Introduction to Surface Acoustic Wave (SAW) Devices

Part 5: Coupling of Modes Theory

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• Colinear Coupling
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Contents

• Colinear Coupling
**Coupling-Of-Modes (COM) Theory**

**Normal Mode Equation**

\[
\frac{\partial u_1}{\partial X} = -j \beta_u u_1 \\
\frac{\partial u_2}{\partial X} = -j \beta_u u_2
\]

[Solution: \( u_i \propto \exp(-j\beta_u X) \)]

\( \beta_u \): Wavevector at Uncoupled State

**Coupling of Modes Equation**

\[
\frac{\partial u_1}{\partial X} = -j \beta_u u_1 - j\kappa u_2 \\
\frac{\partial u_2}{\partial X} = -j \beta_u u_2 - j\kappa' u_1
\]
Loss-Less Condition (Unitary Condition)

\[
\frac{|u_1(X + \Delta X)|^2 - |u_1(X)|^2 + |u_2(X + \Delta X)|^2 - |u_2(X)|^2}{\Delta X} = 0
\]

\(\Delta X \rightarrow 0\) gives

\[
\frac{\partial}{\partial X} \left[ |u_1(X)|^2 + |u_2(X)|^2 \right] = u_1^* \frac{\partial u_1}{\partial X} + u_1 \left( \frac{\partial u_1}{\partial X} \right)^* + u_2^* \frac{\partial u_2}{\partial X} + u_2 \left( \frac{\partial u_2}{\partial X} \right)^* = 0
\]
Substitution of COM Equations Gives

\[ 2 \text{Im}[\beta_u]\left(|u_1|^2 + |u_2|^2\right) + \text{Im}\left[(\kappa' - \kappa^*)u_1^*u_2\right] = 0 \]

To Satisfy for Arbitrary \( u_1, u_2 \) \& \( X \), \( \text{Im}[\beta_u]=0 \) \& \( \kappa' = \kappa^* \)

**Final COM Equations**

\[
\frac{\partial u_1}{\partial X} = -j \beta_u u_1 - j \kappa u_2
\]

\[
\frac{\partial u_2}{\partial X} = -j \beta_u u_2 - j \kappa^* u_1
\]

When Two Waveguides are Exchangable, \( \kappa \) is Real
General Solution

\[ u_1 = A_+ \exp(-j\beta_+X) + A_- \exp(-j\beta_-X) \]
\[ u_2 = rA_+ \exp(-j\beta_+X) - rA_- \exp(-j\beta_-X) \]

Where \( \beta_\pm = \beta_u \pm |\kappa| \) \( \quad r = |\kappa| / \kappa \)

When \( \kappa \) is Real, Two Partial Waves Correspond to

Red: Symmetric Mode,
Blue: Antisymmetric Mode
Application of Boundary Condition

Boundary Condition

\[ u_1(0) = A_i \quad \& \quad u_2(0) = 0 \]

\[ \Rightarrow A_+ = A_- = A_i / 2 \]

\[ u_1 = A_i \exp(- j \beta u X) \cos( |\kappa| X ) \]

\[ u_2 = - j r A_i \exp(- j \beta u X) \sin( |\kappa| X ) \]
Multi-Strip-Coupler (MSC)

Velocity Difference in Short- & Open-Circuited Gratings

Transversal Filter Using MSC
When two waveguides are not equivalent

\[
\frac{\partial u_1}{\partial X} = -j(\beta_u + \delta)u_1 - j\kappa u_2
\]
\[
\frac{\partial u_2}{\partial X} = -j\kappa u_1 - j\beta_u u_2
\]

\(\kappa: \text{real value}\)

General Solution

\[
u_1 = A_+ \exp(-j\beta_+ X) + A_- \exp(-j\beta_- X)\\
u_2 = r_+ A_+ \exp(-j\beta_+ X) + r_- A_- \exp(-j\beta_- X)\\
\]

where
\[
\beta_\pm = \beta_u + \delta / 2 \pm \Delta
\]
\[
r_\pm = (\delta / 2 \mp \Delta) / \kappa
\]
\[
\Delta = \sqrt{(\delta / 2)^2 + \kappa^2}
\]
Boundary Condition

\[ u_1(0) = A_i, \quad u_2(0) = 0 \]

\[ u_1 = A_i \exp\left\{-j(\beta_u + \delta/2)X\right\} \left\{\cos(\Delta X) - j(\delta/2\Delta)\sin(\Delta X)\right\} \]

\[ u_2 = j(\kappa/\Delta)A_i \exp\left\{-j(\beta_u + \delta/2)X\right\}\sin(\Delta X) \]

\[ \Delta = \sqrt{\left(\frac{\delta}{2}\right)^2 + \kappa^2} \]
Relative wavenumber, $\Delta$

$\beta_+ = \beta_u + \delta / 2 + \Delta$

$\beta = \beta_u + \delta$

$\beta_- = \beta_u + \delta / 2 - \Delta$

$\Delta = \sqrt{(\delta / 2)^2 + \kappa^2}$

Influence of Coupling Obvious Only When $\delta$ is Small

Split Width $\propto$ Coupling Strength
Contents

• Reflective Coupling
Due to Periodicity, Eigen Modes in Infinite Periodic Gratings Satisfy

\[ u_{\pm}(X + p) = u_{\pm}(X) \exp(\mp j\beta_0 p) \]

Where \( \beta_0 \) is Wavenumber of Grating Mode

Define \( u_{\pm}(X) = U_{\pm}(X) \exp(\mp j\beta_0 X) \)

Then We Obtain

\[ U_{\pm}(X + p) = U_{\pm}(X) : \text{Periodic Function} \]
Since $U_{\pm}(X)$ is Periodic Function

$$U_{\pm}(X) = \sum_{n=-\infty}^{+\infty} A_{\pm}^{(n)} \exp(\mp nj \beta_G X)$$

Where $\beta_G = 2\pi / p$: Grating Vector

$A_{\pm}^{(n)}$: Amplitude of $n$-th Partial Wave

$$u_{\pm}(X) = \sum_{n=-\infty}^{+\infty} A_{\pm}^{(n)} \exp(\mp j \beta_n X)$$

Where $\beta_n = \beta_0 + n \beta_G$

*Incident Wave with $\beta$ is Spatially Modulated, and Components with $\beta \pm n \beta_G$ are Generated.*
SAW Dispersion in Periodic Structures

Bragg Reflection
2D Expression of Bragg Reflection

When $|\vec{\beta}_S - \vec{\beta}_G| > |\vec{\beta}_S|

When $|\vec{\beta}_S - \vec{\beta}_G| > |\vec{\beta}_S|$
**Lateral Propagation with Bragg Reflection**

![Diagram](image)

When \( |\beta_s - \beta_G| < |\beta_s| \)
When Two SAWs Coupled through Bragg Reflection
COM Analysis for Periodic Structures

**Eigen Mode Equations** [General Solution: \( u_\pm \propto \exp(\mp j \beta_u X) \)]

\[
\begin{align*}
\frac{\partial u_+}{\partial X} &= -j \beta_u u_+ - j \kappa_{12} u_- \exp(-j \beta_G X) \\
\frac{\partial u_-}{\partial X} &= +j \beta_u u_- + j \kappa_{12}^* u_+ \exp( + j \beta_G X )
\end{align*}
\]

**COM Equations for Forward & Backward Waves**

\( \beta_u \): Wavenumber of Uncoupled Wave

\( \beta_G \): Grating Vector \((2\pi/p)\), \( p \): Periodicity

\( \kappa_{12} \): Mutual Coupling Coefficient

= Reflectivity per Unit Length

For Derivation, Loss Less Condition was Applied
Define $U_{\pm}(X) = u_{\pm}(X)\exp(\pm j\beta_G X/2)$.

Since $u_{\pm}(X) = U_{\pm}(X)\exp(\mp j\beta_G X/2)$, 

$$
\frac{\partial U_+}{\partial X} = - j \theta_u U_+ - j \kappa_{12} U_-
$$

$$
\frac{\partial U_-}{\partial X} = + j \kappa_{12}^* U_+ + j \theta_u U_-
$$

where $\theta_u = \beta_u - \beta_G/2$ : Detuning Factor

($\theta_u=0$ corresponds to Bragg Condition)

**Origin of Phase in $\kappa_{12}$**

**Displacement of Reflection Center from Origin**

$$
d_r/p_I = \angle(\kappa_{12})/4\pi
$$
General Solution

\[ U_+(X) = A_+ \exp(-j \theta_p X) + \Gamma_- A_- \exp(+j \theta_p X) \]
\[ U_-(X) = \Gamma_+ A_+ \exp(-j \theta_p X) + A_- \exp(+j \theta_p X) \]

\[ \beta_p = \theta_p + \pi/p: \text{ Wavenumber of } \text{Perturbed} \ \text{Wave} \]

\[ \theta_p = \sqrt{\theta_u^2 - |\kappa_{12}|^2} \]

\[ \Gamma_+ = (\theta_p - \theta_u)/\kappa_{12} \ \& \ \Gamma_- = (\theta_p - \theta_u)/\kappa_{12}^* : \text{ Reflection Coefficient of Semi-Infinite Grating Looking toward } \pm X \ \text{direction} \]

\[ \Rightarrow \kappa_{12} \text{ is Real When Grating is Symmetric} \]

\[ A_\pm: \text{ Amplitude of Partial Wave} \]
**Behavior Near Bragg Frequency**

\[
\theta_p = \sqrt{\theta_u^2 - |\kappa_{12}|^2}
\]

\[|\kappa_{12}| \text{ determines Both Stopband Width & Attenuation Constant}\]
Reflection Coefficient in dB

Reflection Phase in deg.

Relative frequency, $\theta_u p/\pi$

$|\kappa_{12}|p=0.02\pi$
(a) $\Gamma_+ = \frac{A_{\text{ref}}}{A_{\text{in}}}$

(b) $\Gamma_- = \frac{A_{\text{ref}}}{A_{\text{in}}}$
Application of Boundary Condition

\[ U_+ (X) = A_+ \exp(-j \theta_p X) + \Gamma_- A_- \exp(+j \theta_p X) \]
\[ U_- (X) = \Gamma_+ A_+ \exp(-j \theta_p X) + A_- \exp(+j \theta_p X) \]

Since \( U_+(0)=A_{in} \) & \( U_-(L)=0 \),

\[ \Gamma = \frac{A_r}{A_{in}} = \frac{\Gamma_+[1 - \exp(-2j \theta_p L)]}{1 - \Gamma_+ \Gamma_- \exp(-2j \theta_p L)} \]
\[ T = \frac{A_t}{A_{in}} = \frac{\exp(-j \theta_p L)(1 - \Gamma_+ \Gamma_-)}{1 - \Gamma_+ \Gamma_- \exp(-2j \theta_p L)} \]
Reflection Coefficient in dB

Relative frequency, $\theta u p / \pi$

Reflection Phase in deg.

$|\kappa_{12}| p = 0.02 \pi$
(a) Stopband (evanescent field)

(b) Passband (standing wave field due to reflection at edges)
Reflection Coefficient in dB

$L=10p$

$L=30p$

$L=50p$

$|\kappa_{12}|p=0.02\pi$
Contents

- IDT Modeling
COM Equation for SAW Devices

\[ \frac{\partial u_+}{\partial X} = -j\beta_u u_+ - j\kappa_{12} u_- \exp(-j\beta_G X) + j\zeta V_0 \exp(-j\beta_G X / 2) \]

\[ \frac{\partial u_-}{\partial X} = +j\kappa^*_{12} u_+ \exp(j\beta_G X) + j\beta_u u_- - j\zeta^* V_0 \exp(j\beta_G X / 2) \]

\( \zeta \): Transduction Coefficient

\( p_1 (=2p) \): IDT Periodicity

Spatial Components with \( \pm\beta_G / 2 (=\pm2\pi/p_I) \) are Considered
Equation for Current on Bus-Bar

\[ \frac{\partial I}{\partial X} = -j \chi \zeta^* u_+ \exp(+j \beta_G X/2) - j \chi \zeta u_- \exp(-j \beta_G X/2) + j \omega CV_0 \]

C: Static Capacitance per Unit Length

\( \chi = 2 \) for RMS \( I,V \& u \)

\( \chi = 4 \) for peak \( I,V \& RMS \ u \)

Spatial Components with \( \pm \beta_G/2(= \pm 2 \pi/p_I) \) are Considered

For Derivation, Loss Less Condition & Bidirectionality (When Mechanical Reflection is Zero) are Applied
Final COM Equations

\[ \frac{\partial u_+}{\partial X} = -j \theta_u u_+ - j \kappa_{12} u_- \exp(-j \beta_G X) + j \zeta V_0 \exp(-j \beta_G X / 2) \]

\[ \frac{\partial u_-}{\partial X} = j \kappa^*_{12} u_+ \exp(+j \beta_G X) + j \theta_u u_- - j \zeta^* V_0 \exp(+j \beta_G X / 2) \]

\[ \frac{\partial I}{\partial X} = -j \chi \zeta^* u_+ \exp(+j \beta_G X / 2) - j \chi \zeta u_- \exp(-j \beta_G X / 2) + j \omega CV_0 \]

Define \( U_\pm(X) = u_\pm(X) \exp(\pm j \beta_G X / 2) \). Then

Since \( u_\pm(X) = U_\pm(X) \exp(\mp j \beta_G X / 2) \),

\[ \frac{\partial U_+}{\partial X} = -j \theta_u U_+ - j \kappa_{12} U_- + j \zeta V_0 \]

\[ \frac{\partial U_-}{\partial X} = +j \kappa^*_{12} U_+ + j \theta_u U_- - j \zeta^* V_0 \]

\[ \frac{\partial I}{\partial X} = -j \chi \zeta^* U_+ - j \chi \zeta U_- + j \omega CV_0 \]
General Solution

\[ U_+(X) = A_+ \exp(-j \theta_p X) + \Gamma_- A_- \exp (+j \theta_p X) + \xi_+ V_0 \]
\[ U_-(X) = \Gamma_+ A_+ \exp(-j \theta_p X) + A_- \exp (+j \theta_p X) + \xi_- V_0 \]

Where \( \xi_+ = (\zeta \theta_u - \zeta^* \kappa_{12}) / \theta_p^2 \) & \( \xi_- = (\zeta^* \theta_u - \zeta \kappa_{12}^*) / \theta_p^2 \):

Excitation Efficiency toward \( \pm X \) Direction

Origin of Phase in \( \zeta \)

Displacement of Excitation Center from Origin

\[ d_t/p_I = \angle(\zeta)/2\pi \]
Short Circuited (SC) Grating

Since $V_0 = 0$, 

$$
\frac{\partial U_+}{\partial X} = - j \theta_u U_+ - j \kappa_{12} U_-
$$

$$
\frac{\partial U_-}{\partial X} = + j \kappa_{12}^* U_+ + j \theta_u U_-
$$

$$
\theta_p = \sqrt{\theta_u^2 - |\kappa_{12}|^2}
$$

Open Circuited (OC) Grating

Since $\delta I = 0$, 

$$
\frac{\partial U_+}{\partial X} = - j \hat{\theta}_u U_+ - j \hat{\kappa}_{12} U_-
$$

$$
\frac{\partial U_-}{\partial X} = + j \hat{\kappa}_{12}^* U_+ + j \hat{\theta}_u U_-
$$

$$
\hat{\theta}_p = \sqrt{\hat{\theta}_u^2 - |\hat{\kappa}_{12}|^2}
$$

where

$$
\hat{\theta}_u = \theta_u - \chi |\zeta|^2 / \omega C
$$

$$
\hat{\kappa}_{12} = \kappa_{12} - \chi \zeta^2 / \omega C
$$
\[ \chi \zeta^2 p_1 / \omega C = 0.08 \]
\[ \kappa_{12} p_1 = 0.04 \pi \]

**Relative frequency, \( \theta_u p / \pi \)**

- \( \text{Re}(\theta_u p / \pi) \)
- \( \text{Im}(\theta_u p / \pi) \)
- \( \text{Re}(\theta_p p / \pi) \)
- \( \text{Im}(\theta_p p / \pi) \)
Field Distribution at Stopband with $\beta p=\pi$ (Bidirectional Case)

(a) Symmetric Mode

(b) Antisymmetric Mode
• Colinear Coupling
• Reflective Coupling
• IDT Modeling
• IDT Properties
• SAW Device Simulation
• Parameter Extraction
• BAWs and SH SAWs

• IDT Properties
SAW Excitation by IDT
(When $\zeta$ and $\kappa_{12}$ are Real)

Boundary Conditions: $U_+(-L/2)=0$, $U_+(+L/2)=0$, $I(-L/2)=0$

$$A_+ = A_- = \frac{-\xi_0 V_0}{\exp(+j \theta_p L / 2) + \Gamma_0 \exp(-j \theta_p L / 2)}$$

$$Y = V_0^{-1} \int_{-L/2}^{+L/2} \frac{\partial I(X)}{\partial X} dX = \int_{-L/2}^{+L/2} \left[ -j \chi \xi V_0^{-1} (U_+ + U_-) + j \omega C \right] dX$$

$$\Gamma_\pm = (\theta_p - \theta_u)/\kappa_{12} \equiv \Gamma_0$$

$$\xi_\pm = \xi / (\theta_u + \kappa_{12}) \equiv \xi_0$$
\[ Y = \int_{-L/2}^{+L/2} \left[ -2 j \chi \zeta V_0^{-1} A_+ (1 + \Gamma_0) \cos(\theta_p X) - j(2 \chi \xi_0 \zeta - \omega C) \right] dX \]

\[ = \frac{2 j \chi \xi_0 \zeta (1 + \Gamma_0) L \text{sinc} (\theta_p L / 2)}{\exp(+ j \theta_p L / 2) + \Gamma_0 \exp(- j \theta_p L / 2)} - j(2 \chi \xi_0 \zeta - \omega C) L \]

When \( \kappa_{12} = 0, \ \theta_p = \theta_u, \ \Gamma_0 = 0 \) \& \( \xi_0 = \zeta / \theta_u \). Then

\[ Y = \frac{2 j \chi \zeta^2 L}{\theta_u} \left[ \text{sinc}(\theta_u L) - j \text{sinc}(\theta_u L / 2) \sin(\theta_u L / 2) - 1 \right] + j \omega CL \]

\[ = \chi \zeta^2 L^2 \text{sinc}^2(\theta_u L / 2) + \frac{2 j \chi \zeta^2 L}{\theta_u} \left[ \text{sinc}(\theta_u L) - 1 \right] + j \omega CL \]

**Comparison : Delta Function Model Gives**

\[ Y = \chi (\xi p_I)^2 \frac{\sin^2(\theta_u L / 2) + 2^{-1} j \sin(\theta_u L) - jL / p_I \sin(\theta_u p_I / 2)}{\sin^2(\theta_u p_I / 2)} + j \omega CL \]
Relative admittance

Relative frequency, $\theta_u p_1/2\pi$

$\chi \zeta^2 p_1/\omega C = 0.08$

$\kappa_{12} p_1 = 0$

$L = 10p_1$
Relative admittance

Relative frequency, $\theta_p/2\pi$

$\kappa_{12} p_1 = 0.04 \pi$

$\chi \zeta^2 p_1/\omega C = 0.08$

$L = 10 p_1$
Relative admittance vs. Relative frequency, $\theta_u p_1/2\pi$

- Imag
- Real

$k_{12}p_1 = -0.04\pi$
$\chi\zeta^2 p_1/\omega C = 0.08$
$L = 10p_1$
Relative conductance in dB

Relative frequency, $\theta_u p_1/2\pi$

$L=10p_1$

-30 -25 -20 -15 -10 -5 0 5 10

-0.2 -0.15 -0.1 -0.05 0 0.05 0.1 0.15 0.2

$\kappa_{12} p_1 = 0.16\pi$
$\kappa_{12} p_1 = 0.12\pi$
$\kappa_{12} p_1 = 0.08\pi$
$\kappa_{12} p_1 = 0.04\pi$
$\kappa_{12} p_1 = 0$
Input Admittance for Infinite IDT

Since $\partial U_\pm / \partial X = 0$ & $i = p_I \partial I / \partial X$,

\[
0 = -j \theta_u U_+ - j \kappa_{12} U_- + j \zeta V_0 \\
0 = +j \kappa^*_{12} U_+ + j \theta_u U_- - j \zeta^* V_0 \\
i = -j \chi \zeta^* p_I U_+ - j \chi \zeta p_I U_- + j \omega C p_I V_0
\]

Then

\[
\hat{Y} = \frac{i}{V_0} = -j \chi p_I \frac{2 \theta_u |\zeta|^2 - \kappa_{12} \zeta^* - \kappa_{12}^* \zeta^2}{\theta_u^2 - |\kappa_{12}|^2} + j \omega C p_I
\]

\[
= j \omega C p_I \frac{(\theta_u - \theta^+_\text{oc} ) (\theta_u - \theta^-_\text{sc} )}{(\theta_u - \theta^+_\text{sc} ) (\theta_u - \theta^-_\text{sc} )}
\]

Where $\theta^\pm_{\text{oc}} = \chi |\zeta|^2 / \omega C^\pm |\kappa_{12} - \chi \zeta^2 / \omega C|$, $\theta^\pm_{\text{sc}} = \mp |\kappa_{12}|$
COM Parameter Determination by Input Admittance of Infinite IDT

\[ \hat{Y}(\omega) = j\omega C_{p1} \frac{(\omega - \omega_{oc}^+)(\omega - \omega_{oc}^-)}{(\omega - \omega_{sc}^+)(\omega - \omega_{sc}^-)} \]
$\Imag$ Relative admittance

$\Re$ Relative frequency, $\theta_u p_1/2\pi$

$\chi \zeta^2 p_1/\omega C = 0.08$

$\kappa_{12} p_1 = 0.04\pi$
\[ |\kappa_{12}| p_1 = 0.04\pi \]
\[ \chi \zeta^2 p_1 / \omega C = 0.08 \]
\[ \mu = \angle(\kappa_{12} / \zeta^2) = 90^\circ \]
\[ \text{Re}(\theta_u / p_1) \]
\[ \text{Re}(\theta_p / p_1) \]
\[ \text{Im}(\theta_p / p_1) \]
\[ \text{Im}(\hat{\theta}_p / p_1) \]
$|\kappa_{12}|p_1 = 0.04\pi$

$\chi\zeta^2 p_1 / \omega C = 0.08$

$\mu = 90^\circ$

$L = 10p_1$
Relative admittance

imag
real

Relative frequency, \( \theta \)

\[ |\kappa_{12}|p_1 = 0.04\pi \]
\[ \chi|\zeta|^2p_1/\omega C = 0.08 \]
Relative conductance in dB

\[ L = 10p_I \]

Relative frequency, \( \theta \frac{p_I}{2\pi} \)

\[ |\kappa_{12}|p_I = 0.04\pi \]

\[ \chi|\zeta|^2 p_I / \omega C = 0.08 \]

\[ \mu = \angle \left( \frac{\kappa_{12}}{\zeta^2} \right) \]
Directivity in dB

\[ L = 10 \rho_1 \]

Relative frequency, \( \theta \rho_1 / 2\pi \)

- \( |k_{12}| \rho_1 = 0.04\pi \)
- \( \chi |\zeta|^2 \rho_1 / \omega C = 0.08 \)
- \( L = 10 \rho_1 \)

\( \mu = 0^\circ \)
\( \mu = 30^\circ \)
\( \mu = 60^\circ \)
\( \mu = 90^\circ \)
• SAW Device Simulation
Simulation of Complex Structures

- Grating (Bragg) Reflectors
- Piezoelectric substrate
- Interdigital Transducer (IDT)

• Combination of Periodic Structures
SC Grating = Short-Circuited IDT
OC Grating = IDT with Isolated Fingers
Gap = Reflection-less, Excitation-less IDT

*IDT Modeling* ↔ *Device Modeling*
P-Matrix Expression

\[
\begin{align*}
\begin{pmatrix}
P_{11} & P_{12} & P_{13} \\
P_{21} & P_{22} & P_{23} \\
-\chi & -\chi & \chi
\end{pmatrix}
\begin{pmatrix}
a_1 \\
b_1 \\
I
\end{pmatrix}
&=
\begin{pmatrix}
a_2 \\
b_2 \\
V
\end{pmatrix}
\end{align*}
\]

Unitary Condition:

- \(|P_{11}|^2 + |P_{12}|^2 = 1, |P_{22}|^2 + |P_{12}|^2 = 1\)
- \(p_{11}p_{13}^* + p_{12}p_{23}^* + p_{13} = 0\)
- \(p_{12}p_{13}^* + p_{22}p_{23}^* + p_{23} = 0\)
- \(\frac{\chi}{2} \left[ |P_{11}|^2 + |P_{12}|^2 \right] = \Re(p_{33})\)
Use of COM Model Gives

\[ P_{11} = \frac{\Gamma_- (1 - E^2)}{1 - \Gamma_+ \Gamma_- E^2}, \quad P_{22} = \frac{\Gamma_+ (1 - E^2)}{1 - \Gamma_+ \Gamma_- E^2}, \quad P_{12} = \frac{E (1 - \Gamma_+ \Gamma_-)}{1 - \Gamma_+ \Gamma_- E^2} \]

\[ P_{13} = \frac{(1 - E) \{ \xi_- (1 + \Gamma_+ \Gamma_-) - \xi_+ \Gamma_+ (1 + E) \}}{1 - \Gamma_+ \Gamma_- E^2} \]

\[ P_{23} = \frac{(1 - E) \{ \xi_+ (1 + \Gamma_+ \Gamma_-) - \xi_- \Gamma_- (1 + E) \}}{1 - \Gamma_+ \Gamma_- E^2} \]

\[ P_{33} = \frac{\chi (1 - E) \{ ((\xi_+ - \Gamma_- \xi_- E)(\zeta^* + \Gamma_+ \zeta) + (\xi_- - \Gamma_+ \xi_+ E)(\zeta + \Gamma_- \zeta^*) \}}{1 - \Gamma_+ \Gamma_- E^2} \]

\[ - j\chi L (\zeta^* \xi_+ + \zeta \xi_-) + j\omega CL \]

where \( E = \exp(-j\theta_p L) \)
When the unit is symmetrical,

\[ P_{11} = P_{22} = \frac{\Gamma_0 (1 - E^2)}{1 - \Gamma_0^2 E^2}, \quad P_{12} = \frac{E (1 - \Gamma_0^2)}{1 - \Gamma_0^2 E^2} \]

\[ P_{13} = P_{23} = \frac{\xi (1 - E)(1 - \Gamma_0 E)}{1 + \Gamma_0 E} \]

\[ P_{33} = 2 \chi \xi \zeta \left[ \frac{(1 - E)(1 + \Gamma_0)}{\theta_p (1 + \Gamma_0 E)} - jL \right] + j \omega CL \]
Recursive Relation for Unit A (left) + B (right)

\[ P_{11} = P_{11}^A + P_{11}^B \frac{P_{21}^A P_{12}^A}{1 - P_{11}^B P_{22}^A} , \quad P_{22} = P_{22}^B + P_{22}^A \frac{P_{12}^B P_{21}^B}{1 - P_{11}^B P_{22}^A} , \quad P_{12} = \frac{P_{12}^A P_{12}^B}{1 - P_{11}^B P_{22}^A} \]

\[ P_{13} = P_{13}^A + P_{12}^B \frac{P_{13}^B + P_{11}^A P_{23}^A}{1 - P_{11}^B P_{22}^A} , \quad P_{23} = P_{23}^B + P_{21}^B \frac{P_{23}^A + P_{22}^A P_{13}^B}{1 - P_{11}^B P_{22}^A} \]

\[ P_{33} = P_{33}^A + P_{33}^B + P_{32}^A \frac{P_{13}^B + P_{11}^A P_{23}^A}{1 - P_{11}^B P_{22}^A} + P_{31}^B \frac{P_{23}^A + P_{22}^A P_{13}^B}{1 - P_{11}^B P_{22}^A} \]
• Parameter Extraction
Determination of COM Parameters

\[
\frac{\partial U^+}{\partial X} = -j \theta_u U^+ - j \kappa_{12} U^- + j \zeta V_0
\]

\[
\frac{\partial U^-}{\partial X} = +j \kappa_{12}^* U^+ + j \theta_u U^- - j \zeta^* V_0
\]

\[
\frac{\partial I}{\partial X} = -j \chi \zeta^* U^+ - \chi j \zeta U^- + j \omega C V_0
\]

\( \kappa_{12} \): Mutual Coupling Coefficient (Mostly Constant)

\( \zeta \): Transduction Coefficient  (Mostly Constant)

\( C \): Capacitance  (Mostly Constant)

\( \theta_u \): detuning factor (Linearly Changes with \( \omega \))

\[ \Rightarrow \theta_u = \omega V_{\text{ref}} - \pi/p + \kappa_{11} \]

\( V_{\text{ref}} \): Reference SAW Velocity

\( \kappa_{11} \): Self Coupling Coefficient
Physical Mean of $\kappa_{11}$ and $V_{\text{ref}}$

$$\beta_u \approx \frac{\omega}{V_{\text{ref}}} + \kappa_{11}$$

$$\kappa_{11} = \pi/p - \omega_r/V_{\text{ref}}$$
For Short-Circuited (SC) Grating, $V_0=0$

$$\theta_p = \sqrt{\theta_u^2 - |\kappa_{12}|^2}$$

For Open-Circuited (OC) Grating, $\delta I=0$

$$\theta_p = \sqrt{\left(\theta_u - \chi|\zeta|^2/\omega C\right)^2 - |\kappa_{12} - \chi\zeta^2/\omega C|^2}$$
Since $\chi|\zeta|^2/\omega C \pm |\kappa_{12}|^2/\omega C| = \omega_{oc}^\pm V_{ref} - \pi/p + \kappa_{11}$
& $|\kappa_{12}| = \omega_{sc}^\pm V_{ref} - \pi/p + \kappa_{11}$,

\[
\kappa_{11} = \frac{\pi}{p} - \frac{\omega_{sc}^+ + \omega_{sc}^-}{2V_{ref}}
\]

\[
|\kappa_{12}| = \frac{\omega_{sc}^- - \omega_{sc}^+}{2V_{ref}}
\]

\[
\chi \frac{\zeta^2}{\omega C} = \frac{(\omega_{oc}^+ + \omega_{oc}^-) - (\omega_{sc}^+ + \omega_{sc}^-)}{2V_{ref}}
\]

\[
|\chi \frac{\zeta^2}{\omega C}| = \frac{\omega_{oc}^+ - \omega_{oc}^-}{2V_{ref}}
\]

**Sign of $\psi = \angle (\zeta^2/\kappa_{12})$ can not be Determined**
When IDT is Bidirectional, $\zeta^2/\kappa_{12}$ is Real

One of Stopband Edge for OC Grating
Coincides with that for SC Grating
Relation Between Stopband Edges and COM Parameters

\[ \kappa_{11} = \frac{\pi}{p} - \frac{\omega_{sc}^+ + \omega_{sc}^-}{2V_{\text{ref}}} \]
\[ \kappa_{12} = s \frac{\omega_{sc}^+ - \omega_{sc}^-}{2V_{\text{ref}}} \]
\[ \frac{\chi \zeta^2}{\omega C} = \frac{(\omega_{oc}^+ + \omega_{oc}^-) - (\omega_{sc}^+ + \omega_{sc}^-)}{2V_{\text{ref}}} \]

How to Determine \( V_{\text{ref}} \)?

1. Determination of \( |\kappa_{12}| \) by Max[-Im(\( \theta_p \))]  
2. Determination of \( V_{\text{ref}} \) by Stopband Edges

\[ s = \begin{cases} 
1 & (\omega_{sc}^+ = \omega_{oc}^+) \\
-1 & (\omega_{sc}^- = \omega_{oc}^-) 
\end{cases} \]
FEMSDA (Full Wave Simulator)

Finite Element Analysis
For Arbitrary Electrode Cross-Section (+ Analytic Solution not Available)

SAW Propagation Direction

Spectral Domain Analysis
Flat Substrate Surface
Analytic Solution = Fast Analysis

Boundary Condition: Minimization of Radiated Power (Error) from Boundary
Dispersion of Rayleigh SAW on YZ-LN ($h/p=0.07$) Calculated by **FEMSDA**

$V_B=3,590.1$ m/s (Slow-shear SSBW velocity)
Phase Velocity: \( V_p = \frac{\omega}{\text{Re}(\beta_p)} \)

Attenuation: \( \alpha_p = 40\pi \log_{10} e \times \frac{\text{Im}(-\beta_p)}{\text{Re}(\beta_p)} \) [dB/\( \lambda \)]
Existence of Multiple Solutions

Possibility to Jump into Blue Branch
Most Possible near Stopband Edges

Countermeasure: Attacking Upward and/or Downward
Efficient Calculation by Combining FEMSDA and SYNC

Single-Electrode IDT
1. FEMSDA for determination of $\beta$ for OC & SC
2. Fitting after Squared
3. SYNC for determination of $C$

Double-Electrode IDT
1. MSYNC for calculation of input impedance of infinitely long IDT
2. Determination of $C$ & frequencies giving stopband edges by fitting
3. MULTI for determination of $\beta$ for SC
Wavenumber of Rayleigh SAW on YZ-LN ($h/p=0.07$) Calculated by **FEMSDA**

$V_B=3,590.1 \text{ m/s}$ (Slow-shear SSBW velocity)
Squared Wavenumber of Rayleigh SAW on YZ-LN ($h/p=0.07$) Calculated by FEMSUDA
$V_B=3,590.1$ m/s (Slow-shear SSBW velocity)
Input admittance, $Y/\omega(\infty)W$ vs Relative frequency, $fp/V_{\text{ref}}$

Input Admittance of Infinitely long single-electrode IDT on YZ-LN ($h/p=0.07$) Calculated by *SYNC*

$V_B=4,030.8$ m/s (Slow-shear SSBW velocity)
(A) $\zeta$ with same polarity for input and output IDTs

(B) $\zeta$ with opposite polarity for input and output IDTs
Dispersion of Rayleigh SAW on 128-LN
Blue: Analysis by FEMSDA
Red: Conventional COM Analysis

$V_B=4,025$ m/s (Slow-shear SSBW velocity)
Dispersion Relation vs. Al Thickness

Phase velocity (m/sec) vs. Relative frequency, $f_p/V_B$

Relative Frequency, $f_p/V_B$

Attenuation (dB/λ) vs. Relative Frequency, $f_p/V_B$
**Change in COM Parameters with Al Thickness**

![Graph showing relative COM parameters vs. Al thickness/period.](image)

- $K_u^2 = \pi \chi |\zeta|^2 p_I / 4 \omega C$ : Electromechanical Coupling Factor for *Perturbed Mode*

- $c = V_B / V_{\text{ref}}$ : $V_B = 4,025$ m/s (Slow Shear SSBW)
Correction of Simulation Parameters

• Uncertainties in Substrate Material Constants (Supplier and Lot Dependent)

• Uncertainties in Film Material Constants (Fab. Process Dependent)

• Electrode Cross-Section (Fab. Process Dependent)

Although their Absolute Values may be Doubtful, Dependencies on Device Parameters might be held
Effective Velocity of Rayleigh-type SAW on 128-LN vs. Metallization Ratio

Solid Lines: FEMSDA, +×: Experiment
Reflectivity of Rayleigh-type SAW on 128-LN vs. Metallization Ratio

Solid Lines: FEMSDA,  +×: Experiment
Behavior in Ultimate Situations

(a) When \( w/p \approx 0 \)

(b) When \( w/p \approx 1 \)

\( w/p \rightarrow 1 \) is not Equivalent to Flat Metallization!
Relation of COM Parameters with Resonance Characteristics

Each Parameter Independently Relates Each Property ⇒ Easy to Fit with Experiments
• BAWs and SH SAWs
Excitation and Propagation of BAWs

Propagation of Cylindrical Wave

\( P \propto r^{-1} \Rightarrow u \propto r^{-0.5} \)

Rapid Attenuation When Influence of Surface is Significant
Radiation Pattern of BAWs

Angler Dependence of BAW Power Flow in Far Field

L & SV Do not Satisfy Surface Boundary Condition

Non-Radiative Parallel to Surface?

BAW Radiated to Surface Changes into SAW
Coupling of SH & $\Phi$ Components

$SH+\phi \Rightarrow SH$-Type SAW

Efficient SAW Radiation $\Leftrightarrow$ Suppression of SSBW Radiation
SSBW: Surface Skimming Bulk Wave

BAW Propagating on Surface

IDT → BAW → IDT

Excitation and Propagation: Very Sensitive to Surface Condition

Characterized by Velocity Difference Between SAW and BAW
Frequency Response of BAW Radiation

- Cutoff Nature
- Radiation Peak just above the Cutoff Freq.
$F(\theta) = \sin \theta$

$F(\theta) = 1$

$G_B$ (blue dashed line)

$B_B$ (red dashed line)

$F(\theta)$: $\theta$ Dependence of BAW Radiation
Back-Scattering to BAW

For SH-type SAW,
Cutoff for BAW Back-Scattering ≈ SAW Resonance Frequency
**SAW Bragg Reflection to Bulk Waves**

Phase Matching Condition

\[- \beta_s p - \beta_B p \cos \theta_B = -2n\pi\]

\[\beta_s - 2n\pi / p = -\beta_B \cos \theta_B\]

\[f = \frac{n p^{-1}}{S_s + S_B \cos \theta_B}\]
When $S_S > S_B$

Cutoff Freq. $f_{BC}$ for BAW Back-Scattering

$$f_{BC} = \frac{n}{(S_S + S_B)p} > \frac{n}{2S_S p}$$

SAW Bragg Freq.

BAW Bean Scan with $f$

$$\cos \theta_B = \frac{f_{BC}}{f} \left( S_S S_B^{-1} + 1 \right) - S_S S_B^{-1}$$
At Frequency Below $f_{Bc}$

Evanescent Field via Non-Radiated BAW

Energy Storage (SAW Velocity Reduction) Effect
Dispersion in phase velocity and attenuation of \textit{SH-type SAW} on the SC grating ($h/\lambda=0.1$) on 42-LT. Blue lines: calculated by FEMSDA.
SAW Wavenumber Derived by Conventional COM Theory

\[ \beta_s = \pi / p + \sqrt{\theta_u^2 - \kappa^2} \]

where \( \theta_u \) is detuning factor
(linearly dependent on frequency)
\( \kappa \): mutual coupling factor (const.)

\( p \): grating period
Dispersion in phase velocity and attenuation of Rayleigh-type SAW on gratings on 128-LN. Blue lines: calculated by FEMSDA, and red lines: calculated by conventional COM.
Dispersion in phase velocity & attenuation of *SH-type SAW* on SC grating on 42-LT.

Blue lines: calculated by FEMSDA, and red lines: calculated by conventional COM.
Frequency
Amplitude

SAW Response

BAW Radiation

Backscattered BAW cutoff

Origin of dispersion near stopband

Stored Energy

($\propto$ Velocity Reduction)
Characterisation by Plessky’s Model

- Dispersion Relation

\[ \beta_s = \pi / p + \sqrt{\Delta^2 - \left( \varepsilon_s^2 / 2 + \eta_s \sqrt{\Delta_{Bs} - \Delta} \right)^2} \]

where \( \Delta = c(\omega / V_B - \pi / p) \) : normalised frequency
\( \Delta_{Bs} = \varepsilon_s^2 / 2 - \eta_s^2 / 4 \) : normalised BAW-cutoff frequency

\( \varepsilon_s, \eta_s, c \) : parameters for SC grating (constant)
Phase velocity of *SH-type SAW* for grating structure on 42-LT.

Blue lines: calculated by FEMSDA, and red lines: calculated by Plessky's model.
Attenuation of *SH-type SAW* for grating structure on 42-LT.

Blue lines: calculated by FEMSDA, and red lines: calculated by Plessky's model.
What will Happen When Periodicity Breaks?

Discontinuities

BAW Radiation + Additional Phase Shift (Frequency Dependent)