Introduction to Surface Acoustic Wave (SAW) Devices

Part 3: Coupling-Of-Modes Theory

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Contents

• Colinear Coupling
• Reflective Coupling
• IDT Modeling
• IDT Properties
• SAW Device Simulation
• Parameter Extraction
• Parameter Extraction of Bidirectional IDTs
• Parameter Extraction of Directional IDTs
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Coupling-Of-Modes (COM) Theory

Normal Mode Equation
\[
\frac{\partial u_1}{\partial X} = - j \beta_u u_1 \\
\frac{\partial u_2}{\partial X} = - j \beta_u u_2
\]

[Solution: \( u_i \propto \exp(-j\beta_u X) \)]

\( \beta_u \): Wavevector at Uncoupled State

Coupling of Modes Equation
\[
\frac{\partial u_1}{\partial X} = - j \beta_u u_1 - j \kappa u_2 \\
\frac{\partial u_2}{\partial X} = - j \beta_u u_2 - j \kappa' u_1
\]
Loss-Less Condition (Unitary Condition)

\[
\frac{|u_1(X + \Delta X)|^2 - |u_1(X)|^2 + |u_2(X + \Delta X)|^2 - |u_2(X)|^2}{\Delta X} = 0
\]

\(\Delta X \to 0 \) gives

\[
\frac{\partial}{\partial X} \left( |u_1(X)|^2 + |u_2(X)|^2 \right) = u_1^* \frac{\partial u_1}{\partial X} + u_1 \left( \frac{\partial u_1}{\partial X} \right)^* + u_2^* \frac{\partial u_2}{\partial X} + u_2 \left( \frac{\partial u_2}{\partial X} \right)^* = 0
\]
Substitution of COM Equations Gives
\[ 2 \text{Im}[\beta_u] \left( |u_1|^2 + |u_2|^2 \right) + \text{Im}[(\kappa - \kappa^*)u_1^*u_2] = 0 \]

To Satisfy for Arbitrary \( u_1, u_2 & X \), \( \text{Im}[\beta_u] = 0 \) & \( \kappa' = \kappa^* \)

**Final COM Equations**
\[
\frac{\partial u_1}{\partial X} = -j \beta_u u_1 - j \kappa u_2 \\
\frac{\partial u_2}{\partial X} = -j \beta_u u_2 - j \kappa^* u_1
\]

When Two Waveguides are Exchangable, \( \kappa \) is Real
**General Solution**

\[ u_1 = A_+ \exp(-j\beta_+X) + A_- \exp(-j\beta_-X) \]
\[ u_2 = rA_+ \exp(-j\beta_+X) - rA_- \exp(-j\beta_-X) \]

Where \( \beta_\pm = \beta_u \pm |\kappa| \) \quad \( r = |\kappa| / \kappa \)

When \( \kappa \) is Real, Two Partial Waves Correspond to

Red: Symmetric Mode,
Blue: Antisymmetric Mode
Application of Boundary Condition

Boundary Condition

\[ u_1(0) = A_i \quad \text{and} \quad u_2(0) = 0 \]

\[ \Rightarrow A_+ = A_- = A_i / 2 \]

\[ u_1 = A_i \exp(-j\beta_u X) \cos(\kappa |X|) \]

\[ u_2 = -jrA_i \exp(-j\beta_u X) \sin(\kappa |X|) \]
Multi-Strip-Coupler (MSC)

Velocity Difference in Short- & Open-Circuited Gratings

Transversal Filter Using MSC
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Due to Periodicity, Eigen Modes in Infinite Periodic Gratings Satisfy

\[ u_\pm(X + p) = u_\pm(X) \exp(\mp j \beta_0 p) \]

Where \( \beta_0 \) is Wavenumber of Grating Mode

Define \( u_\pm(X) = U_\pm(X) \exp(\mp j \beta_0 X) \)

Then We Obtain

\[ U_\pm(X + p) = U_\pm(X) \text{ : Periodic Function} \]
Since $U_{\pm}(X)$ is Periodic Function

$$U_{\pm}(X) = \sum_{n=-\infty}^{+\infty} A_{\pm}^{(n)} \exp(\mp nj \beta_G X)$$

Where $\beta_G = \frac{2\pi}{p}$: Grating Vector

$A_{\pm}^{(n)}$: Amplitude of $n$-th Partial Wave

$$u_{\pm}(X) = \sum_{n=-\infty}^{+\infty} A_{\pm}^{(n)} \exp(\mp j \beta_n X)$$

Where $\beta_n = \beta_0 + n \beta_G$

*Incident Wave with $\beta$ is Spatially Modulated, and Components with $\beta+n\beta_G$ are Generated.*
SAW Dispersion in Periodic Structures

Bragg Reflection
COM Analysis for Periodic Structures

**Eigen Mode Equations** [General Solution: \( u_\pm \propto \exp(\mp j \beta_u X) \)]

\[
\frac{\partial u_+}{\partial X} = - j \beta_u u_+ - j \kappa_{12} u_- \exp( - j \beta_G X )
\]

\[
\frac{\partial u_-}{\partial X} = + j \beta_u u_- + j \kappa_{12}^* u_+ \exp( + j \beta_G X )
\]

**COM Equations for Forward & Backward Waves**

\(\beta_u\): Wavenumber of Uncoupled Wave

\(\beta_G\): Grating Vector \((2\pi/p)\), \(p\): Periodicity

\(\kappa_{12}\): Mutual Coupling Coefficient

\(=\) Reflectivity per Unit Length

For Derivation, Loss Less Condition was Applied
Define $U_\pm(X) = u_\pm(X)\exp(\pm j \beta_G X/2)$.

Since $u_\pm(X) = U_\pm(X)\exp(\mp j \beta_G X/2)$,

$$\frac{\partial U_+}{\partial X} = -j \theta_u U_+ - j \kappa_{12} U_-, \quad \frac{\partial U_-}{\partial X} = +j \kappa^*_{12} U_+ + j \theta_u U_-,$$

where $\theta_u = \beta_u - \beta_G/2$ : Detuning Factor

($\theta_u=0$ corresponds to Bragg Condition)

**Origin of Phase in $\kappa_{12}$**

**Displacement of Reflection Center from Origin**

$$d_r/p_I = \angle(\kappa_{12})/4\pi$$
General Solution

\[ U_+(X) = A_+ \exp(-j \theta_p X) + \Gamma_- A_- \exp(+j \theta_p X) \]
\[ U_-(X) = \Gamma_+ A_+ \exp(-j \theta_p X) + A_- \exp(+j \theta_p X) \]

\[ \beta_p = \theta_p + \pi/p: \text{Wavenumber of } \textbf{Perturbed} \text{ Wave} \]
\[ \theta_p = \sqrt{\theta_u^2 - |\kappa_{12}|^2} \]

\[ \Gamma_+ = (\theta_p - \theta_u)/\kappa_{12} \text{ & } \Gamma_- = (\theta_p - \theta_u)/\kappa_{12}^* \]: Reflection Coefficient of Semi-Infinite Grating Looking toward \( \pm X \) direction

\[ \Rightarrow \kappa_{12} \text{ is Real When Grating is Symmetric} \]

\[ A_\pm: \text{Amplitude of Partial Wave} \]
Relative Wavenumber

Relative frequency, $\theta_u p / \pi$

$|\kappa_{12}| p = 0.02 \pi$

$\text{Re}(\theta_u/p)$

$\text{Re}(\theta_p/p)$

$\text{Im}(\theta_p/p)$
Behavior Near Bragg Frequency

\[ \theta_p = \sqrt{\theta_u^2 - |\kappa_{12}|^2} \]

\[ |\kappa_{12}| \text{ determines Both Stopband Width & Attenuation Constant} \]
Reflection Coefficient in dB

Reflection Phase in deg.

Relative frequency, $\theta_u p/\pi$
(a) $\Gamma_+ = \frac{A_{\text{ref}}}{A_{\text{in}}}$

(b) $\Gamma_- = \frac{A_{\text{ref}}}{A_{\text{in}}}$
Application of Boundary Condition

\[ U_+(X) = A_+ \exp(-j\theta_p X) + \Gamma_+ A_- \exp(+j\theta_p X) \]
\[ U_-(X) = \Gamma_+ A_+ \exp(-j\theta_p X) + A_- \exp(+j\theta_p X) \]

Since \( U_+(0) = A_{in} \) & \( U_-(L) = 0 \),

\[ \Gamma = \frac{A_r}{A_{in}} = \frac{\Gamma_+ [1 - \exp(-2j\theta_p L)]}{1 - \Gamma_+ \Gamma_- \exp(-2j\theta_p L)} \]
\[ T = \frac{A_t}{A_{in}} = \frac{\exp(-j\theta_p L)(1 - \Gamma_+ \Gamma_-)}{1 - \Gamma_+ \Gamma_- \exp(-2j\theta_p L)} \]
Reflection Coefficient in dB

Reflection Phase in deg.

Relative frequency, $\theta_{u}p/\pi$

$|\kappa_{12}|p = 0.02\pi$
(a) Stopband (evanescent field)

(b) Passband (standing wave field due to reflection at edges)
Reflection Coefficient in dB

$L=10\rho$

$L=30\rho$

$L=50\rho$

$|\kappa_{12}|\rho=0.02\pi$
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COM Equation for SAW Devices

\[ \frac{\partial u_+}{\partial X} = -j \beta_u u_+ - j \kappa_{12} u_- \exp(-j \beta_G X) + j \zeta V_0 \exp(-j \beta_G X / 2) \]

\[ \frac{\partial u_-}{\partial X} = +j \kappa^*_{12} u_+ \exp(+j \beta_G X) + j \beta_u u_- - j \zeta^* V_0 \exp(+j \beta_G X / 2) \]

\( \zeta \): Transduction Coefficient

\( p_1(=2p) \): IDT Periodicity

Spatial Components with \( \pm \beta_G / 2(= \pm 2 \pi / p_1) \) are Considered
Equation for Current on Bus-Bar

\[ \delta I \ast p_I \quad I(X) \]

C: Static Capacitance per Unit Length
\[ \chi = 2 \] for RMS I, V & u
\[ \chi = 4 \] for peak I, V & RMS u

\[ \frac{\partial I}{\partial X} = -j \chi \xi^* u_+ \exp( + j \beta_G X / 2 ) - j \chi \xi u_- \exp( - j \beta_G X / 2 ) + j \omega CV_0 \]

Spatial Components with \( \pm \beta_G / 2 = (\pm 2 \pi / p_I) \) are Considered

For Derivation, Loss Less Condition & Bidirectionality (When Mechanical Reflection is Zero) are Applied
Final COM Equations

\[
\begin{align*}
\frac{\partial u_+}{\partial X} &= -j \theta_u u_+ - j \kappa_{12} u_- \exp(-j \beta_G X) + j \zeta V_0 \exp(-j \beta_G X / 2) \\
\frac{\partial u_-}{\partial X} &= j \kappa^*_{12} u_+ \exp(+j \beta_G X) + j \theta_u u_- - j \zeta^* V_0 \exp(+j \beta_G X / 2) \\
\frac{\partial I}{\partial X} &= -j \chi \zeta^* u_+ \exp(+j \beta_G X / 2) - j \chi \zeta u_- \exp(-j \beta_G X / 2) + j \omega CV_0
\end{align*}
\]

Define \( U_\pm(X) = u_\pm(X) \exp(\pm j \beta_G X / 2) \). Then
Since \( u_\pm(X) = U_\pm(X) \exp(\mp j \beta_G X / 2) \),

\[
\begin{align*}
\frac{\partial U_+}{\partial X} &= -j \theta_u U_+ - j \kappa_{12} U_- + j \zeta V_0 \\
\frac{\partial U_-}{\partial X} &= +j \kappa^*_{12} U_+ + j \theta_u U_- - j \zeta^* V_0 \\
\frac{\partial I}{\partial X} &= -j \chi \zeta^* U_+ - j \chi \zeta U_- + j \omega CV_0
\end{align*}
\]
**General Solution**

\[ U_+(X) = A_+ \exp(-j\theta_p X) + \Gamma_- A_- \exp(+j\theta_p X) + \xi_+ V_0 \]
\[ U_-(X) = \Gamma_+ A_+ \exp(-j\theta_p X) + A_- \exp(+j\theta_p X) + \xi_- V_0 \]

Where \( \xi_+ = (\zeta u - \zeta^* \kappa_{12})/\theta_p^2 \) & \( \xi_- = (\zeta^* u - \zeta \kappa_{12}^*)/\theta_p^2 \):

Excitation Efficiency toward \( \pm X \) Direction

**Origin of Phase in \( \zeta \)**

**Displacement of Excitation Center from Origin**

\[ d_t/p_I = \angle(\zeta)/2\pi \]
Relative frequency, $\theta p_1/2\pi$

- $\chi \zeta^2 p_1/\omega C = 0.08$
- $\kappa_{12} p_1 = 0.04\pi$

$\text{Re}(\theta_u/p_1)$
$\text{Im}(\theta_u/p_1)$
$\text{Re}(\theta_p/p_1)$
$\text{Im}(\hat{\theta}_p/p_1)$
$\text{Re}(\hat{\theta}_p/p_1)$
$\text{Im}(\theta_p/p_1)$
Field Distribution at Stopband with $\beta p = \pi$ (Bidirectional Case)

(a) Symmetric Mode

(b) Antisymmetric Mode
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SAW Excitation by IDT
(When $\zeta$ and $\kappa_{12}$ are Real)

$$\Gamma_{\pm} = (\theta_p - \theta_u)/\kappa_{12} \equiv \Gamma_0$$
$$\xi_{\pm} = \zeta / (\theta_u + \kappa_{12}) \equiv \xi_0$$

Boundary Conditions: $U_+(-L/2)=0$, $U_+(+L/2)=0$, $I(-L/2)=0$

$$A_+ = A_- = -\xi_0 V_0$$
$$\exp(+j \theta_p L / 2) + \Gamma_0 \exp(-j \theta_p L / 2)$$

$$Y = V_0^{-1} \int_{-L/2}^{+L/2} \frac{\partial I(X)}{\partial X} dX = \int_{-L/2}^{+L/2} [-j \chi \zeta V_0^{-1} (U_+ + U_-) + j \omega C] dX$$
\[
Y = \int_{-L/2}^{+L/2} \left[ -2 j \chi \zeta V_0^{-1} A_+ (1 + \Gamma_0) \cos(\theta_p X) - j(2 \chi \xi_0 \zeta - \omega C) \right] dX \\
= \frac{2 j \chi \xi_0 \zeta (1 + \Gamma_0) L \text{sinc}(\theta_p L / 2)}{\exp(+j\theta_p L / 2) + \Gamma_0 \exp(-j\theta_p L / 2)} - j(2 \chi \xi_0 \zeta - \omega C) L
\]

When \( \kappa_{12}=0, \ \theta_p=\theta_u, \ \Gamma_0=0 \ & \ \xi_0=\xi/\theta_u \). Then

\[
Y = \frac{2 j \chi \zeta^2 L}{\theta_u} [\text{sinc}(\theta_u L) - j\text{sinc}(\theta_u L / 2) \sin(\theta_u L / 2) - 1] + j\omega CL
\]

\[
= \chi \zeta^2 L^2 \text{sinc}^2(\theta_u L / 2) + \frac{2 j \chi \zeta^2 L}{\theta_u} [\text{sinc}(\theta_u L) - 1] + j\omega CL
\]

**Comparison : Delta Function Model Gives**

\[
Y = \chi (\zeta p_1)^2 \frac{\sin^2(\theta_u L / 2) + 2^{-1} j\sin(\theta_u L) - jL / p_j \sin(\theta_u p_j / 2)}{\sin^2(\theta_u p_j / 2)} + j\omega CL
\]
Relative admittance

\[ \frac{\chi \zeta^2 p_1}{\omega C} = 0.08 \]

\[ \kappa_{12} p_1 = 0 \]

\[ L = 10 p_1 \]
Relative admittance

- Imag
- Real

Relative frequency, $\theta u p_1/2\pi$

$\kappa_{12} p_1 = 0.04\pi$

$\chi \zeta^2 p_1 / \omega C = 0.08$

$L = 10 p_1$
\[ \kappa_{12} p_1 = -0.04\pi \]
\[ \chi \zeta^2 p_1 / \omega C = 0.08 \]

\( L = 10 p_1 \)
Relative conductance in dB

$\kappa_{12}p_1=0.04\pi$
$\kappa_{12}p_1=0.08\pi$
$\kappa_{12}p_1=0.12\pi$
$\kappa_{12}p_1=0.16\pi$

$L=10p_1$

Relative frequency, $\theta_u p_1/2\pi$
Input Admittance for Infinite IDT

Since \( \partial U_{\pm}/\partial X = 0 \) and \( i = p_I \partial V / \partial X \),

\[
0 = -j \theta_u U_+ - j \kappa_{12} U_- + j \zeta V_0
\]

\[
0 = +j \kappa_{12}^* U_+ + j \theta_u U_- - j \zeta^* V_0
\]

\[
i = -j \chi \zeta^* p_I U_+ - j \chi \zeta p_I U_- + j \omega Cp_I V_0
\]

Then

\[
\hat{Y} = \frac{i}{V_0} = -j \chi p_I \frac{2 \theta_u |\zeta|^2 - \kappa_{12} \zeta^* - \kappa_{12}^* \zeta^2}{\theta_u^2 - |\kappa_{12}|^2} + j \omega Cp_I
\]

\[
= j \omega Cp_I \frac{(\theta_u - \theta_{oc}^+)(\theta_u - \theta_{oc}^-)}{(\theta_u - \theta_{sc}^+)(\theta_u - \theta_{sc}^-)}
\]

Where \( \theta_{oc}^\pm = \chi |\zeta|^2/\omega C \pm |\kappa_{12} - \chi \zeta^2/\omega C| \), \( \theta_{sc}^\pm = \pm |\kappa_{12}| \).
COM Parameter Determination by Input Admittance of Infinite IDT

\[
\hat{Y}(\omega) = j\omega C_p \frac{(\omega - \omega_{oc}^+)(\omega - \omega_{oc}^-)}{(\omega - \omega_{sc}^+)(\omega - \omega_{sc}^-)}
\]
\[ \chi \zeta^2 p_1 / \omega C = 0.08 \]
\[ \kappa_{12} p_1 = 0.04 \pi \]
Relative Wavenumber

$|\kappa_{12}| p_1 = 0.04 \pi$

$\chi \xi^2 p_1 / \omega C = 0.08$

$\mu = 90^\circ$

$\text{Re}(\theta_u / p_1)$

$\text{Re}(\theta_p / p_1)$

$\text{Im}(\theta_p / p_1)$

Relative frequency, $\theta_u p_1 / 2\pi$
Relative admittance

Imag

Real

\[ |\kappa_{12}| p_1 = 0.04\pi \]

\[ \chi |\zeta|^2 p_1 / \omega C = 0.08 \]

Relative frequency, \( \theta_ω p_1 / 2\pi \)
\[ L = 10p_I \]

\[ |\kappa_{12}| p_I = 0.04\pi \]

\[ \chi \frac{\zeta^2}{p_I / \omega C} = 0.08 \]

\[ \mu = \angle \left( \frac{\kappa_{12}}{\zeta^2} \right) \]
Directivity in dB

\[ L = 10p_1 \]

\[ \lambda_1 |p_1| = 0.04\pi \]

\[ \chi |\zeta|^2 p_1/\omega C = 0.08 \]

\[ L = 10p_1 \]

\[ \mu = 0^\circ \]

\[ \mu = 30^\circ \]

\[ \mu = 60^\circ \]

\[ \mu = 90^\circ \]
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Simulation of Complex Structures

- Piezoelectric substrate
- Grating (Bragg) Reflectors
- Interdigital Transducer (IDT)

• Combination of Periodic Structures
Cascade-Connection of Elements

SC Grating = Short-Circuited IDT
OC Grating = IDT with Isolated Fingers
Gap = Reflection-less, Excitation-less IDT

IDT Modeling  ⇌  Device Modeling
P-Matrix Expression

\[
\begin{pmatrix}
 a_1 \\
 b_1
\end{pmatrix}
\xrightarrow{[P]}
\begin{pmatrix}
 b_2 \\
 a_2
\end{pmatrix}
\]

\[
\begin{pmatrix}
 b_1 \\
 b_2 \\
 I
\end{pmatrix} = \begin{pmatrix}
 P_{11} & P_{12} & P_{13} \\
 P_{21} & P_{22} & P_{23} \\
 -\chi P_{13} & -\chi P_{23} & P_{33}
\end{pmatrix}
\begin{pmatrix}
 a_1 \\
 a_2 \\
 V
\end{pmatrix}
\]

Unitary Condition:

\[
|P_{11}|^2 + |P_{12}|^2 = 1, |P_{22}|^2 + |P_{12}|^2 = 1
\]

\[
p_{11}p_{13}^* + p_{12}p_{23}^* + p_{13} = 0
\]

\[
p_{12}p_{13}^* + p_{22}p_{23}^* + p_{23} = 0
\]

\[
\frac{\chi}{2} \left[ |P_{11}|^2 + |P_{12}|^2 \right] = \Re(p_{33})
\]
Use of COM Model Gives

\[ P_{11} = \frac{\Gamma_-(1-E^2)}{1-\Gamma_+\Gamma_-E^2}, \quad P_{22} = \frac{\Gamma_+(1-E^2)}{1-\Gamma_+\Gamma_-E^2}, \quad P_{12} = \frac{E(1-\Gamma_+\Gamma_-)}{1-\Gamma_+\Gamma_-E^2} \]

\[ P_{13} = \frac{(1-E)\{\xi_-(1+\Gamma_+\Gamma_-E) - \xi_+\Gamma_+(1+E)\}}{1-\Gamma_+\Gamma_-E^2} \]

\[ P_{23} = \frac{(1-E)\{\xi_+(1+\Gamma_+\Gamma_-E) - \xi_-\Gamma_-(1+E)\}}{1-\Gamma_+\Gamma_-E^2} \]

\[ P_{33} = \frac{\chi(1-E)\{(\xi_+ - \Gamma_-\xi_-E)(\zeta^* + \Gamma_+\zeta) + (\xi_- - \Gamma_+\xi_+E)(\zeta + \Gamma_-\zeta^*)\}}{1-\Gamma_+\Gamma_-E^2} \]

\[ - j\chi L(\zeta^*\xi_+ + \zeta\xi_-) + j\omega CL \]

where \( E = \exp(- j \theta_p L) \)
When the unit is symmetrical,

\[ P_{11} = P_{22} = \frac{\Gamma_0 (1 - E^2)}{1 - \Gamma_0^2 E^2}, \quad P_{12} = \frac{E(1 - \Gamma_0^2)}{1 - \Gamma_0^2 E^2} \]

\[ P_{13} = P_{23} = \frac{\xi (1 - E)(1 - \Gamma_0 E)}{1 + \Gamma_0 E} \]

\[ P_{33} = 2 \chi \zeta \zeta \left[ \frac{(1 - E)(1 + \Gamma_0)}{\theta_p (1 + \Gamma_0 E)} - jL \right] + j \omega CL \]
Recursive Relation for Unit A (left) + B (right)

\[
P_{11} = P_{11}^A + P_{11}^B \frac{P_{21}^A P_{12}^A}{1 - P_{11}^B P_{22}^A}, \quad P_{22} = P_{22}^B + P_{22}^A \frac{P_{12}^B P_{21}^B}{1 - P_{11}^B P_{22}^A}, \quad P_{12} = \frac{P_{12}^A P_{12}^B}{1 - P_{11}^B P_{22}^A}
\]

\[
P_{13} = P_{13}^A + P_{12}^B \frac{P_{13}^B + P_{11}^B P_{23}^A}{1 - P_{11}^B P_{22}^A}, \quad P_{23} = P_{23}^B + P_{21}^B \frac{P_{23}^A + P_{22}^A P_{13}^B}{1 - P_{11}^B P_{22}^A}
\]

\[
P_{33} = P_{33}^A + P_{33}^B + P_{32}^A \frac{P_{13}^B + P_{11}^B P_{23}^A}{1 - P_{11}^B P_{22}^A} + P_{31}^B \frac{P_{23}^A + P_{22}^A P_{13}^B}{1 - P_{11}^B P_{22}^A}
\]

再帰的
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Determination of COM Parameters

\[
\frac{\partial U^+}{\partial X} = - j \theta_u U^+ - j \kappa_{12} U^- + j \zeta V_0
\]

\[
\frac{\partial U^-}{\partial X} = + j \kappa^{*}_{12} U^+ + j \theta_u U^- - j \zeta^{*} V_0
\]

\[
\frac{\partial I}{\partial X} = - j \chi \zeta^{*} U^+ - \chi j \zeta U^- + j \omega CV_0
\]

\(\kappa_{12}\): Mutual Coupling Coefficient (Mostly Constant)

\(\zeta\): Transduction Coefficient (Mostly Constant)

\(C\): Capacitance (Mostly Constant)

\(\theta_u\): detuning factor (Linearly Changes with \(\omega\))

\[
\Rightarrow \theta_u = \omega V_{\text{ref}} - \pi/p + \kappa_{11}
\]

\(V_{\text{ref}}\): Reference SAW Velocity

\(\kappa_{11}\): Self Coupling Coefficient
Physical Mean of $\kappa_{11}$ and $V_{\text{ref}}$

\[ \kappa_{11} = \frac{\pi}{p} - \frac{\omega_r}{V_{\text{ref}}} \]

\[ \beta_u \approx \frac{\omega}{V_{\text{ref}}} + \kappa_{11} \]
For Short-Circuited (SC) Grating, $V_0=0$

$$\theta_p = \sqrt{\theta_u^2 - |\kappa_{12}|^2}$$

For Open-Circuited (OC) Grating, $\delta I=0$

$$\theta_p = \sqrt{(\theta_u - \chi|\zeta|^2/\omega C)^2 - |\kappa_{12} - \chi\zeta^2/\omega C|^2}$$

\[ \theta_{oc}^{\pm} = \chi |\zeta|^2/\omega C \pm |\kappa_{12} - \chi\zeta^2/\omega C| \]

\[ \theta_{sc}^{\pm} = \pm |\kappa_{12}| \]
Since $|\chi\zeta|^2/\omega C \pm |\kappa_{12}|$ $\chi^2/\omega C = \omega_{oc}^\pm N_{ref} - \pi/p + \kappa_{11}$

$\kappa_{11} = \frac{\pi}{p} - \frac{\omega_{sc}^+ + \omega_{sc}^-}{2V_{ref}}$

$|\kappa_{12}| = \frac{\omega_{sc}^+ - \omega_{sc}^-}{2V_{ref}}$

$|\frac{\chi\zeta}{\omega C}|^2 = \frac{(\omega_{oc}^+ + \omega_{oc}^-) - (\omega_{sc}^+ + \omega_{sc}^-)}{2V_{ref}}$

$|\frac{\kappa_{12} - \chi^2/\omega C}{\omega C}|^2 = \frac{\omega_{oc}^+ - \omega_{oc}^-}{2V_{ref}}$

**Sign of $\psi = \angle \left(\zeta^2/\kappa_{12}\right)$ can not be Determined**
When IDT is Bidirectional, $\zeta^2/\kappa_{12}$ is Real

One of Stopband Edge for OC Grating Coincides with that for SC Grating
Relation Between Stopband Edges and COM Parameters

\[ \kappa_{11} = \frac{\pi}{p} - \frac{\omega_{sc}^+ + \omega_{sc}^-}{2V_{\text{ref}}} \]

\[ \kappa_{12} = s \frac{\omega_{sc}^+ - \omega_{sc}^-}{2V_{\text{ref}}} \]

\[ \frac{\chi\zeta}{\omega C} = \frac{(\omega_{oc}^+ + \omega_{oc}^-) - (\omega_{sc}^+ + \omega_{sc}^-)}{2V_{\text{ref}}} \]

\[ s = \begin{cases} 1 & (\omega_{sc}^+ = \omega_{oc}^\pm) \\ -1 & (\omega_{sc}^- = \omega_{oc}^-) \end{cases} \]

How to Determine \( V_{\text{ref}} \)?

1. Determination of \(|\kappa_{12}|\) by \( \text{Max}[-\text{Im}(\theta_p)] \)

2. Determination of \( V_{\text{ref}} \) by Stopband Edges

\[ \theta_u \]

\[ \text{Re}[\theta_p] \]

\[ \text{Im}[-\theta_p] \]

\[ 0 \quad \beta_p = \theta_p + \pi/p \]
FEMSDA (Full Wave Simulator)

Finite Element Analysis
For Arbitrary Electrode Cross-Section (+ Analytic Solution not Available)

SAW Propagation Direction

Piezoelectric Substrate

Metal Electrode

Spectral Domain Analysis
Flat Substrate Surface

Analytic Solution = Fast Analysis

Boundary Condition: Minimization of Radiated Power (Error) from Boundary
Dispersion of Rayleigh SAW on YZ-LN\((h/p=0.07)\) Calculated by **FEMSDA**

\[ V_B = 3,590.1 \text{ m/s} \] (Slow-shear SSBW velocity)
Fitting with Full Wave Analysis

Phase Velocity: $V_p = \omega / \text{Re}(\beta_p)$

Attenuation: $\alpha_p = 40\pi \log_{10} e \times \text{Im}(-\beta_p) / \text{Re}(\beta_p)$ [dB/λ]
Existence of Multiple Solutions

Possibility to Jump into Blue Branch
Most Possible near Stopband Edges

Countermeasure: Attacking Upward and/or Downward
Contents

- Colinear Coupling
- Reflective Coupling
- IDT Modeling
- IDT Properties
- SAW Device Simulation
- Parameter Extraction
- Parameter Extraction of Bidirectional IDTs
- Parameter Extraction of Directional IDTs
Efficient Calculation by Combining FEMSDA and SYNC

Single-Electrode IDT

1. FEMSDA for determination of $\beta$ for OC & SC
2. Fitting after Squared
3. SYNC for determination of $C$

Double-Electrode IDT

1. MSYNC for calculation of input impedance of infinitely long IDT
2. Determination of $C$ & frequencies giving stopband edges by fitting
3. MULTI for determination of $\beta$ for SC
Wavenumber of Rayleigh SAW on YZ-LN ($h/p=0.07$) Calculated by *FEMSDA*

$V_B=3,590.1$ m/s (Slow-shear SSBW velocity)
Squared Wavenumber of Rayleigh SAW on YZ-LN \((h/p=0.07)\) Calculated by **FEMSDA**

\[\left(\frac{\beta p}{2\pi - 1/2}\right)^2\]

\(V_B=3,590.1\) m/s (Slow-shear SSBW velocity)
Input admittance, $Y/\omega \varepsilon(\infty) W$

Relative frequency, $f_p/V_{\text{ref}}$

Input Admittance of Infinitely long single-electrode IDT on YZ-LN ($h/p=0.07$) Calculated by $\text{SYNC}$

$V_B=4,030.8$ m/s (Slow-shear SSBW velocity)
(A) $\zeta$ with same polarity for input and output IDTs

(B) $\zeta$ with opposite polarity for input and output IDTs
Dispersion of Rayleigh SAW on 128-LN
Blue: Analysis by **FEMSDA**
Red: Conventional COM Analysis

\[ V_B = 4,025 \text{ m/s (Slow-shear SSBW velocity)} \]
Dispersion Relation vs. Al Thickness

Phase velocity (m/sec) vs. Relative frequency, $f_p/V_B$

Attenuation (dB/$\lambda$) vs. Relative Frequency, $f_p/V_B$
**Change in COM Parameters with Al Thickness**

\[ K_u^2 = \frac{\pi \chi |\zeta|^2 p_I}{4\omega C} \] : Electromechanical Coupling Factor for *Perturbed Mode*

\[ c = \frac{V_B}{V_{\text{ref}}} \quad V_B = 4,025 \text{ m/s (Slow Shear SSBW)} \]
Correction of Simulation Parameters

- Uncertainties in Substrate Material Constants (Supplier and Lot Dependent)
- Uncertainties in Film Material Constants (Fab. Process Dependent)
- Electrode Cross-Section (Fab. Process Dependent)

Although their Absolute Values may be Doubtful, Dependencies on Device Parameters might be held
Effective Velocity of Rayleigh-type SAW on 128-LN vs. Metallization Ratio

Solid Lines: FEMSDA, +×: Experiment
Reflectivity of Rayleigh-type SAW on 128-LN vs. Metallization Ratio

Solid Lines: FEMSDA, +×: Experiment
Behavior in Ultimate Situations

(a) When w/p~0

(b) When w/p~1

$w/p \rightarrow 1$ is not Equivalent to Flat Metallization!
Relation of COM Parameters with Resonance Characteristics

Each Parameter Independently Relates Each Property $\Rightarrow$ Easy to Fit with Experiments
Contents

• Colinear Coupling
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• IDT Modeling
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• SAW Device Simulation
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• Parameter Extraction of Bidirectional IDTs
• Parameter Extraction of Directional IDTs
Analysis of Double-Electrode IDT

How to Determine Equivalent Open-Circuited Grating?

Subdivide Each Period While Keeping Mutual Connection Within Period

(a) Short-circuited Grating  (b) Open-circuited Grating
Input Admittance of Infinitely long double-electrode IDT on 128-LN ($h/p=0.04$) Calculated by MSYNC

$\varepsilon(\infty)=55.64\varepsilon_0$

$V_B=4,030.8$ m/s (Slow-shear SSBW velocity)
\[ \hat{Y} = -j \chi p_1 \frac{2\theta_u |\zeta|^2 - \kappa_{12} \zeta^* - \kappa_{12}^* \zeta^2}{\theta_u^2 - |\kappa_{12}|^2} + j \omega C p_1 \]

Since \( \kappa_{12} = 0 \),

\[ \hat{Y} = -j \chi p_1 \frac{2|\zeta|^2}{\theta_u} + j \omega C p_1 = j \omega C p_1 \frac{\theta_u - 2 \chi |\zeta|^2 / \omega C}{\theta_u} \]

Since \( C \) and \( |\zeta| \) will be determined by fitting provided \( \omega \) dependence of \( \theta_u \) is determined.
Dispersion of SAW velocity on Infinitely long double-electrode IDT on 128-LN ($h/p=0.04$) Calculated by \textit{MULTI} \\
\(V_B=4,030.8 \text{ m/s} \) (Slow-shear SSBW velocity)
Electrode-Width-Controlled Single-Phase Unidirectional Transducer (EWC/SPUDT)

$\kappa_{12}: \text{pure real (Structural Symmetry)}$

(a) Short-circuited grating
(b) Open-circuited grating
Input Admittance of Infinitely long EWC/SPUDT on ST-quartz ($h/p=0.02$) Calculated by $\textit{MSYNC}$

$V_B=3,???. \text{ m/s (Slow-shear SSBW velocity)}$
Input Admittance for Infinite IDT

\[ \hat{Y} = j \omega C p I \frac{(\theta_u - \theta_{oc}^+)(\theta_u - \theta_{oc}^-)}{(\theta_u - \theta_{sc}^+)(\theta_u - \theta_{sc}^-)} \]

Where \( \theta_{oc}^\pm = \frac{\chi|\zeta|^2/\omega C \pm |\kappa_{12} - \chi \zeta^2/\omega C|}{|\kappa_{12}|} \)

Since \( C, |\kappa_{12}|, |\zeta| \) and \( \angle(\zeta^2/\kappa_{12}) \) will be determined by fitting provided \( \omega \) dependence of \( \theta_u \) is determined.