Design Considerations on Wideband Longitudinally-Coupled Double-Mode SAW Filters

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Abstract—This paper discusses, both qualitatively and quantitatively, the operation and the design principle of current longitudinally-coupled double-mode SAW (DMS) resonators where the internal reflection within interdigital transducers (IDTs) is not negligible and lower capacitance ratio is necessary. For the purpose, the $p$-matrix expression is used skillfully with a help of the coupling-of-modes theory. The internal reflection causes a) deformation of the IDT passband shape, and 2) frequency dependence of the effective SAW velocity within IDTs, and 3) suppression of higher-order resonances. It is shown that these features can be effectively used to enhance performances of DMS filters. In addition, under proper designs taking the internal reflection into account, most of all structural discontinuities can be removed, and is most preferable for the reduced scattering loss at the discontinuity. Design criteria are also presented for DMS filters of wide bandwidth, and it is demonstrated how device performances are improved by proper design taking account of the criteria.

I. INTRODUCTION

From the first proposal in 1970[1], SAW resonators have been extensively studied, and now are widely used as filtering and/or frequency control elements in various consumer and communication equipments[2], [3], [4], [5].

For long years, the Fabry-Perot model[6], [7], [8] has been widely used to explain basic behaviors of SAW resonators. Although it offers physical insights for the operation of the resonators and their fundamental design guidelines, actual operation of major SAW resonators is considerably different from that of this model. Namely, the internal reflection within interdigital transducers (IDTs) is not negligible, and it plays an important role for the operation because IDTs are placed within the resonance cavity. Especially, in current RF SAW filters, the internal reflection is significant, because the thickness of the grating electrodes becomes relatively large at high frequencies.

Let us consider longitudinally-coupled double-mode SAW (DMS) filters[9] which have widely been used in RF sections of recent handy phones. Since their passband shape is conformed by overlapped responses of multiple resonances, the location of resonance frequencies is very important for device design. Although the location is basically determined by the distance between the grating reflectors, it is also influenced by the IDT design. Clearly this fact makes the design of DMS filters complex, and behavior of their characteristics on design parameters does not seem well understood.

For taking the internal reflection into account, the numerical techniques have been applied extensively, and presently seem to be well-established. However, results given by the procedure were expected to be too complex to understand their operation.

This paper is aimed at revealing the operation and the design principle of present longitudinally-coupled DMS filters where the internal reflection within IDTs is not negligible. To discuss its influence both qualitatively and quantitatively, the $p$-matrix[10] expression is used skillfully with a help of the coupling-of-modes (COM)[11] theory.

First, one-port SAW resonators are discussed to explain the influence of the internal reflection. It is shown that due to the deformation of the IDT passband shape and the phase characteristic caused by the internal reflection, the resonator performance becomes inferior when the resonance frequency of the whole structure is designed to coincide with the synchronous (Bragg) frequency of the IDT. The minimum capacitance ratio $\gamma$ is available provided that the IDT electrode pitch is chosen so that the whole structure resonates at the Bragg frequency for the grating reflectors, and that the gaps between the IDT and grating reflectors are removed. This is most preferable for the reduced scattering loss at the discontinuity.

Secondly, two-port DMS filters are discussed. It is shown that although the internal reflection in the IDTs limits the achievable passband width and should usually be suppressed, it is effectively used to control the location of the resonance frequencies in the conventional design. This suggests that the proper handling of the internal reflection would be crucial in realizing high performance DMS filters.

Finally, design criteria are presented for DMS filters of wide bandwidth, and it is demonstrated how device performances are improved by proper design taking account of the criteria.

II. ANALYSIS

A. $p$-matrix

Consider the behavior of an IDT with the acoustic length of $L$ (see Fig. 1). Since the IDT is equivalent to the short-circuited (SC) grating when $V = 0$, the result of the following analysis on an IDT can be also applicable to short-circuited grating reflectors.

It is known that due to the reciprocity, the normalized
direction, the applied voltage \( V \) and the current \( I(L) \) are related to each other by the \( p \)-matrix[10]:

\[
\begin{bmatrix}
U_-(0) \\
U_+(L)
\end{bmatrix} = \begin{bmatrix}
p_{11} & p_{12} & p_{13} \\
p_{12} & p_{22} & p_{23} \\
-\chi p_{13} & -\chi p_{23} & p_{33}
\end{bmatrix}
\begin{bmatrix}
U_+(0) \\
U_-(L) \\
V
\end{bmatrix},
\]

where \( p_{11} \) and \( p_{22} \) are the SAW reflectivities at the acoustic ports “1” \((x = 0)\) and “2” \((x = L)\), respectively, for the SC grating, \( p_{12} \) is the transmission coefficient, and \( p_{33} \) is the IDT input admittance \( Y_1 = G_1 + j B_1 \) when the IDT is acoustically isolated. The factor \( \chi \) is four when \( V \) and \( I \) are defined in the peak amplitude while \( U_\pm(x) \) by the RMS value, whereas \( \chi = 2 \) when all of them are defined in the peak amplitude[13].

In the following analysis, the IDT is assumed to be bidirectional and symmetric for simplicity. Under the assumption, the relations \( p_{22} = p_{11} \) and \( p_{23} = p_{13} \) hold because the ports “1” and “2” are equivalent to each other.

In addition, the system is assumed to be unitary (lossless). Then the coefficients appearing in Eq. (1) should satisfy the following relations:

\[
|p_{11}|^2 + |p_{12}|^2 = 1,
\]

\[
G_1 = \chi |p_{13}|^2,
\]

and

\[
p_{12}^2 - p_{11}^2 = p_{12}/p_{12} - p_{11}/p_{11}.
\]

Here we define \( \phi_\pm = \phi(p_{12} \pm p_{11}) \). Since \( \phi_+ \) and \( \phi_- \) correspond to the transmission phases when the acoustic ports of the SC grating are acoustically driven in-phase and 180° out-of-phase, respectively[14], the relation \( |p_{12} \pm p_{11}| = 1 \) holds and thus \( \phi_\pm \) is always pure real independent of the driving frequency. Then we get the following relation from Eqs. (2) and (4):

\[
p_{13}^2 = -|p_{13}|^2 \exp(j \phi_+).
\]

**B. Coupling-of-mode equations**

The behavior of the \( p \)-matrix elements is discussed by the following COM equation for bidirectional IDTs[11]:

\[
\frac{\partial U_+(x)}{\partial x} = -j \theta_p U_+(x) - j \kappa_{12} U_-(x) + j \zeta V,
\]

and

\[
\frac{\partial I(x)}{\partial x} = -j \chi (U_+(x) - j \chi U_-(x)) + j \omega CV.
\]

Here, \( \theta_p = \beta_p - 2\pi/p_1 \) is the detuning factor corresponding to the mismatch of the wavenumber \( \beta_p \) of the unperturbed SAW mode under the SC grating from the Bragg (synchronous) condition \( \beta_p = 2\pi/p_1 \), where \( p_1 \) is the periodic length of an IDT. And \( C, \kappa_{12} \) and \( \zeta \) are the static capacitance per period, mutual coupling coefficient and transduction coefficient, respectively. It should be noted that \( \pi \zeta^2 p_1/4\omega C \) corresponds to the conventional electromechanical coupling factor for unperturbed SAW mode[16].

Solving the simultaneous linear equations (6) and (7) with appropriate boundary conditions, we obtain

\[
p_{11} = \frac{\Gamma_0(1 - E^2)}{1 - \Gamma_0 E^2},
\]

\[
p_{12} = \frac{E(1 - \Gamma_0^2)}{1 - \Gamma_0 E^2},
\]

\[
p_{13} = \frac{\xi_0(1 - \Gamma_0)}{1 + \Gamma_0 E},
\]

and

\[
p_{33} = -2j \chi \xi_0 \xi L \left[ \frac{(1 - E)(1 + \Gamma_0)}{j \theta_p L(1 + \Gamma_0 E)} \right] + j \omega C L,
\]

where \( E = \exp(-j \theta_p L) \), \( \theta_p = \sqrt{\theta_a^2 - \kappa_{12}^2} \), \( \Gamma_0 = (\theta_p - \theta_a)/\kappa_{12} \) and \( \xi_0 = \zeta/(\theta_a + \kappa_{12}) \).

**C. IDT properties**

Fig. 2 shows the normalized radiation conductance \( G_1/\chi(\zeta L)^2 \) as a function of the normalized frequency \( \theta_p p_1/2\pi \). In the calculation, \( \chi^2 p_1/\omega C = 0.08 \) and \( L = 10 p_1 \). When \( \kappa_{12} p_1 \) is very small, the main lobe in \( G_1 \) exhibits typical \( (\sin x/x)^2 \) dependence. With an increase in \( \kappa_{12} p_1 \), the main lobe becomes peaky and its peak moves toward low frequencies. Although now shown, with an increase in \( L \), the peak amplitude becomes steep and large while the position is scarcely changed.

It is clear from Eq. (11), \( G_1 \) takes a first zero at \( \theta_p L = -2\pi \). On the other hand, \( G_1 \) takes the maximum value at \( \theta_p L \approx -\pi (\theta_a \approx -\sqrt{(\pi L/2)^2 + \kappa_{12}^2}) \) when \( \kappa_{12} L \gg 1 \). Since \( \theta_p \) is real for these frequencies, the SAW amplitude distributes as shown in Fig. 3. For the case (b), the SAW amplitude is antisymmetric with respect to the geometrical center, and SAWs can not be excited and detected by the IDT.

Fig. 4 shows \( p_{11} \) calculated by Eq. (9) for \( \kappa_{12} p_1 = 0.08\pi \) and \( L = 10 p_1 \). Comparison of Fig. 4 with Fig. 2 indicates
that \( p_{11} \) is zero at \( \theta_pL = -\pi \) where \( G_1 \) is close to its maximum. Within the stopband (\( |\theta_p| < \kappa_{12} \)), \( |p_{11}| \cong 1 \) and change in the phase is very gradual. On the other hand, out of the stopband, small ripples appear and the phase change is rapid.

For the case where \( \kappa_{12} < 0 \), \( G_1 \) shown in Fig. 2 becomes mirror image with respect to \( \theta_p = 0 \), and the peak appears at \( \theta_pL \cong +\pi \). And \( |p_{11}| \) shown in Fig. 4 is unchanged, but \( \angle(p_{11}) \) shifts by \( -\pi \).

Fig. 5 shows \( \phi_\pm (= \angle(p_{12} \pm p_{11})) \) for \( L = 10p_1 \). It is seen that \( \phi_+ \) is almost equal to \( \phi_- \), namely, the influence of the internal reflection is not significant for the SAW propagation out of the stopband (\( |\theta_p| > \kappa_{12} \)). On the other hand, \( \phi_+ - \phi_- \cong \pi \) within the stopband.

Note that \( -\partial(\phi_\pm)/\partial\omega \) corresponds to the effective time delay passing through the structure, and the value is small in the stopband. In other words, the effective SAW velocity is large in this region. This is because incident SAW is reflected near the structural edge and hardly penetrate into the structure. On the other hand, just below the stopband, the effective time delay for \( \phi_- \) becomes very large due to the multiple reflection between the edges.
For various \( L \) and \( \kappa_{12} \), \( G_1 \) and \( \phi_{3k} \) were also calculated. The result indicated that influence of the internal reflection is characterized by \( \kappa_{12}L \); the influence significant and the IDT behaves like a resonator when \( \kappa_{12}L > 1 \) near the stopband whereas the influence is insignificant when \( \kappa_{12}L < 1 \) and/or a little far from the stopband.

### III. One-port SAW Resonators

Let us discuss the behavior of one-port SAW resonators having a mirror symmetry structure (see Fig. 7). In the figure, \( L_r, L_g \) denote the grating reflector length and the gap length between the IDT and grating reflectors, respectively. The grating reflector length and the gap length are chosen to be \( 0.5 \lambda \) and \( 0.1 \lambda \), respectively; the variation of \( \phi_+ \) within the stopband is less than \( 3 \pi \). Thus the single-mode resonance is realized when the variation of \( \phi_+ \) within the stopband is less than \( 3 \pi \). Since \( \phi_+ = -2 \pi \omega / \omega_r \), \( I_1/p_1 \) for this case, the single-mode resonance condition gives allowable maximum \( L_1 \):

\[
L_1/p_1 < 1.5 (\Delta \omega / \omega_r)^{-1},
\]

where \( \Delta \omega \) is the stopband width. When \( \kappa_{12}p_1 = 0.08 \pi \) for example, \( L_1/p_1 < 18.75 \) since \( \Delta \omega / \omega_r = 0.08 \).

In this case, the traditional design[5], [6], [7], [8] gives the best result for both \( \gamma \) and \( Q \), where the reflector position is displaced so that the standing wave pattern within the cavity coincides with the IDT finger location \( L_g = -\lambda/8 \) and \( p_1/p_1 = 1 \).

Fig. 9 shows the frequency response of a design example. Here \( L_1 \) and \( L_r \) are chosen to be \( 16 \lambda/15 \) and \( 50 \lambda/15 \), respectively. Here, \( L_1 \) is chosen smaller than the value given by Eq. (16) so as to suppress adjacent ripples. Further suppression is possible by decreasing \( L_1 \), though this results in increased \( \gamma \). The ripple is suppressed also by weighting grating finger lengths appropriately with a penalty of increased grating length[17].

Next, let us discuss the resonator design when \( \kappa_{12} \) for the IDT is identical with that for the reflectors. Since \( \Delta \omega / \omega_r = \kappa_{12}p_1 / \pi \), maximum \( L_1 \) given by Eq. (16) is \( 1.5 \pi / \kappa_{12} \). This means that provided small \( \gamma \) is necessary, the internal reflection within the IDT is not always negligible because \( \kappa_{12}L_1 \approx 5 \).

In the following three designs, \( L_r \) is fixed to be \( 50 \lambda /15 \) and \( L_1/p_1 \) and \( p_1/p_r \) are adjusted to minimize \( \gamma \) for giving \( L_g \).

**Case 1** When the device is designed to resonate at \( \phi_1 = \pm \pi \) (\( \theta_2, L_1 \approx -\pi \)): This design offers the largest \( G_1 \) and then \( \phi_1 \) is close to \( \pi /2 \). However, Fig. 5 shows that
the gradient of $\phi_0$ is steep. As a result, $\gamma$ becomes considerably large.

Fig. 10 shows a frequency response of a device with $L_g = \lambda/8$ designed so as to resonate at $\phi_0 \cong \pi$. Comparing Fig. 10 with Fig. 9, it is seen that $\gamma$ is increased and the ripple in the higher side becomes strong.

**Case 2** When the device is designed to resonate when $\phi_0 \cong \pi/2$ ($\theta_{p1}L_1 \cong -\pi/2$): Unless the peak value of $G_1$ is not very steep, $\phi_1$ remains close to $\pi/2$. In addition, the gradient of $\phi_0$ is relatively small. Then, $\gamma$ obtained is usually smaller than that of Case 1.

It should be noted that $\angle\Gamma_1 = -\pi/2$ at $\theta_u = 0$. This means that $L_g$ can be set to be zero when the device is designed so that the resonance takes place at the Bragg frequency for the reflectors. Making $L_g = 0$ is desirable for minimizing the SAW scattering to bulk waves[18] and, therefore, enables to realize high $Q$ resonators[19].

Fig. 11 shows a frequency response of a device with $L_g = 0$ designed so as to resonate at $\phi_0 \cong \pi/2$. Comparing Fig. 11 with Figs. 9 and 10, it is seen that the resonator possess small $\gamma$ and out-of-band ripples appear in the lower side.

**Case 3** When the device is designed to resonate when $\phi_0 \cong 0$ ($\theta_{p1}L_1 \cong 0$): From the same reason for Case 2, it is expected that smaller $\gamma$ can be realized. Nevertheless, $\gamma$ is scarcely improved because of small $\phi_1$. Furthermore, since $\phi_0$ changes rapidly by $2\pi$ in lower frequencies, $L_1$ must be reduced significantly, and this results in increase in $\gamma$.

Fig. 12 shows a frequency response of a device with $L_g = -\lambda/8$ designed so as to resonate at $\phi_0 \cong 0$. Comparing Fig. 12 with Fig. 11, it is seen that achieved $\gamma$ is limited and strong out-of-band ripples appear.

*Fig. 9. Design example when the internal reflection within the IDT is ignored ($L_g = -\lambda/8$).*

*Fig. 10. Design example when $L_g = \lambda/8$ ($L_1 = 8p_1$ and $p_r/p_1 = 1.056$).*

*Fig. 11. Design example when $L_g = 0$ ($L_1 = 13p_1$ and $p_r/p_1 = 1.048$).*

*Fig. 12. Design example of when $L_g = -\lambda/8$ ($L_1 = 8p_1$ and $p_r/p_1 = 1.040$).*
The discussion suggests that the optimal design could make \( L_g \) zero and adjust \( p_I \) so that the whole structure resonates at the Bragg frequency for the SC gratings; all of our numerical attempts confirmed this suggestion.

When higher \( Q \) and/or lower motional resistance are desirable rather than lower \( \gamma \), resonators are sometimes designed in the same way with Case 1 where the resonance takes place at the frequency giving the peak in \( G_I \), where the steepest gradient of \( \phi_+ \) and the largest \( p_I \) are simultaneously achievable. However, since \( L_g \) is nonzero under this design, the SAW scattering to bulk waves occurs, and this may cause significant reduction in achievable resonance \( Q \).

By the way, as shown in Fig. 5, variation of \( \phi_+ \) within the stopband is less than \( \pi \) independent of \( L_I \). Hence, less than one resonance occurs within the stopband even when variation of \( \phi_T \) is taken into account. This means that the internal reflection within the IDT can be used to suppress higher-order resonances.

For example, when the stopband for the IDT is chosen to almost coincide with that of the grating reflectors, only one resonance appears no matter how large \( L_I \) is. Since \( \phi_T \) increases with \( L_I \), small \( \gamma \) and extremely low input impedance are simultaneously achievable by simply increasing \( L_I \). In addition, \( L_g \) can be set to be zero to suppress the BAW scattering because of \( \phi_+ \equiv 0 \) and \( \phi_T \equiv 0 \) near the lower edge of the stopband. This type is called the IDT resonator[19], [20].

IV. TWO-PORT DMS FILTERS

Next, let us discuss the behavior of longitudinally-coupled two-port DMS filters designed so as to support multiple resonances (see Fig. 13). In the figure, \( L_T \) denotes the gap length between the two IDTs. Because of the structural symmetry, the resonances are categorized into two types: one is the even-modes with symmetrical field distribution, and another is the odd-modes with anti-symmetrical one (see Fig. 13).

Fig. 13. Two-port DMS resonator

Fig. 14 shows the basic structure and electrical connection of three-port DMS filters. Due to the electrical connection, only the even-modes are excitable for this case. Then this property enables to extend the distance between the resonance frequencies, and in general this results in wider passband width than the Two-port case. Further increase in the number of IDTs may enable to further increase in the passband width through the increased number of degrees of freedom in the design parameters[22].

Hereafter only 2-port DMS filters will be discussed for simplicity since the basic procedures for the analysis and the design of three-port DMS filters are the same with that of the 2-port ones.

Fig. 15 shows its equivalent circuit near the resonances for the 2-port DMS resonators. In the figure, \( Y_{me} \) and \( Y_{mo} \) are the motional admittances due to even- and odd-modes, respectively. Although the circuit is similar to that for the one-port SAW resonator, the resonance circuit is involved as a shunt element between two IDTs. This is because the structure is equivalent to a one-port SAW resonator if two IDTs are electrically parallel-connected. When two IDTs are simply parallel-connected, only the even-mode resonance occurs whereas only the odd-mode resonance occurs when two IDTs are parallel-connected with polarity inversion.

When the relation between the currents \( I_n \) and voltage \( V_n \) at the electric port \( n \) is expressed in the admittance matrix form:

\[
\begin{pmatrix}
I_1 \\
I_2
\end{pmatrix} = 
\begin{pmatrix}
Y_1 & Y_1 \\
Y_1 & Y_1
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2
\end{pmatrix}.
\]

(17)
If \( V_1 = V_2 \), the field distribution in the structure is symmetric, and the following relation holds;
\[
\frac{I_1}{V_1} = Y_l + Y_t = Y_e.  \tag{18}
\]
On the other hand, if \( V_1 = -V_2 \), the field distribution in the structure is anti-symmetric, and the following relation holds;
\[
\frac{I_1}{V_1} = Y_l - Y_t = Y_o.  \tag{19}
\]
By \( Y_e \) and \( Y_o \), the scattering parameter \( S_{21} \) for the corresponding two-port SAW resonator is given by
\[
S_{21} = -\frac{G_L(Y_e - Y_o)}{(Y_e + G_L)(Y_o + G_L)},  \tag{20}
\]
where \( G_L^{-1} \) is the characteristic impedance of the measurement system.

It is clear that the equivalent circuit shown in Fig. 15 can be modified as shown in Figs. 16(a) and (b) by the use of \( Y_e (= Y_{me} + j\omega C_0) \) and \( Y_o (= Y_{mo} + j\omega C_0) \).

\[
Y_e = p_{33} + \chi p_{34}^2 \frac{2p_{11} - 2p_{12} - \Gamma^{-1} + T^{-1}}{(p_{11} - \Gamma^{-1})(p_{11} - T^{-1}) - p_{12}^2} = jB_1 + jG_1 \frac{\cos \psi - a_p}{\sin \psi - b_p},  \tag{21}
\]
and
\[
Y_o = p_{33} + \chi p_{34}^2 \frac{2p_{11} - 2p_{12} - \Gamma^{-1} + T^{-1}}{(p_{11} - \Gamma^{-1})(p_{11} - T^{-1}) - p_{12}^2} = jB_1 + jG_1 \frac{\cos \psi - a_o}{\sin \psi - b_o},  \tag{22}
\]
where \( T = \exp(-j\beta L_T) \).

Basic idea for DMS filter design is very similar to that of ladder-type SAW filter design[12]. That is, it is clear from Fig. 16 that the resonance frequency of \( Y_e \) (or \( Y_o \)) is set to coincide with the anti-resonance frequency of \( Y_o \) (or \( Y_e \)). If we assume \( |B_1| \ll G_1 \) in Eqs. (21) and (22), this requirement is met when \( a_p = b_o \) or \( a_o = -b_p \). Accordingly, from Eqs. (23)-(26), the condition reduces to
\[
\psi = \frac{\phi_+ + \phi_-}{2} + \frac{\phi_T + \phi_T}{2} = n\pi.  \tag{28}
\]

At first, let us consider the case where the influence of the internal reflection is not significant. Since \( \phi_+ \cong \phi_- \), we obtain \( b_p \cong 0 \) and \( b_o \cong 0 \) from Eqs. (24) and (26). Therefore, the resonance condition is given by \( \psi \cong n\pi/2 \) from Eqs. (21) and (22), requiring \( \phi_T - \phi_T = n\pi \) to satisfy the condition of Eq. (28). This means that the difference between the resonance frequencies is governed by \( L_1 \) and their position is controlled by \( L_T \).

On the other hand, when the influence of the internal reflection is significant, the difference between the resonance frequencies can be adjusted by not only \( L_1 \) but also \( L_T \) because \( b_p \) and \( b_o \) are not small. Nevertheless, this design is not desirable because of poorer resonance characteristics resulted from smaller \( G_1 \).

For the operation over wide bandwidth, current RF DMS filters are designed to satisfy the condition of Eq. (28) by more than two resonances. In practice, the first and second resonances are placed in the frequency region where the internal reflection is not significant, and \( L_1 \) and \( L_T \) are used.

\[
(a) \text{ Circuit A} \quad Y_e - Y_o \quad Y_o - Y_e \quad 1:-1
\]

\[
(b) \text{ Circuit B} \quad Y_o - Y_e \quad Y_e - Y_o
\]

Fig. 16. Modified equivalent circuits for two-port double-mode SAW resonator.
for adjusting their location and distance. On the other hand, the third resonance is placed close to the IDT stopband, and its location is adjusted by the IDT and grating reflector pitches. Since $\phi_+ - \phi_- \text{ and } G_I/B_I$ are finite and frequency dependent, fine tuning for design parameters such as $L_I$ and $L_T$ are necessary.

One may also be able to adjust the location of resonances by $L_g$. The same discussion in Section 3, however, suggests that making $L_g = 0$ could be preferable when the behaviors of $G_I/B_I$ and $\phi_+ + \phi_- \text{ are taken into consideration.}$

As an example, Fig. 17 shows $Y_e$ and $Y_o$ of the designed DMS resonator with the condition of $\chi_3^2 p_1/\omega C = 0.08 \text{ and } \kappa_1 p_1 \equiv 0.08\pi \text{.}$ The optimization resulted in $\kappa_1 p_1 = 0.0824\pi, L_1/p_1 = 13.5, L_o/p_o = 55, L_T = 0.36\lambda \text{ and } p_o/p_1 = 1.03 \text{ and } G_L = 0.93\omega CL_1.$ It is seen that two resonance frequencies coincide with two antiresonance frequencies.

![Fig. 17. Admittance characteristics of designed DMS filter](image)

Fig. 17 shows $S_{21}$ when two DMS resonators are cascaded. It is seen that a very flat and relatively wide passband is realized.

By the way, Eq. (20) indicates that $S_{21} = 0$ when $Y_e = Y_o.$ From Eqs. (21) and (22), the condition of $Y_e = Y_o$ can be met when $\phi_+ + \phi_T = (2\pi + 1)\pi.$ This shows the situation in which the SAW excited at one end of the IDT is canceled out by the SAW excited and reflected at the other end. In Fig. 18, a notch response due to this can be seen at $\theta_n p_1/2\pi \equiv -0.03.$

Note that the strong ripple at the rejection band is due to the multiple reflection at the grating reflectors, and is suppressed also by weighting grating finger lengths appropriately with a penalty of increased grating length[17].

V. ADVANCED DMS FILTER DESIGN

The above consideration gives the following criteria for further improvements:

1. A number of degrees of freedom should be increased in the design parameters which are effective in adjusting the location of resonance and/or anti-resonance frequencies.
2. The IDT stopband width, namely the internal reflection in the IDT, should be reduced in order to place these resonances out of the IDT stopband.
3. The IDTs should be designed so that $G_I$ is almost flat and relatively large over the filter passband.

From these criteria, it is clear that precise control of the internal reflection in the IDTs is crucial for the optimal design of DMS filters.

Current fabrication and design technologies enable us to employ non-uniform IDTs and/or gratings for increasing the number of degrees of freedom. Since the main task is to allocate small number of resonance and/or anti-resonance frequencies properly, course and global optimization is sufficient.

As an example, Fig. 19 shows $Y_e$ and $Y_o$ of the DMS resonator redesigned according to these criteria. The design conditions are identical with those for the original one shown in Fig. 17. It is seen that three resonance frequencies coincide with three antiresonance frequencies.

Fig. 20 shows $S_{21}$ when two re-designed DMS resonators are cascaded. Comparing Fig. 20 with Fig. 18, one can see that the passband is made wider and the out-of-band rejection is improved.

It should be noted that due to a sufficient number of degrees of freedom, not only $L_g$ but also $L_T$ are able to be made zero in this design. This enables further suppression of the bulk wave radiation from structural discontinuities, and this results in further reduction of the device insertion loss.

Of course, the analysis and design principle discussed here are also applicable to more complex resonator struc-
Fig. 20. Frequency response of the re-designed DMS filter (two-stage cascaded)

Fig. 19. Admittance characteristics of re-designed DMS filter

Design criteria are also presented for DMS filters of wide bandwidth, and it is demonstrated how device performances are improved by proper design taking account of the criteria through increasing the number of degrees of freedom in resonator design.

VI. CONCLUSIONS

This paper has discussed the operation and the design of current SAW resonators where the internal reflection in IDTs is not negligible and lower capacitance ratio is necessary. In the qualitative and quantitative discussion, the p-matrix expression was used skillfully with a help of the COM theory.

It was shown that the internal reflection causes a) deformation of the IDT passband shape, and 2) frequency dependence of the effective SAW velocity within IDTs, and 3) suppression of higher-order resonances, and these features can be effectively used to enhance performances of both one-port SAW resonators and two-port DMS filters. In addition, under proper designs taking the internal reflection into account, most of all structural discontinuities can be removed, and this is most preferable for the reduced scattering loss at the discontinuity.

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