulation and to compare, for example, the solutions given by electrodes obtained either by dry or wet etch.

The status today is that both numerically intensive [7-23;31-33] and physical approximated models are used [24-26;28]. We refer as “finite transducer model” very general model based on a low level of physical approximations where the simulation of a SAW device is obtained from the knowledge of its geometrical characteristics [9;13;17;18] Because of the computational requirement of this kind of model, we are not aware of any optimization procedure using it as analysis tool. A more standard approach is to use scalar models like COM, P-matrix or equivalent circuit as a basis for the design while the general models are used to validate some design choices or to check the results at the final stage. The difficulty is then transferred into feeding scalar models with accurate propagation parameters (reflection coefficient, velocity, transduction strength...). Two approaches are used here. The first is to manufacture test devices, often simply resonators, and to extract the propagation parameters from measurements. The second consists in extracting the parameters from a numerical model assuming an infinite periodic grating of electrodes (referred here as “periodic model”) [11-14;19]. Both methods have advantages and drawbacks. The first method has the advantage of using parameters coming from real life devices manufactured with real life technology. It includes naturally all effects related to the electrode shapes for example. No absolute knowledge of the geometry and deposited materials is required but only technological good reproducibility is requested. The drawback is that specific test devices are needed each time a design uses a new operating point (in term of metal thickness, mark to pitch...).
ratio or even frequency range). In addition, technology modifications can imply large variations on devices performances difficult to control. Even if parameters are coming from virtual devices, we prefer, the second approach. It allows a very large flexibility since it is possible for example to design successfully a filter, on a new material or cut, without any prior manufacturing of test device. The drawback is that an absolute geometrical and material characterization is required, but we consider that it is already needed for a good control of the process. Using unstructured mesh of 5 wavelengths depth, it gives approximately 1000 nodes per wavelength along the propagation direction. To give an example a 200 electrodes transducers (ie 100 wavelengths long) will result in a minimum of 1000x100 nodes in the substrate If 5 nodes are used in the thickness of each electrode, it will results in 100 000x4+200x40x5x3 = 520 000 dof for a general problem solving every polarizations (u₁, u₂, u₃).

**Figure 1**: general organization of SAW models

### 2 FEM/BEM METHODS

#### 2.1 IS A STANDARD FINITE ELEMENT MODEL APPLICABLE?

The problem of modeling a SAW device is mathematically equivalent to the resolution of the differential equations of the piezoelectricity with a given excitation. Various numerical methods are possible to solve these kinds of equations. For example, finite element methods (FEM) are a very popular method and commercial software’s including elasticity and piezoelectricity even exist. It may seem more efficient to use these software instead of spending huge resources to develop specialized models. We believe that standard FEM is not the optimal choice because of the SAW devices particularities. The first is the large number of necessary degrees of freedom (dof). All the electrodes have to be meshed as well as the substrate. Numerical tests have shown that spatial discretisation of the interface using P1 finite elements requires at least 80 nodes per wavelength to approach convergence. Using unstructured mesh of 5 wavelengths depth, it gives approximately 1000 nodes per wavelength along the propagation direction. To give an example a 200 electrodes transducers (ie 100 wavelengths long) will result in a minimum of 1000x100 nodes in the substrate If 5 nodes are used in the thickness of each electrode, it will results in 100 000x4+200x40x5x3 = 520 000 dof for a general problem solving every polarizations (u₁, u₂, u₃). This is a very large number, even if only band matrices are involved. In addition, boundary conditions at the limit of the meshed substrates will be very critical and typically, numerical plate modes as well as resonance in the length of the substrate will appear.

#### 2.2 BOUNDARY ELEMENT METHOD, GREEN’S FUNCTIONS

**Figure 2**: typical geometry for a periodic grating on stratified substrate

**2.2.1 Definition of Green’s functions**

The boundary element method [1] is a typical method when infinite media have to be analyzed. For example, it exists now numerous commercial electromagnetic simulators based on this approach. The principle is to replace the differential equations in the volume by an integral equation at the boundary. The principle is to replace the differential equations in the volume by an integral equation at the boundary. The principle is to replace the differential equations in the volume by an integral equation at the boundary. Main advantages are a drastic reduction of the number of dof and the possibility to consider infinite homogeneous media, suppressing the apparition of numerical spurious modes. The principal feature in BEM is to represent the substrate behavior by mean of Green’s functions[2-6] The piezoelastic field is completely defined by the three components of the displacement uᵢ and the electrical potential φ. Boundary conditions at the surface will involve these four
components and the normal components of the stress and of the electrical displacement \(D\). From the piezoelectric equations, it is possible to express a linear relation between these components in the slowness’ or wave numbers space, i.e. for a harmonic dependence of every components in

\[
\begin{bmatrix}
u(s_1) \\
\phi(s_1)
\end{bmatrix} = \tilde{G}(s_1) \begin{bmatrix}
u_1(s_1) \\
\phi_1(s_1)
\end{bmatrix}
\] (1)

In equation (1) all values are tilded meaning they are expressed in the slowness domain. The index \(i\) varies from 1 to 3 so that the Green’s function matrix \(\tilde{G}(s_1)\) is a 4X4 matrix. The system of coordinates chosen is given if figure 2. The equation (1) can be Fourier transformed to the x domain and the multiplication becomes a convolution :

\[
\begin{bmatrix}u_i(x_1) \\
\phi(x_1)
\end{bmatrix} = \int_{-\infty}^{\infty} \tilde{u}_i(s_1) \exp(2 \pi j f x_1) ds_1
\] (2)

The displacement, stresses, electrical displacement and potential are transform to the x domain by the usual inverse Fourier transform (for example for \(u_1\)) :

\[
u_1(x_1) = \int_{-\infty}^{\infty} \tilde{u}_1(s_1) \exp(2 \pi j f x_1) ds_1
\]

The equation (2) is an integral form of the system of differential equations of the piezoelectricity.

An important point to note here is that there is a possible dual form for the integral equation (2). Obviously, the equation (1) can be also used in the inverse form :

\[
\begin{bmatrix}\nu_2(s_1) \\
\phi_2(s_1)
\end{bmatrix} = \tilde{G}^{-1}(s_1) \begin{bmatrix}\nu_1(s_1) \\
\phi_1(s_1)
\end{bmatrix}
\] (3)

Which gives in the \(x_1\) space the equation :

\[
\begin{bmatrix}\nu_2(x_1) \\
\phi_2(x_1)
\end{bmatrix} = \int_{-\infty}^{\infty} \tilde{G}(x_1-x') \begin{bmatrix}\nu_1(x') \\
\phi(x')
\end{bmatrix} dx'
\] (4)

In equation (4), \(\tilde{G}(s_1)\) is the inverse Fourier transform of \(\tilde{G}^{-1}(s_1)\). Since boundary conditions outside the electrodes are usually free surface conditions (ie null normal stresses and charge), the integral form (2) allows to restrict the problem to the boundary located under the electrodes.

The different steps in any BEM are the following :

- choice of a basis for the description of the unknowns. This choice is a critical point.
- transformation of the integral equation into a system of equations

2.2.2 Green’s functions for an semi-infinite homogeneous material

![Figure 3](image-url)

**Figure 3:** variation of the Green’s function \(G_{44}\) for \(Y+36^\circ\) lithium tantalate

![Figure 4](image-url)

**Figure 4:** variation of the Green’s function \(G_{44}\) for a layer of 10 wavelength of \(Y+36^\circ\) lithium tantalate on fused silica

This Green’s function matrix is computed from the material constants. Assuming plane wave propagation in the substrate, the distribution of the electromechanical fields is obtained as a decomposition over a set of 8 eigenmodes. If propagation occurs along \(x_1\) axis, those eigenmodes are the eight possible bulk modes in the
substrate having the same wave number component along x₁. The second step consists in selecting among these four modes the four physically acceptable modes (i.e. with no energy at infinite depth or Poynting vector going into the substrate). The knowledge of these four modes, by linear combination, gives the relation (1) to find the displacements and potential from the normal stresses and the normal electrical displacement. The procedure to find this Green’s function matrix for an homogeneous substrate is published by several authors [2-6] and is now well known. An example of Green’s function is given in figure 3. As seen on this figure, it is possible to isolate in this function the most important contribution for a SAW device.

A true SAW, i.e. a surface acoustic wave, propagating without propagation loss with the slowness s_p gives in the slowness domain a pole of the form :

\[ G_{SAW}(s_1) = \frac{2G_p s_1}{s_1^2 - s_p^2} \] (5)

For a pseudo SAW, propagating with propagation loss, the slowness s_p is no more real and have an imaginary part. The other contributions are SSBWs which are discontinuities in \( \sqrt{s} \) and for the electrical term \( G_{el} \) an electrostatic of the form :

\[ G_{el}(s_1) = \frac{G_{el}}{s_1} \] (6)

2.2.3 Green’s functions for a stratified material

It is also possible to determine the Green’s function matrix for a stratified media with an arbitrary numbers of homogeneous layers. As for as semi-infinite substrate, the electromechanical problem is solved as an eigenvalue problem for each layer. In that way, all mechanical and electrical fields are a linear combination of the 8 eigen modes. By application of the boundary condition at each interface of the stratified media, it is then possible to find linear relations between the fields from an interface to another: this is the so-called transfer matrix, initially developed by Fahmy and Adler [2]. With such relations, the Green’s function matrix can be finally computed at the substrate surface. In the simple case when the metal thickness is neglected, all normal stresses at the surface are null and the potential is constant under the electrodes. Then, the equation becomes a scalar integral equation relating the electrical potential \( \phi(x) \) to the electrical displacement \( D_2(x) \) under the electrodes. In the general cases when finite thickness electrodes are accounted, the mechanical continuity equations at the interface between the substrate and the electrodes must apply. In our formulation, we consider only the mechanical contribution of the electrode, meaning that we neglect charges located at electrodes vertical sides.

In the following, we define the generalized displacement vector \( u_g \) as a 4 components vector formed by the three components of the mechanical displacement and the electrical potential \( u_1 \) and the generalized stresses vector \( t \) as the four components vector formed by the three normal stresses \( t_{ij} \) and the electrical displacement \( D_2 \). The integral equation (2) can be rewritten:

\[ u_{g1}(x) = \sum_{k=1}^{N_e} \sum_{j=1}^{4} G_{kj}(x-x') y_j(x') dx' \] (7)

where \( N_e \) is the number of electrodes and \( \Gamma_k \) the interface of the \( k^{th} \) electrode. To solve this integral equation, it is necessary to chose a basis to project the generalized displacements and stresses in order to obtain a finite dimension system of equations. The simplest way is to use the so called collocation method. The problem is that the charges present singularities at the edge of the electrodes which would result in a large number of samples and so in a very large system. A good choice of the basis is very important to reduce as much as possible the dimension of the system. Several choices are possible [4;7;15]. Due to the form of the singularity in \( 1/\sqrt{1-x^2} \), an expansion involving Chebyshev polynomials is well suited [13] for a proper description of the charges with a limited number of unknowns. Since there is stress accumulation at the edge of the electrodes, we choose the same basis for the normal
stresses. Under the electrode k, the generalized stress vector is developed in the form:

\[ t_{j,k}(x) = \sum_{n=0}^{N_{ch}} t_{j,k,n} T_n(x_k) \frac{2(x - X_k)}{a_k} \] (8)

for each electrode k (0 ≤ k ≤ N_e) where \( T_n \) is the nth Chebyshev polynomial of the first kind, \( X_k \) is the position of the electrode center and \( a_k \) the electrode width.

Similarly, the generalized displacement vector under the ith electrode is projected on the Chebyshev polynomials basis by:

\[ u_{g,1,i}(x) = \sum_{n=0}^{N_{ch}} u_{g,1,i,n} T_m(x_l) \frac{2(x - X_l)}{a_l} \] (9)

Thanks to the orthogonality properties of the Chebyshev polynomials, the components \( u_{g,1,i,n} \) can be obtained by:

\[ u_{g,1,i,n} = \frac{1}{\pi} \int_{-1}^{1} u_{g,1,i}(x_l) T_m(x_l) \frac{dx_l}{\sqrt{1 - x_l^2}} \] (10)

The integral equation is then transformed into:

\[ u_{g,1,i}(x) = \sum_{k=1}^{N_e} \sum_{j=1}^{N_e} a_k j_{j,k,n} \frac{1}{2} \int_{-1}^{1} G_{ij}(x - X_k - a_k x_k) \frac{dx_k}{\sqrt{1 - x_k^2}} \] (11)

And inserting (10) into (11), one gets:

\[ u_{g,1,i,m} = \frac{1}{2\pi} \sum_{k=1}^{N_e} \sum_{l=1}^{N_e} j_{j,k,n} \frac{1}{2} \int_{-1}^{1} G_{ij}(a_l x_l - a_k x_k + \Delta_{l,k}) \frac{dx_l}{\sqrt{1 - x_l^2}} \] (12)

The relation (12) is a linear relation relating the \( N_{tot} \) coefficients of the generalized stresses \( t_{j,k,n} \) to the \( N_{tot} \) coefficient of the generalized displacement \( u_{g,1,i,n} \). The last step is to complete this set of equations by boundary conditions, projected to the Chebyshev polynomial basis.

In the simple case when the electrodes are considered infinitely thin, the normal stresses at the surface are null. Equation (12) is reduced to a relation between the coefficients of the electrical potential \( u_{g,1,i,m} \) and those of the normal electrical displacement \( t_{j,k,n} \) with the Green’s function \( G_{e} \). The other boundary condition is the imposed constant potential \( V_i \) under the electrode l. This implies that all terms of the expansion of the electrical potential on the Chebyshev basis are null except the first one which equals \( V_i \):

\[ u_{g,1,i,m} = V_i \delta_{m} \delta_{0} = 0 \] (13)

The system of linear equations (12) is numerically solved. The current flowing in each electrode is obtained by integrating the charge density \( \sigma_i(x) \) in each electrode. This is simply given by the 0th order Chebyshev coefficient:

\[ I_k = j a \int_{-1}^{1} u_{g,1,i}(x_l) \frac{dx_l}{\sqrt{1 - x_l^2}} \] (14)

In practice, to evaluate the integrals involved in (12), the Green’s function is transformed from the slowness domain to the x domain by inverse Fourier transform. Due to the rapid variations of \( G \), it is necessary, for a good precision, to separate it into a sum of the various contributions (SAW, pseudo SAW, dielectric and other discontinuous functions). The inverse Fourier transform is done analytically for these contributions and by FFT for the residue.

For homogeneous substrates it is possible to reduce the required CPU time by using the property of the Green’s functions to be only slowness dependent. The knowledge of the functions at one frequency is sufficient to know it for any frequency. Then, for a given substrate orientation, it is necessary to compute the Green’s function only one time. If this result is stored with a dense sampling, it can be used for all problems on the same substrate simply by an interpolation. This precomputation of the Green’s functions is not possible for stratified media since the Green’s functions depend also of the layers thickness.

### 2.3.2 Electrode mechanical effect

When taking into account the mechanical loading of the electrodes, the mechanically free surface boundary condition is no more valid under the electrode and the BEM formalism has to be extended to the entire 4x4 Green’s function. Using the Chebyshev polynomial basis functions (eq. (8)) for the components of the normal stress in conjunction with spatial Green functions, one can rewrite the BEM part of the system (12) to be solved under the schematic form:

\[
\begin{bmatrix}
Z_m \\
Z_{me} \\
\end{bmatrix} \begin{bmatrix}
D_2 \\
D_1 \\
\end{bmatrix} = \begin{bmatrix}
\phi_l \\
\phi_i \\
\end{bmatrix}
\] on \( \Gamma \) (15)
where \( \{t_2\}, \{D_2\}, \{u\}, \{\phi\} \) stand respectively for the coefficients of the Chebyshev discretization of the normal stress components \((t_{j,k,n}: i=1\ldots 3; k = 1\ldots N_e, n = 0\ldots N_{k,A}^{ch})\)

electrical displacement \((u_{4,k,n}: i=1\ldots N_e, n = 0\ldots N_{k,A}^{ch})\)

mechanical displacements \((u_{6,1,m}: i=1\ldots 3; l = 1\ldots N_e, m = 0\ldots N_{i,l}^{ch})\)

and potentials \((\varphi_{6,1,m}: l = 1\ldots N_e, m = 0\ldots N_{i,l}^{ch})\). \( \Gamma \) stands for the propagating surface restricted to the domain located under the electrodes. On \( \Gamma \), the boundary conditions of stress and displacement continuity and of imposed potential apply:

\[
\begin{align*}
\{t_2\}_{\Gamma} &= \{t_{2,FEM}\}; \\
\{u\}_{\Gamma} &= \{u_{FEM}\}; \\
\{\phi\}_{\Gamma} &= \{\phi_b\}
\end{align*}
\]

where \( \{t_{2,FEM}\} \) and \( \{u_{FEM}\} \) represent respectively the normal stress and displacement coming from the FEM part and projected on the Chebyshev polynomial basis and \( \{\phi_b\} \) the components of the potentials imposed by the voltages boundary conditions on the electrodes. The aim of the method is now to eliminate the displacement unknowns at the interface. Owing to the fact that most SAW devices use limited number of electrode types, i.e., electrodes with the same geometrical aspect, there is only a limited number of FEM problem to solve. One way to suppress the displacement unknowns at the interface is to solve, for each electrode type, the following system derived from classical FEM analysis using variational formulation:

\[
\left(K - \omega^2 M\right)v_{i,k} = 0 \quad \text{in } \Omega_{FEM}
\]

\[
T_{FEM}^{\Gamma} = F_i^k \quad \text{on } \Gamma
\]

\[
0 \quad \text{elsewhere}
\]

where \( K \) and \( M \) stand for stiffness and mass matrices, \( x \) the nodal displacement unknowns and where \( F_i^k \) represents the projection onto the FEM basis of the \( k^{th} \) Chebyshev polynomial distribution of the \( i^{th} \) stress component. The system (17) is solved for \( N_{mec}^{ch} \) right hand sides, where \( N_{mec}^{ch} \) is the total number of Chebyshev components for the mechanical stresses. Using the computed solutions of the nodal displacements \( x_{i,k} \) and the right hand sides \( F_i^k \), one can construct the so-called mechanical impedance matrix linking the displacement and normal stress due to the FEM contribution expressed in the Chebyshev basis through:

\[
\{u\}_{FEM} = Z_{FEM}\{t_2\}_{FEM}
\]

Using boundary conditions (16) and relation (18), the system (15) is now ready to be solved and reads:

\[
\begin{pmatrix}
Z_m - Z_{FEM}^{ad} & Z_{me} \\
Z_{em} & Z_e
\end{pmatrix}\begin{pmatrix}
\{t_2\}_2 \\
\{D_2\}_2
\end{pmatrix} = \begin{pmatrix}
0 \\
\{\phi_b\}_1
\end{pmatrix} \tag{19}
\]

The system (19) is then solved for as many electrical ports used in the SAW device under simulation. Moreover, ohmic losses can be easily implemented by adding a diagonal matrix whose non-zero terms represent the serial resistance of each electrode.

On a numerical side, the impedance matrix of system (19) is complex and symmetric. We use either a direct Cholesky factorization method for the small cases arising with transverse wave on short transducers or a parallel solver using distributed data algorithm with block Cholesky factorization for large cases such as ones arising from SPUDT using Rayleigh wave. The overall goal while constructing this model was accuracy and low CPU time. This is achieved by taking advantages of:
- only one spatial Green function computation for any cut and propagation angle, computation independent of the SAW device simulation,
- using stress and electrical displacement as variables to reduce the problem under the electrodes,
- using well suited polynomial to reduce the number of degree of freedom,
- solving a very few number of small FEM problems by taking advantage of the use of limited distinct electrodes type in most SAW devices.

With this approach, a transducer made of 200 electrodes using the PSAW on LiTaO3 42 is accurately simulated in 15 s of elapsed time per frequency point on a DELL 800 MHz bi-processor PC.

2.4 INFINITE PERIODIC GRATING

2.4.1 FEM/BEM computation of the harmonic admittance

A way to reduce the computation time is to consider an infinite periodic grating instead of a finite transducer. In this case, it is possible to analyze only one period. Due to the linearity, it is sufficient to compute the admittances \( Y_n \) giving the current in the \( n \)th electrode when the voltage in the electrode 0 is 1 and all the other electrodes are short circuited. This problem can be equivalently solved by using Fourier analysis \([7; 8; 13; 15; 19]\). This means that an harmonic excitation is assumed ie the voltages \( V_n \) in the \( n \)th electrode are in the form \( V_n = V_0 \exp(-j2\pi\gamma) \). Since the problem is invariant by translation, the current \( I_n \) in the form \( I_n = I_0(\gamma) \exp(-j2\pi\gamma) \) and all quantities are \( \gamma \) periodic and so that the following equations are obtained:

\[
u_i(x_1 + np) = u_i(x_1)\exp(-j2\pi\gamma n)
\]
\[ \phi(x_1 + np) = \phi(x_1) \exp(-j2\pi n) \]
\[ I_2(x_1 + np) = I_2(x_1) \exp(-j2\pi n) \]
\[ D_2(x_1 + np) = D_2(x_1) \exp(-j2\pi n) \]
(20)

The integral equation (2) becomes:

\[ \left[ u_i(x_1) \right] = \frac{a_i}{\gamma} \int_{-\gamma/2}^{\gamma/2} \left[ \frac{2}{D_2(x_1 - x')} \left[ \frac{I_2(x_1 - x')}{f(x') \Gamma(x')} \right] \right] dx' \]

(21)

\[ \left[ u_j(x_1) \right] = \frac{a_j}{\gamma} \int_{-\gamma/2}^{\gamma/2} \left[ G_p(x') \left[ \frac{f(x')}{f(x)} \right] \right] dx' \]

(22)

where \( G_p(x, \gamma) = \sum_{n=-\infty}^{\infty} G(x + n.p) \exp(j2\pi n \gamma) \)

Thus, the resolution of the periodic problem is equivalent to the resolution of the finite transducer problem, except that the Green’s function have to be changed to \( G_0(x, \gamma) \). In addition, it should be necessary to write the \( \gamma \) periodicity condition at the limit of the period, ie to write that the dof at both ends of the period differ only by a phase term. Since this formulation consider only the fields under the electrode, no dof has to be found at the edge of the period and this condition does not apply here.

To express the system of equations it is convenient to express the Fourier transform of \( G_0(x, \gamma) \):

\[ \hat{G}_p(k, f) = \sum_{n=-\infty}^{\infty} G \left( \frac{\gamma + n.p}{fp} \right) \left( k - 2\pi \frac{\gamma + n.p}{p} \right) \]

(23)

Then, the linear system of equations becomes, assuming for simplicity infinitely thin electrodes:

\[ \sum_{k=0}^{N_k} \sum_{n=0}^{N_n} \int_{-\gamma/2}^{\gamma/2} \left[ G \left( \frac{\gamma + n.p}{fp} \right) \right] \left( \pi \frac{\gamma + n.p}{p} \right) \delta_m = 4j^m \gamma \]

(24)

To account for the finite thickness of the electrodes, the same procedure as for the finite element matrix is used (see §2.4.2).

The result of the periodic FEM/BEM is the harmonic admittance \( Y(\gamma) = \frac{I_0(\gamma)}{V_0} \) which is by definition the Fourier transform of mutual admittances \( Y_{\omega} \). It has to be noticed that the harmonic admittance is equal to the admittance of an infinite transducer with alternate polarities (+V, -V) when \( \gamma \) is chosen equal to \( \frac{\pi}{2} \).

The formalism described above can be applied as well for homogeneous substrates as for stratified substrates without any modification except the evaluation of the Green’s functions.

2.4.2 Obliquely propagating waves

Using the periodic FEM/BEM, it is also possible to analyze waves propagating obliquely [21-23]. The principle is to assume that all fields have also an harmonic dependence \( \exp(-jk_3x_3) \) in the \( x_3 \) direction. Abruptly, the voltage transverse dependence in \( V(k_3) \exp(-jk_3x_3) \) seems unrealistic. Nevertheless, one may consider that a finite aperture transducer can be modeled using the angular spectrum of waves, in which case the electrical excitation is expanded in Fourier components along the transverse direction. Obviously, only the superposition of these elementary conditions can be considered to have a physical existence.

The difference from classic \( x_3 \) propagation, is that for a given wave number \( k_3 \), the Green’s function have to be evaluated for two slowness’ \( s_1 \) and \( s_3 = \frac{k_3}{2\pi} \). It means that waves propagating obliquely are considered in the Green’s functions evaluation. In addition, the finite element matrices have to be modified slightly in order to consider that the displacements and stresses have an harmonic dependence in \( x_3 \).

2.5 Periodic model for waves propagating partially into an inhomogeneous layer

A limitation of the above FEM/BEM models is that they are applicable only for the case of electrodes on plane substrates. If this was not such a limitation when they were developed, the apparition of more complex structures was the motivation to examine new more general formulations. This following model allows simulating an inhomogeneous domain comprised between two homogeneous domains [33]. It can be used to simulate a variety of devices, such as
Interface Acoustic Wave [29], influence of passivated layer over SAW [30], impact of etched grooves or medical imaging transducer radiating in layered media [31]. The general addressed problem is described in figure 5. This software has been initially developed for ultrasonic medical imaging applications [32], it uses a free FEM package MODULEF [34] which has been extended to address piezoelectricity. Further developments have led to the implementation of radiation conditions into fluid, dielectric, solid or piezoelectric material through the a general Green functions package [6] and a proper interface.

The main differences between this model and the classical one described previously are:
- the unknowns are displacement and potential instead of normal stress components and charge.
- The problem is mainly driven by the FEM part instead of the BEM part. The boundary integral method is included in the FEM problem through a boundary condition.
- The model can be used with an arbitrary number of electrodes and with true periodic boundary conditions, ie excitation parameter \( \gamma \) being an integer without specific development.
- The approximation of electrically infinitely thin electrode with respect to dielectric radiation in air can be removed by filling the space between electrode of dielectric finite elements and applying a radiation condition above.

The basic equations governing these periodic FEM computations are now briefly recalled. It consists first in writing the harmonic boundary condition relating all the degree of freedom (dof) on boundary \( \Gamma_A \) to those on boundary \( \Gamma_B \), (defined in figure 5), yielding the linear relation:

\[
\begin{bmatrix}
\mu \\
\phi \\
\end{bmatrix}_A = \begin{bmatrix}
\mu \\
\phi \\
\end{bmatrix}_B e^{-j2\pi \gamma}
\]

This relation is then used to reduce the number of independent dof of the FEM model. This is performed without changing the total number of dof of the problem, simply by using a variable change operator \( C \). This provides the following form of the FEM algebraic system to be solved:

\[
C_u^T \begin{bmatrix}
K - \omega^2 M - X(\omega, \gamma) \end{bmatrix} C_u \begin{bmatrix}
\mu \\
\phi \\
\end{bmatrix} = 0 \quad \text{in } \Omega
\]

\[
\phi = \phi_b \quad \text{on } \Gamma_r
\]

with the same notations as previous. Since \( K \) can be complex to take into account lossy material, the left hand side matrix in (26) is general (hermitic if \( K \in \mathbb{R} \), but sparse. These properties are considered when solving the problem.

Let us now consider the case of acoustic radiation on one boundary of the meshed domain referred as \( \Gamma \). In that purpose, the general variational equation is considered, limited to the purely elastic problem without any loss of generality:

\[
\int_{\Gamma} \left[ \left( -\frac{\partial \hat{\delta}_{\Gamma}^*}{\partial \xi_j} - C_{ijkl} \frac{\partial u_i}{\partial \xi_k} + \rho \omega^2 u_i \right) \right] dS - \int_{\Omega} \delta_{\Gamma}^* T_j n_j dS = 0
\]

in which \( \hat{\delta}_{\Gamma} \) is the variational unknown and \( n_j \) the normal to boundary \( \Gamma \) on which the radiation boundary condition is applied. Equation (27) is written in 3D but of course its restriction to 2D problems does not induce any fundamental difficulty. To define the right hand side of (27), one can relate the stress \( T_j \) to the displacement \( u_i \) in the spectral domain through equation (3). Using the now well-established periodic Green’s function formalism, the right hand side of eq.(27) is expressed as :

\[
\int_{\Gamma} \hat{\delta}_{\Gamma}^* T_j n_j dS = \int_{\Gamma} \hat{\delta}_{\Gamma}^* [x] \sum_{l=-\infty}^{+\infty} G_{jl}^{\text{eq}}(\gamma + 1, \omega) n_j e^{-\frac{j2\pi}{p}(\gamma + 1)(x-x')} u_k(x') dx' dx
\]

The classical FEM interpolation procedure is then applied to eq. (28), yielding a frequency and excitation parameter dependent matrix \( X(\alpha, \gamma) \) related to both dof and variational unknowns. More detailed description of this matrix construction can be found in [33]. This matrix \( X(\alpha, \gamma) \) is dense in contrast to the classical band type FEM matrices. It is moved to the right hand side of equation (26) which now reads, along with boundary conditions of imposed potential:

\[
C_u^T \begin{bmatrix}
K - \omega^2 M - X(\omega, \gamma) \end{bmatrix} C_u \begin{bmatrix}
\mu \\
\phi \\
\end{bmatrix} = 0 \quad \text{in } \Omega
\]

\[
\phi = \phi_b \quad \text{on } \Gamma_r
\]

where the boundary \( \Gamma_r \) is defined by the electrode periphery. The resolution of linear system (29) is performed with the aid of the UMFPACK package [35]. Once displacements and potential are known, electrical displacement and stress are computed with a simple matrix multiplication and the integral of charge density on \( \Gamma_r \) is computed leading to the harmonic admittance or harmonic admittance matrix in the case of multi-electrodes cell.
The FEM approach of this model induces at the same
time an advantage and a drawback.. The main advantage is
its versatility and the drawback is the larger CPU time and
memory space requirement in comparison to classical
periodic FEM/BEM. This is induced by the choice of
Lagrange polynomial (P1 or P2) for the discretization
which are less suited to represent the charge distribution
than Chebishev polynomial. Therefore the convergence of
the discrete problem towards the continuous solution
require a dense mesh which in turn imply the use of a large
number of harmonics of the spectral Green function. The
overall increase of discretization (mesh and number of
harmonics) leads to a larger requirement in term of CPU
time and memory space. Nevertheless, this software allows
to simulate accurately a passivated mono-electrode cell in
less than 10 CPU’s per frequency point on a standard 800
MHz PC.

3 APPLICATION OF FEM/BEM METHODS
TO PRACTICAL SAW DEVICES
3.1 P MATRIX MODEL AND PARAMETER
EXTRACTION
As shown in figure 1, an approach to the analysis of
SAW devices is to extract the P matrix parameters from the
results of the infinitely periodic transducer model.
Obviously, a similar approach can be used to determine the
COM parameters. The main approximation in the P matrix
model is to consider that it is sufficient, in order to describe
a device, to consider two waves propagating in opposite
directions. With this assumption, one period can be
modeled by a matrix giving the current and the output wave
amplitudes from the voltage on the electrode and the input
wave amplitudes. An other assumption is that it is possible
to cascade the period, i.e that their is no wave polarization
change from one transducer section to the other and that it
is possible to assume that the output wave amplitude for
one section is the input wave amplitude for the neighboring
section. In practice, this assumption which seems very
strong is found correct for a wide range of devices
explaining the success of these approximate scalar
methods. If $s_g$ and $s_d$ are respectively the outcoming
waves at the left and right side, $e_g$ and $e_d$ the incoming
waves at the left and right side, the P matrix for one period

\[
\begin{bmatrix}
Y & -\alpha_g & -\alpha_d \\
\alpha_g & r_g & t \\
\alpha_d & t & r_d \\
\end{bmatrix} = \begin{bmatrix}
1 \\
\end{bmatrix}
\]

(30)

where $Y$ is the admittance for one period, $\alpha_g$ and $\alpha_d$ the
transduction strength for the left and right side, $t$ the
transmission coefficient for one period and $r_g$ and $r_d$ the
reflection coefficients at the left and right sides. The different elements of the P matrix verifies the following
relations:

\begin{align*}
Y &= G + jB \\
r_g &= -j \sin \Delta \exp(-j(\varphi + \psi_r)) \\
r_d &= -j \sin \Delta \exp(-j(\varphi - \psi_r)) \\
t &= \cos \Delta \exp(-j\varphi)
\end{align*}

(31)

\begin{align*}
\alpha_g &= j\sqrt{G} \exp \left[-j \frac{\varphi + \psi_r}{2}\right] \begin{bmatrix}
\cos \delta e^{j\frac{\Delta}{2}} + j \sin \delta e^{j\frac{\Delta}{2}} \\
\end{bmatrix} \\
\alpha_d &= j\sqrt{G} \exp \left[-j \frac{\varphi - \psi_r}{2}\right] \begin{bmatrix}
\cos \delta e^{-j\frac{\Delta}{2}} - j \sin \delta e^{-j\frac{\Delta}{2}} \\
\end{bmatrix}
\end{align*}

(32)

In the preceding equations, $\sin \Delta$ is the amplitude of the
reflection coefficient for one period and $\varphi$ is the phase of
propagation for one period. $\psi_r$ represents the phase
between the reflection center and the center of the period
while $\delta$ is at first order [11;13] the phase difference
between the transduction center and the reflection center.
$\delta$ is null for non directive cuts while it is equal to $\pi/4$ for
NSPUDT cuts.

From the P matrix, it is possible to find an expression for
the harmonic admittance $YH(\gamma)$:

\begin{equation}
YH(f, \gamma) = j \left( \frac{G_S + G_A}{G_S - G_A} \sin \psi - \frac{G_S - G_A}{G_S + G_A} \sin \Delta \cos(2\pi \gamma) \right) + jB
\end{equation}

(33)

It is interesting to notice that the harmonic admittance
does not depend on the phase $\psi$, between the center of the
period and the reflection. The only parameter involved in the
directivity meaning it is only possible to determine the
relative phase between reflection and transduction from the
harmonic admittance and not the absolute phase of the
reflection (or the transduction).

For the usual case of non directive cuts, the harmonic
admittance becomes:

\begin{equation}
YH(f, \gamma) = j \frac{G[S \sin \psi - \sin \Delta \cos(2\pi \gamma)]}{\cos(\varphi) - \cos \Delta \cos(2\pi \gamma)} + jB
\end{equation}

(34)

Equation (33) shows that the harmonic admittance
presents a pole for $\gamma = \gamma_{cc}$ where

\[\cos(2\pi \gamma_{cc}) = \frac{\cos(\varphi)}{\cos(\Delta)}\]
The variation of the position of the pole $\gamma_{ce}(f)$ gives the dispersion curve of the grating. This pole becomes complex in the stop band.

It is impossible to determine the phase $\varphi$ and the reflection $\Delta$ from this dispersion curve without adding an extra hypothesis about their frequency variations. The simplest hypothesis, which is very efficient for Rayleigh waves is to assume that the phase $\varphi$ varies linearly with the frequency while the reflection $\Delta$ is independent of the frequency. Under these assumptions, it is sufficient to determine the lower and the upper edge of the stop band $f_1$ and $f_2$ to determine $\varphi$ and $\Delta$. Since the phase $\varphi$ is $\pi$ at the center of the stop band and the relative width of the stop band is proportional to $\Delta$, the following relation can be used:

$$\varphi = \pi \frac{f}{f_1 + f_2} \quad |\Delta| = \pi \frac{f_2 - f_1}{f_1 + f_2}$$

Similarly, under these assumptions it is possible to determine the conductance $G$ and the directivity $G_f$ from the pole amplitude $Y_p[13]$.

In the case of PSAW, this method becomes very difficult to use since the upper edge of the stop band may be difficult to detect due to vicinity of the SSBW. Furthermore, the velocity is not found constant with the frequency and the above assumptions are no more valid. An approach proposed by Koskela et al. to detect the upper edge of the stop band is based on the variation of the harmonic admittance when $\gamma$ is close but not equal to 1/2. In this case, a resonance appears at the upper edge of the stop band allowing to estimate the reflection coefficient.

Our approach is based on an assumption of the form of the frequency variation of $\Delta$. With this assumption, it is possible to find the best fit parameters defining the variation of $\Delta$. Then, from this knowledge it is possible to find at any frequency the remaining parameters. An important point to understand is that any choice for the determination of $\varphi$ and $\Delta$ is valid until only periodic transducers are analyzed. The difficulty appears when we use the $P$ matrix model in order to juxtapose several sections having different periods. In fact, the choice of the determination is equivalent to a basis choice for the $P$ matrix model, i.e., a choice for the definition of the amplitude waves. To validate our choice, the only possibility is to examine non-periodic structures, obviously not covered by the periodic model. For a base of comparison, it is very interesting to have the finite transducer model.

### 3.2. RESULTS

#### 3.2.1 Rayleigh waves on quartz

![Graph](image)

**Figure 6:** Short and open-circuit aluminum strip reflectivity for a $\lambda/2$ periodic grating on 38° Y rotated quartz: $h/\lambda = 0.72 \%$.

![Graph](image)

**Figure 7:** Short and open-circuit aluminum strip reflectivity for a $\lambda/2$ periodic grating on 38° Y rotated quartz: $h/\lambda = 2.5 \%$.
Figure 8: Plot of the relative velocity shift (compared to free surface) under a 4 fingers per λ versus mark to pitch ratio and electrode thickness. The two different curves sets correspond to rectangular electrode shape and wet etching with 10% overetch.

Figure 9: Plot of the reflection coefficient on one electrode versus mark to pitch ratio and electrode thickness. The two different curves sets correspond to rectangular electrode shape and wet etching with 10% overetch.

Figures 6 and 7 show comparisons of measured and simulated reflection coefficient on 38° Y rotated quartz for two metallization thicknesses. The agreement is very good. It was one of the first demonstration of the ability of FEM/BEM to be used in practical SAW designs [12]. An interesting result is the shift of the electrode width for maximum reflectivity from \( a/p = 0.5 \) for low metallization thickness to \( a/p = 0.8 \) for higher thickness. One practical consequence, when designing a SPUDT based filter on quartz is that DART cells using \( 3\lambda/8 \) reflector widths are much more efficient than EWC cells using \( \lambda/4 \) widths for metallization thickness above 1.5%.

Another interesting characteristic of FEM/BEM is its ability to take into account for the electrode shape. This was used to compute the effect of technology on SAW parameters and filter performance. In practice, when using lift off or dry etch process, rectangular electrodes are considered. This is no more true when using wet etch technology. It is generally considered that the etching is isotropic, resulting in circular electrode sidewall, the circle radius being the metallization thickness \( h \) multiplied by a factor proportional to the overetch. Figures 8 and 9 show for example a comparison between dry etch and wet etch for phase velocity and reflectivity. It can be seen that SAW devices are process sensitive and any change of process may require a redesign of the device.

3.2.2 \( P \) matrix parameters for Leaky SAW

Figure 10: frequency variations of the velocity for leaky SAW on Y+64° lithium niobate estimated from periodic FEM/BEM for \( h/(2p) = 1\% \)

Figure 11: frequency variations of the propagation losses \( y \) for leaky SAW on Y+64° lithium niobate estimated from periodic FEM/BEM for \( h/(2p) = 1\% \)
If for Rayleigh waves on quartz, it is sufficient to consider that the reflectivity and velocity do not depend on the frequency, as explained above, this no more true for more complex waves as leaky SAWs on lithium tantalate or lithium niobate. In this case, it is possible to determine from the harmonic admittance the frequency variation of the propagation parameters. Figures 10;11 show the extracted velocity and propagation losses frequency variations for PSMAW on Y+64° lithium niobate showing for example the large increase of propagation losses due to the bulk mode and the large impact of the mark to pitch ratio on the frequency variations of velocity.

![Figure 12a](image12a.png)

**Figure 12a**: comparison of results of the finite transducer and the P matrix for a DMS on Y+64° lithium niobate

For validation, fig.12 shows a comparison between the results of the finite transducer FEM/BEM and the P matrix model when using this approach. The correspondence is very good except in the ssbw frequency band, which validates this approach. Figure 13 show the same kind of comparison for a resonator on Y+42° lithium tantalate with a 10 periods length gap. In this case, the P matrix is much less accurate due to transition between the sections.

![Figure 13](image13.png)

**Figure 13**: comparison of results of P matrix model and finite transducer for a delay line on Y+42° LiTaO₃ substrate (free substrate of 10 periods)

### 3.2.3 Multielectrode periodic FEM/BEM

SPUDT based devices use cells comprising several electrodes having different widths. It is important for the design, to have a good prediction of the directivity and velocity and all P matrix parameters of these cells. For this purpose, an approach consists to generalize the periodic FEM/BEM for the case when the period comprise several electrodes. This analysis was reported by Hashimoto [4;15;16] using Aoki’s theory. A similar approach for infinitely thin electrodes was published by Lin et al.[36]. On our side, we developed the same kind of model, but using Chebychev polynomials. Compared to the monoelectrode periodic FEM/BEM the difference is that the harmonic admittance is now replace by an harmonic admittance matrix relating the harmonic currents in the electrodes to the harmonic voltage on the electrodes (which may be different each other). From this matrix, it is possible to extract the P matrix parameters of the cell and to use it for the device analysis. Figure 14 shows an example of analysis for a DART cell.

### 3.2.4 Oblique propagating waves periodic FEM/BEM

From the oblique waves propagating model, it is possible to determine the effect of the grating on the slowness curve by the extraction of the pole of the harmonic admittance for different propagation angles.
These results are very important when analyzing guided modes devices [28]. Figure 15 shows the results for Rayleigh waves on ST quartz and figures 16,17 show the slowness curve and the variation of propagation with the propagation angle for leaky SAW on Y+36° lithium tantalate. These figures show the strong influence of the surface condition on the shape of the slowness curves as well as the dramatic increase of propagation losses for oblique angles.

**Figure 14a**

Figure 14 : DART cell and extracted reflection coefficient and directivity versus d

All distances are in periods. \( P=17 \ \mu m, \) thickness=0.326 \( \mu m \)

**Figure 14b**

**Figure 15**. Velocity of the SAW of 42.75°YX quartz under an aluminum layer for some values of \( f h \), and in a periodic grating with \( f p=1000 \) m/s, \( a/p=0.5 \) and for some values of \( h/(2p) \); \( f \) is the frequency, \( p \) the period, \( a \) the width, and \( h \) the thickness.

**Figure 16**. Velocity of the leaky SAW of 36°YX lithium tantalate under an aluminum layer for some values of \( f h \), and in a periodic grating with \( f p=1000 \) m/s, \( a/p=0.5 \) and for some values of \( h/(2p) \).
3.2.5 Analysis of temperature effects in a grating

![Figure 18](image1.png)

**Figure 18**: Frequency vs. temperature dependence of a resonator on (YZ) lithium Niobate ($a/p=0.7$, $h/2p=8\%$) LLSAW.

3.2.6 Effect of passivating layer

![Figure 20](image2.png)

**Figure 20**: Mesh used for passivation.

It is also possible to include the effects of the temperature variation in the periodic FEM/BEM. The principle described in [27] consists in assuming that the thermal expansion in the $x_1$ direction is imposed by the substrate while the electrodes geometry variation with temperature in the $x_2$ direction are given by the thermal expansion coefficient of the aluminium. This approach allows to derive the temperature variations for waves guided by the metal grating like STWs or longitudinal leaky saw (fig. 18).

It allows also to predict the variations of the P-matrix parameters with the temperature. This can shown by fig. 19 where the temperature variations of resonance frequency and antiresonance frequency for a resonator on $Y+42^\circ$ lithium tantalate are plotted and compared to measurements.

![Figure 19](image3.png)

**Figure 19**: Resonators on LiTaO$_3$ ($YX$)/42 – Frequency-temperature laws for resonance and anti-resonance ($h/2p=8\%$, $a/p=0.6$).

It is also possible to include the effects of the temperature variation in the periodic FEM/BEM. The principle described in [27] consists in assuming that the thermal expansion in the $x_1$ direction is imposed by the substrate while the electrodes geometry variation with temperature in the $x_2$ direction are given by the thermal expansion coefficient of the aluminium. This approach allows to derive the temperature variations for waves guided by the metal grating like STWs or longitudinal leaky saw (fig. 18).

It allows also to predict the variations of the P-matrix parameters with the temperature. This can shown by fig. 19 where the temperature variations of resonance frequency and antiresonance frequency for a resonator on $Y+42^\circ$ lithium tantalate are plotted and compared to measurements.
It is very interesting to analyze the effect of a passivating layer on a device. This analysis is possible only by using the new formulation described in section 2.5.3. This formulation allows in particular to describe the passivating layer covering the complete period. The mesh is given in figure 20. Figure 21 shows the effect of passivation on the harmonic admittance for a device on lithium tantalate. The difficulty here is to have a good estimation of the elastic properties of the layer.

![Figure 21](image)

*Figure 21 – Harmonic admittance (S/m) on Tantalate*  
42: (1) New FEA/BIM without passivation;  
(2) Classical FEM/BEM;  
(3) New FEA/BIM with SiO$_2$ layer

### 3.2.7 Complex devices

Even if they are relatively difficult to manufacture, devices using stratified structure with at least one piezoelectric layer on top or interfacial waves may find more applications in future. In particular, these kinds of devices may be an alternative to FBARs when trying to integrate filters with ICs. The stratified structure devices need layered Green function models to be analyzed using one or the other FEM/BEM formulation. The interfacial waves devices present two radiating interface and cannot be analyzed with the “standard FEM/BEM”. Figure 22 shows as an example the harmonic admittance for a thick layer of lithium tantalate bonded on while figure 23-24 shows the influence of the interface realization on the predicted performances of interface wave devices.

![Figure 22](image)

*Figure 22 Harmonic admittance $Y(\frac{1}{2})$ for a LiTaO$_3$ (YX1)/36° layer (thickness=20 periods) bounded on glass. $h/(2p)=5\%$; $a/p=0.5$ where $h$ is the thickness, $p$ the period and $a$ the electrode width*

![Figure 23](image)

*figure 23: one period of a (YX) LiTaO$_3$/SiO$_2$/ (YX) LiTaO$_3$ structure with aluminium electrodes*

![Figure 24](image)

*figure 24: coupling and attenuation estimated for structure of fig 23 as a function of the cut angle for $h/(2p)=2\%$*
3.3 ANALYSIS OF A COMPLETE FILTER

To analyze a complete filter two different approaches are possible. The first is to use directly the finite transducer model. Then it is possible to obtain an admittance matrix directly from the electrodes geometry. To obtain the filter frequency response it is then necessary to account for the electrical circuit used to measure the filter and also for the stray elements which are very important for RF filters. The second possibility is to use the P matrix model, where the parameters are extracted from one or the other periodic model. Obviously this approach is less CPU consuming and is well suited to the design procedure. In addition, it is possible to include some 3D effect in the P matrix model while it is not today for the finite transducer model. For these reasons, the P matrix model gives better results suited for devices sensitive to diffraction effects like 2D-RSPUDTs. Even if the results are not sufficiently illustrative to be presented here, for this kind of devices, the finite transducer model is very interesting as a tool to test some refinements of the P matrix model. We made some analysis for very long devices (several thousands of electrodes). This is made possible by the implementation of our model on a network of PCs allowing memory and CPU sharing. Figure 25 shows an example of comparison between measurement and simulation for a 2D R SPUDT at 183.6 MHz. A very good accuracy is obtained.
For DMS filters using leaky SAW on lithium tantalate and lithium niobate, the finite transducer is used as a verification on a day basis for all our designs. This is much easier than quartz due to smaller number of electrodes and the transverse property of the wave, reducing the mechanical unknowns to $u_3$. The results, obviously combined with the EM simulations are very accurate. This is shown by figure 26, 27 and 28.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure28.png}
\caption{Simulated (solid) and optically measured (dotted) mean vertical displacement profiles for one track of a CDMA DMS filter (see [37] for more detail).}
\end{figure}

4 CONCLUSION

Numerical methods are now widely used for the simulation of SAW devices. If the FEM allows to account for real electrodes geometry, it results in numerical spurious and in very long CPU times. FEM/BEM models use finite elements to account for electrode shapes while boundary elements models allow to analyze semi infinite substrates allowing tremendous computation time reduction. By an adequate choice of the interpolation functions, the gain is further increased. Since first papers, the kind of devices accessible to these methods has been dramatically enlarged. Thanks to the increase power of computers and the effort on numerical implementation, hundreds electrodes finite transducers are analyzed on day to day basis. Devices comprising thousands of electrodes are now accessible using distributed networks of computers. Combined with EM models, very accurate predictions of devices performances are obtained without any previous test devices measurements.

To optimize the devices, scalar models like P matrix (or COM) are still necessary. If the parameters for these models are extracted from periodic FEM/BEM, the comparison with the finite transducer FEM/BEM allows to validate the chosen hypothesis, in particular for the form of frequency variations. For SPUDTs, the periodic model has been generalized for periods comprising several electrodes. Thermal sensitivity have been included in order to simulate the effect of the grating on temperature dependence. The possibility of stratified media was included in the Green’s functions. For more complex structures not compatible with the usual formulation, a generalized periodic approach was developed. It allows to model devices having a continuous non homogeneous interface like passivated interface waves devices.

Limitations are now often coming from the knowledge of the elastic properties of the deposited materials and from the impact of 3D effects like diffraction or mode guiding. If the oblique wave analysis allows to improve the 3D scalar models, next steps should be in the direction of including the 3rd dimension in FEM/BEM.

REFERENCES
