Introduction to Surface Acoustic Wave (SAW) Devices

Part 4: Resonator-Based RF Filters

Ken-ya Hashimoto
Chiba University
k.hashimoto@ieee.org
http://www.te.chiba-u.jp/~ken
Contents

• Basic Filter Design
• Ladder Type Filter Design
• Lattice Filter Design
• Stacked Resonator Filter Design
• Coupled Resonator Filter Design
Contents

• Basic Filter Design
**N-th Order Butterworth Filter**

Maximally Flat Response: $H^{(n)}(0) = 0$ for $n=1,2,\ldots,N$

\[
|H(\omega)|^2 = \frac{1}{1 + (\omega / \omega_0)^{2N}}
\]

Gradual Skirt & Group Delay Characteristics
**N-th Order Chebyshev Filter**

Equal-Ripple Characteristics ($\varepsilon$: parameter)

$$|H(\omega)|^2 = \frac{1}{1 + \varepsilon^2 T_N(\omega / \omega_0)^2}$$

**Chebyshev polynomial**

$$T_N(x) = \begin{cases} 
\cos(N \cos^{-1} x) & |x| \leq 1 \\
\cosh(N \cosh^{-1} x) & |x| \geq 1 
\end{cases}$$

**Graphs**

- **Relative frequency**
- **$S_{21}(dB)$**
- **Good Skirt but Bad Group Delay Characteristics**
- **$N=3$**
- **$N=5$**
- **$N=7$**
- **$N=9$**

- **$0.2dB$**
7 Element T-type Butterworth LPF

Various values

\[ R_0 = 50\Omega, \quad f_0 = 1\text{MHz} \]
• Original LPF

(Cut-off $\omega_c$)

• LPF to HPF

$$\omega_c C_1' = 1 / \omega_c L_1$$
$$\omega_c L_2' = 1 / \omega_c C_2$$

• LPF to BPF

$$\omega_0^2 L_1' C_1' = 1, \quad \omega_0^2 L_2' C_2' = 1$$
$$(\omega_0 + \omega_c) L_1' - 1 / (\omega_0 + \omega_c) C_1' = \omega_c L_1$$
$$(\omega_0 + \omega_c) C_2' - 1 / (\omega_0 + \omega_c) L_2' = \omega_c C_2$$

• LPF to BEF

$$\omega_0^2 L_1' C_1' = 1, \quad \omega_0^2 L_2' C_2' = 1$$
$$(\omega_0 + \omega_c) C_1' - 1 / (\omega_0 + \omega_c) L_1' = 1 / \omega_c L_1$$
$$(\omega_0 + \omega_c) L_2' - 1 / (\omega_0 + \omega_c) C_2' = 1 / \omega_c C_2$$
$R_0 = 50\, \Omega$, $\Delta = 1\%$, $f_c = 100\, \text{MHz}$
$Q=100$ for
Parallel-L

Round Left
Shoulder

Increased Insertion Loss

$Q=100$ for
Series-L

Round Right
Shoulder
Influence of Mutual $L$

\[ V_1 = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt} = (L_1 - M) \frac{dI_1}{dt} + M \frac{d}{dt} (I_1 + I_2) \]

\[ V_2 = M \frac{dI_1}{dt} + L_2 \frac{dI_2}{dt} = (L_2 - M) \frac{dI_2}{dt} + M \frac{d}{dt} (I_1 + I_2) \]
Null Generation!

Equivalent to $L_4$ & $L_6$ Coupling

Equivalent to $L_1$ & $L_3$ Coupling
**Constant K Filter**

Cascade-Connection with Inversion

(Equivalent Circuit for Transmission Line)

Uniform Impedance Values
|H| = 1 When Equivalent Transmission Line Length is \( n\lambda/2 \)

Similar Response to Butterworth Filter
Fabry-Perot Resonator

Interference Between Two Reflected Waves

100% Transmit When $L = n\lambda/2$
Image Parameter $Z_i, Z_o \& \varphi$

Cascade Connection with Inversion

\[
F = \begin{bmatrix}
\xi^{-1} \cosh \varphi & Z_i \xi^{-1} \sinh \varphi \\
Z_i^{-1} \xi \sinh \varphi & \xi \cosh \varphi
\end{bmatrix}
\]

where $\xi = Z_i/Z_o$

\[
Z_i = \sqrt{Z_s(Z_s + Z_p)}
\]

\[
Y_o = Z_o^{-1} = \sqrt{Y_p(Y_p + Y_s)}
\]

\[
\varphi = \sinh^{-1}\left(\sqrt{Z_s/Z_p}\right)
\]
Cascade Connection with Inversion

\[
F = \begin{cases} 
    \xi^{-1} \cosh M\phi & Z_i \xi \sinh M\phi \\
    Z_i^{-1} \xi \sinh M\phi & \xi \cosh M\phi \\
    \cosh M\phi & Z_i \sinh M\phi \\
    Z_i^{-1} \sinh M\phi & \cosh M\phi 
\end{cases} 
\]

\( (M \text{ : odd}) \)

\( (M \text{ : even}) \)
\[ S_{21} = \frac{2}{F_{11} + F_{22} + R_0 F_{21} + G_0 F_{12}} \]

\[
= \begin{cases} 
\frac{2}{(\xi + \xi^{-1}) \cosh M\phi + (Z_i \xi^{-1} / R_0 + R_0 \xi / Z_i) \sinh M\phi} & (M : \text{odd}) \\
\frac{2}{2 \cosh M\phi + (Z_i / R_0 + R_0 / Z_i) \sinh M\phi} & (M : \text{even})
\end{cases}
\]

When \( \phi = jn\pi / M \), \( |S_{21}| = 1 \)
When $\omega < \omega_c$, $\varphi$ Imag., $Z_i$ & $Z_o$ Real $\Rightarrow$ Passband

When $\omega > \omega_c$, $\varphi$ Real, $Z_i$ & $Z_o$ Imag. $\Rightarrow$ Rejection band

Zero Insertion Loss

$\varphi = jn\pi/M \Rightarrow \omega/\omega_c = \sin(n\pi/M)$
**Derived m Filter**

\[ R_0^2 = \frac{L}{C} \]

**Constant K Filter**

Null Due to Series Resonance

Null Due to Parallel Resonance

\[ R_0^2 = \frac{L}{C} \]

\[ 0 \leq m \leq 1 \]
Steep Skirt and Good Out-Band Rejection

Combination of Derived $m$ and Constant $K$ Filters

Steep Skirt and Good Out-Band Rejection

$$H(m) = 0.9 - 0.8 - 0.7 - 0.6 - (m^{-1} - m)L$$

$$H(mL) = 1 - 1 - 1 - 1 - (m^{-1} - m)L$$
Contents

- Ladder Type Filter Design
Ladder-Type Filter

Configuration (Resonator-Based Constant K Filter)

- Fabrication of Multiple Resonators on a Chip
- Low Loss
- High Power Durability
- Moderate Out-of-Band Rejection
Performance of Ladder-Type SAW Filter

![Graph showing Scattering Parameter $S_{21}$ (dB) vs Frequency (MHz) for W-CDMA-Rx with Fujitsu FAR-F6CP-2G1400-L21M filter.]
Scattering Parameter, $S_{21}$ [dB]

Frequency [GHz]
Scattering Parameter, $S_{21}$ [dB]

Frequency [GHz]

-3.0
-2.0
-1.0
0.0
0.9
0.95
1.0
1.05
1.1

$\omega_a^p$

$\omega_r^p$

$Y_f$

Diagram of a circuit component.
Frequency [GHz]

Scattering Parameter, $S_{21}$ [dB]

$\omega_r^s = \omega_a^p$

$\omega_r^p$

$\omega_a^s$
**Resonator Model for \(Y_p\) & \(Y_s\)**

\[
Y = j\omega C_0 \frac{(j\omega/\omega_a)^2 + 1 + (j\omega/\omega_a)/Q_r}{(j\omega/\omega_r)^2 + 1 + (j\omega/\omega_r)/Q_r}
\]

**Assumption 1.** \(\gamma\) & \(Q_r\) Identical for \(Z_s\) & \(Z_p\)

2. \(\omega r^s = \omega a^p\)

\[
Z_i = \sqrt{Z_s (Z_s + Z_p)}
\]

\[
Y_o = Z_o^{-1} = \sqrt{Y_p (Y_p + Y_s)}
\]

\[
\phi = \sinh^{-1}\left(\sqrt{Y_p / Y_s}\right)
\]

Passband When \(-1 < Y_p/Y_s < 0\) (Either \(Y_s\) or \(Y_p\) Inductive)
Relative admittance

\[ \frac{Y_p}{Y_s} \] offers wider bandwidth when\[-1 < \frac{Y_p}{Y_s} < 0\] (either \(Y_s\) or \(Y_p\) inductive).

Passband when\[-1 < \frac{Y_p}{Y_s} < 0\](Either \(Y_s\) or \(Y_p\) inductive)

Larger \(Y_s\) offers wide bandwidth.
$$S_{21} = \begin{cases} \frac{2}{(\xi + \xi^{-1}) \cosh M\varphi + (Z_i\xi^{-1} / R_0 + R_0\xi / Z_i) \sinh M\varphi} \\ \frac{2}{2 \cosh M\varphi + (Z_i / R_0 + R_0 / Z_i) \sinh M\varphi} \end{cases}$$

(M : odd)

(M : even)

At the Resonance ($\varphi = 0$)

$$S_{21}|_{\omega = \omega_r^s} \approx \frac{2}{2 + N (Z_s / R_0 + R_0 / Z_p)}$$

where $\eta = R_0 \omega \sqrt{C_0^s C_0^p}$

$$r = \sqrt{C_0^p / C_0^s}$$

$$M = Q_r / \gamma$$ (FOM)

**Loss Minimum Condition** $\eta = 1$

$$S_{21}|_{\omega = \omega_r^s, \eta = 1} \approx \frac{1}{1 + NrM^{-1}}$$

*Low Loss When $NrM^{-1} \ll 1$*
Bandwidth

\[
\frac{2\delta \omega}{\omega_r^s} \approx \sqrt{1 + \frac{1}{\sqrt{\gamma(1+\gamma)(1+r^2)}}} - \sqrt{1 - \frac{1}{\sqrt{\gamma(1+\gamma)(1+r^2)}}} \\
\approx \frac{1}{\sqrt{\gamma(1+\gamma)(1+r^2)}} \approx \frac{1}{\gamma \sqrt{1+r^2}} \quad \text{where} \quad r = \sqrt{\frac{C_0^p}{C_0^s}}
\]

Out-of-Band Rejection

When \( \eta=1, \quad S_{21} \approx \frac{1}{\cosh(N\varphi)} = \frac{1}{T_N\left(\sqrt{r^2 + 1}\right)} \)

where \( T_N(x) = \cosh(N \cosh^{-1} x) \): Chebyshev Polynomial

\( N \) Influences IL and OoB Rejection

\( r \) Influences IL, Bandwidth and OoB Rejection
Scattering Parameter, $S_{21}$ [dB]

Frequency [GHz]

$N=5$

$r=0.2$

$r=0.4$

$r=0.6$

$r=0.8$

$r=1.0$
Scattering Parameter, $S_{21}$ [dB]

Frequency [GHz]

$N=5$

$r=0.2$
$r=0.4$
$r=0.6$
$r=0.8$
$r=1.0$
Scattering Parameter, $S_{21}$ [dB]

$N=5$

$r=0.4$

Frequency [GHz]

- $\delta=0$ MHz
- $\delta=2$ MHz
- $\delta=4$ MHz
- $\delta=6$ MHz
- $\delta=8$ MHz
Scattering Parameter, $S_{21}$ [dB]

- $N=5$
- $r=0.4$
- $d=2\text{MHz}$

- $Q_p=1000, Q_s=1000$
- $Q_p=1000, Q_s=\infty$
- $Q_p=\infty, Q_s=1000$
- $Q_p=Q_s=\infty$
Influence of Common Impedance

Equivalent Circuit

SAW Device Chip

Origin of $L_c$
$r = 0.4$

$\delta = 2 \text{MHz}$

$N = 5$
$r = 0.4$

$\delta = 2\text{MHz}$

$N = 5$

Scattering Parameter, $S_{21}$ [dB]
Design of Ladder-type Filters

• Resonator $\gamma$ Limits Achievable Band Width
• For IL Minimization, $\omega_r^2 C_0^p C_0^s R_S^2 \approx 1$
• For Wide bandwidth, $\omega_r^s$ should be a little larger than $\omega_a^p$
• With smaller $r=\sqrt{C_0^p/C_0^s}$, IL and Band Width Improved but OoB Rejection Deteriorated.
• With $N$, OoB Rejection Improved but IL Increases
• IL is Very Sensitive to Motional Resistance of $Y_s$
• Very Sensitive to Parasitics
• $Q$ Influences IL and Shoulder Characteristics
Contents

- Lattice Filter Design
Modified Equivalent Circuit

Equivalent to 2-Stage Cascaded $L$-type (or $\pi$-type) Filter

Res. Freq. for Series Arm

Anti-Res. Freq. for Parallel Arm

\[ Y_e = \infty, \quad Y_o = 0 \]

\[ \text{or} \]

\[ Y_o = \infty, \quad Y_e = 0 \]
Resonance Condition

Res. Freq. for $Y_o$
\[
| \quad | 
\]
Anti-Res. Freq. for $Y_e$

Res. Freq. for $Y_e$
\[
| \quad | 
\]
Anti-Res. Freq. for $Y_o$

\[
S_{21} = \frac{R_0(Y_o - Y_e)}{(1 + Y_e R_0)(1 + Y_o R_0)}
\]

At Resonance
\[
S_{21}\big|_{\omega = \omega_r} \approx \frac{\eta(M^{-1} - M)}{\eta^2 + \eta(M^{-1} + M) + 1}
\]

Where $M = Q_r / \gamma$, $\eta = \omega C_0 R_0$

**IL Min. Condition** $\eta = 1$
\[
S_{21}\big|_{\omega = \omega_r} \approx \frac{1 - M}{1 + M}
\]
Frequency Response

Good OoB Rejection
Wideband When $\omega_r^o > \omega_a^e$

$S_{21}(\text{dB})$

Relative frequency

$\Delta=0$
$\Delta=0.2\%$
$\Delta=0.4\%$
$\Delta=0.6\%$
Better OoB Rejection When $C_0^o > C_0^e$

Null Generation Near Passband
Relative admittance

\[ S_{21} = \frac{R_0 (Y_o - Y_e)}{(1 + Y_e R_0)(1 + Y_o R_0)} \]

Null Generation When

\[ Y_e = Y_o \]
Influence of Mutual Coupling

Very Small Values but May Not Negligible
Influence of Inductive Coupling

(a) $C_{c}=0.5\text{fF}$

(b) $C_{c}=0.5\text{fF}$, $L_{34}=0.5\text{pH}$

Tiny Coupling Gives Significant Influence
• Basic Filter Design
• Ladder Type Filter Design
• Lattice Filter Design
• Stacked Resonator Filter Design
Stacked FBAR Filter

Electrodes

- Good OoB Rejection
- Moderate IL

Equivalent Circuit for Fundamental Resonance

Static Capacitance When Parallel-Connected

1-Port Resonator When Parallel-Connected with Inversion
Frequency Response of Stacked Resonator Filter

From TFR Technologies
\[
\begin{align*}
\mathbf{S}_{21} &= \frac{2}{(1 + j\omega C_0 R_0) \{ZR_0^{-1} (1 + j\omega C_0 R_0) + 2\}} \\
\text{Ideal Case (} R_m = 0), \quad 2 + ZR_0^{-1} (1 + j\omega C_0 R_0) &= 2/(1 - j\omega C_0 R_0) \\
\text{At } \omega_0 \text{ Satisfying } Z &= \frac{2j\omega_0 C_0 R_0^2}{1 + (\omega_0 C_0 R_0)^2} \\
\mathbf{S}_{21} &= \frac{1 - j\omega_0 C_0 R_0}{1 + j\omega_0 C_0 R_0} \quad |\mathbf{S}_{21}| = 1 \quad (\text{Loss Less})
\end{align*}
\]
\[ Z \approx 2j(\omega - \omega_0)L + R_m + \frac{2j\omega_0C_0R_0^2}{1 + (\omega_0C_0R_0)^2} \quad \text{Gives} \]

\[ S_{21} \approx \frac{G_0}{(G_0 + j\omega C_0)^2[j(\omega - \omega_0)L + R_m/2 + G_0/\{G_0^2 + (\omega C_0)^2\}]} \]

\[ |S_{21}|_{\omega=\omega_0} \approx \frac{1}{R_m \{1 + (\omega_0C_0R_0)^2\}/2R_0 + 1} \]

**IL Min. Condition** \( \omega_0C_0R_0 = 1 \)

\[ |S_{21}|_{\omega=\omega_0} \approx \frac{1}{1 + \omega_0C_0R_m} = \frac{1}{1 + M^{-1}} \]

*Determined by M*

\[ \gamma = \frac{C_0}{C_m} \]
\[ Q = \frac{1}{\omega_0C_mR_m} = \frac{\omega_0L_m}{R_m} \]
\[ M = \frac{Q}{\gamma} \]
When $\omega_0C_0 = G_0$ 

$$S_{21} \approx \frac{1}{j[2j\gamma(\omega - \omega_0)/\omega_0 + M^{-1} + 1]}$$

When $\omega = \omega_\pm = \omega_0[1 \pm \gamma^{-1}(1 + M^{-1})/2]$

$$\left|S_{21}\right| \approx \frac{1}{\sqrt{2}} \frac{1}{[1 + M^{-1}]}$$

-3dB Bandwidth

$$\frac{\omega_+ - \omega_-}{\omega_0} = \gamma^{-1}(1 + M^{-1})$$

$$\left|S_{21}\right|_{\omega = \omega_0} \cdot \frac{\omega_+ - \omega_-}{\omega_0} \approx \gamma^{-1}$$

\[\begin{align*}
\gamma &= \frac{C_0}{C_m} \\
Q &= \frac{1}{\omega_0C_mR_m} = \frac{\omega_0L_m}{R_m} \\
M &= \frac{Q}{\gamma}
\end{align*}\]

Determined by $M$ and $\gamma$
Response of Stacked Resonator Filter

ZnO ($L=1.6 \, \mu m$, $S=160 \times 160 \, \mu m^2$)

$S$ is Set to $Z$-Match at 2GHz
Cascaded Stacked Resonator Filters

\[ \mathbf{F}^N = \begin{pmatrix} \cos \theta & jR_e \sin \theta \\ jR_e^{-1} \sin \theta & \cos \theta \end{pmatrix}^N \]

where

\[
\cos \theta = 1 + j\omega C_0 Z \\
jR_e \sin \theta = Z \\
jR_e^{-1} \sin \theta = j\omega C_0 (2 + j\omega C_0 Z) \\
Z = j\omega L_m + R_m + 1/j\omega C_m
\]
Widened Passband
Steeper Skirt Characteristics

\[ |S_{21}| \text{ [dB]} \]

ZnO \((L=1.6 \mu m, S=160 \times 160 \mu m^2)\)

Widened Passband
Steeper Skirt Characteristics
Contents

• Coupled Resonator Filter Design
Uncoupled State

\[ \omega_r = \left( \frac{k}{M} \right)^{0.5} \]

Coupled State

\[ \omega_{rs} = \left( \frac{k+2k_c}{M} \right)^{0.5} \]
\[ \omega_{ra} = \left( \frac{k}{M} \right)^{0.5} \]
Coupled Resonator

\[ M \frac{dv_1}{dt} + \eta v_1 + k \int v_1 dt + k_c \int (v_1 + v_2) dt \propto V_1 \]

\[ M \frac{dv_2}{dt} + \eta v_2 + k \int v_2 dt + k_c \int (v_1 + v_2) dt \propto V_2 \]

(a) Electric + Mechanical Circuit
(a) Electric + Mechanical Circuit

(b) Mechanical Circuit
(a) Symmetric Resonance

(b) Antisymmetric Resonance
Transversally Coupled Double-Mode Filter

Electrodes

Symmetrical & Antisymmetrical Resonances

1-Port Resonator When Parallel-Connected

Another 1-Port Resonator When Parallel-Connected with Inversion
Equation:

\[ 2Y_e = 2(Y_{me} + j\omega C_0) \]

\[ 2Y_o = 2(Y_{mo} + j\omega C_0) \]

Variables:
- \( Y_{me} \): Motional admittance
- \( Y_{mo} \): Motional admittance
- \( j\omega C_0 \): Static capacitance

Diagram:

Equivalent Circuit of Double-Mode Filter

Equivalent to Lattice Filter
Design Principle of Wideband DMS Filters

Coincidence of Multiple Resonance $\omega$ for $Y_e$ (or $Y_o$) with Antiresonance $\omega$ for $Y_o$ (or $Y_e$)

Employing More Resonances Results in More Bandwidths
Double Mode SAW (DMS) Filter

Symmetrical & Anti-symmetrical Resonances

Fujitsu FAR-F5EB-942M50-B28E
ZnO ($L=1.6 \ \mu\text{m}$, $S=220 \times 220 \ \mu\text{m}^2$)

Coupler $R=2\text{GRayls}$ (Very hard), $0.18\lambda$

How to Realize?
Cascaded CRF

ZnO ($L=1.6 \, \mu m$, $S=220 \times 220 \, \mu m^2$)

Coupler 2 GRayls, 0.18 $\lambda$
Extremely hard

ZnO ($L=1.6 \, \mu m$, $S=160 \times 160 \, \mu m^2$)

Coupler 1 MRayls, 0.1 $\lambda$
Extremely soft
Design of Coupled Resonator Filters

• Band Width Governed by $\gamma$
• For Loss Minimization, $\omega_r C_0 R_s \approx 1$
• Coupling Layer Should be Designed to Adjust Multiple Poles to Coincide with Nulls (But Very Critical!)
• Slight Difference Between Poles & Nulls Effective
• Cascade-Connection is Effective
• IL is Sensitive to Motional Resistances & Parasitics