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Introduction to Surface Acoustic Wave (SAW) Devices

Part 5: Coupling of Modes Theory

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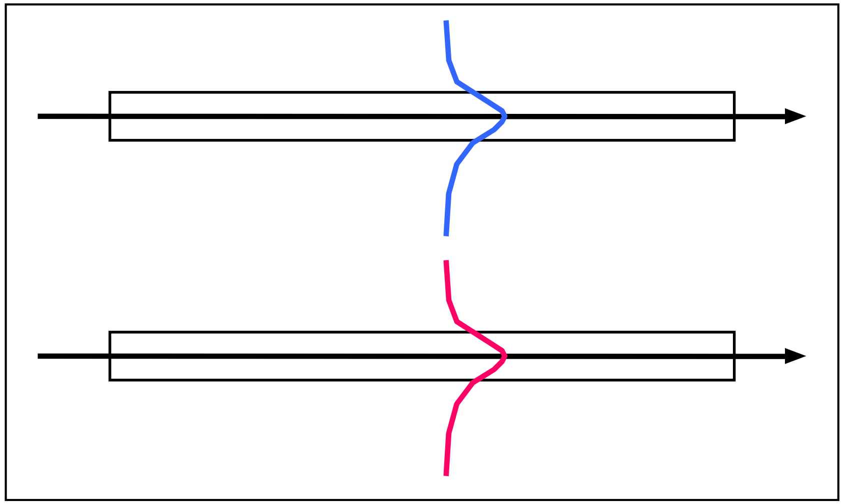
- Colinear Coupling
- Reflective Coupling
- IDT Modeling
- IDT Properties
- SAW Device Simulation
- Parameter Extraction
- BAWs and SH SAWs

Contents

- Colinear Coupling

Coupling-Of-Modes (COM) Theory



Uncoupled

Normal Mode Equation

$$\frac{\partial u_1}{\partial X} = -j\beta_u u_1$$

$$\frac{\partial u_2}{\partial X} = -j\beta_u u_2$$

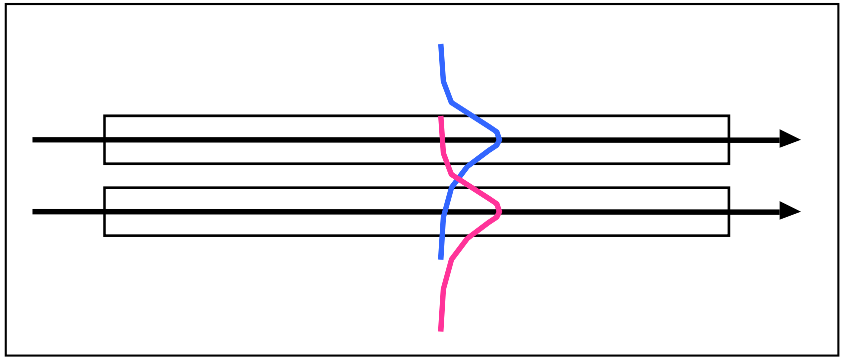
[Solution: $u_i \propto \exp(-j\beta_u X)$**]**

β_u: Wavevector at Uncoupled State

Coupling of Modes Equation

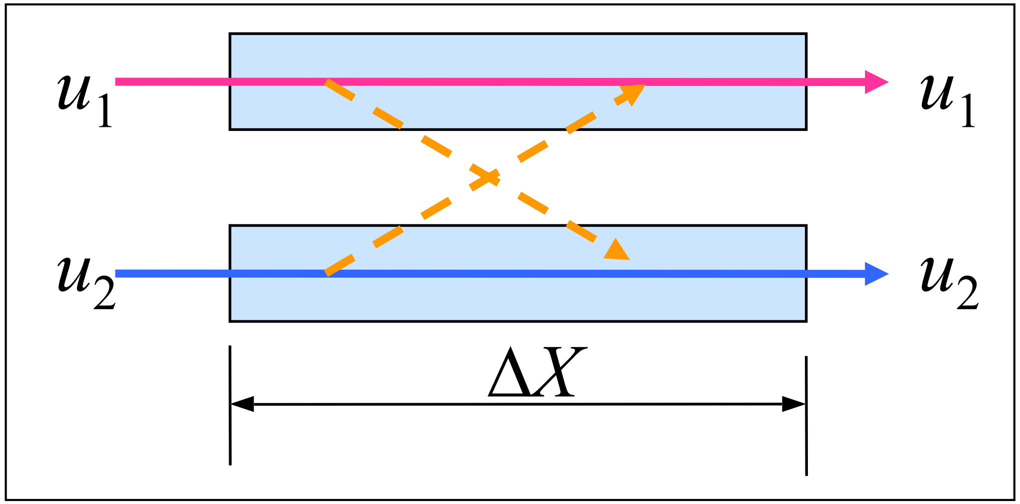
$$\frac{\partial u_1}{\partial X} = -j\beta_u u_1 - j\kappa u_2$$

$$\frac{\partial u_2}{\partial X} = -j\beta_u u_2 - j\kappa' u_1$$



Coupled

Loss-Less Condition (Unitary Condition)



$$\frac{|u_1(X+\Delta X)|^2 - |u_1(X)|^2 + |u_2(X+\Delta X)|^2 - |u_2(X)|^2}{\Delta X} = 0$$

$\Delta X \to 0$ gives

$$\frac{\partial\left[|u_1(X)|^2 + |u_2(X)|^2\right]}{\partial X} = u_1^* \frac{\partial u_1}{\partial X} + u_1\left(\frac{\partial u_1}{\partial X}\right)^* + u_2^* \frac{\partial u_2}{\partial X} + u_2\left(\frac{\partial u_2}{\partial X}\right)^* = 0$$

Substitution of COM Equations Gives

$$2\,\mathrm{Im}[\beta_u]\left(|u_1|^2+|u_2|^2\right)+\mathrm{Im}[(\kappa-\kappa'^*)u_1^*u_2]=0$$

To Satisfy for Arbitrary u_1, u_2 & X, Im[β_u]=0 & $\kappa'=\kappa^*$

Final COM Equations

$$\frac{\partial u_1}{\partial X}=-j\beta_u u_1-j\kappa u_2$$

$$\frac{\partial u_2}{\partial X}=-j\beta_u u_2-j\kappa^* u_1$$

When Two Waveguides are Exchangable, κ is Real

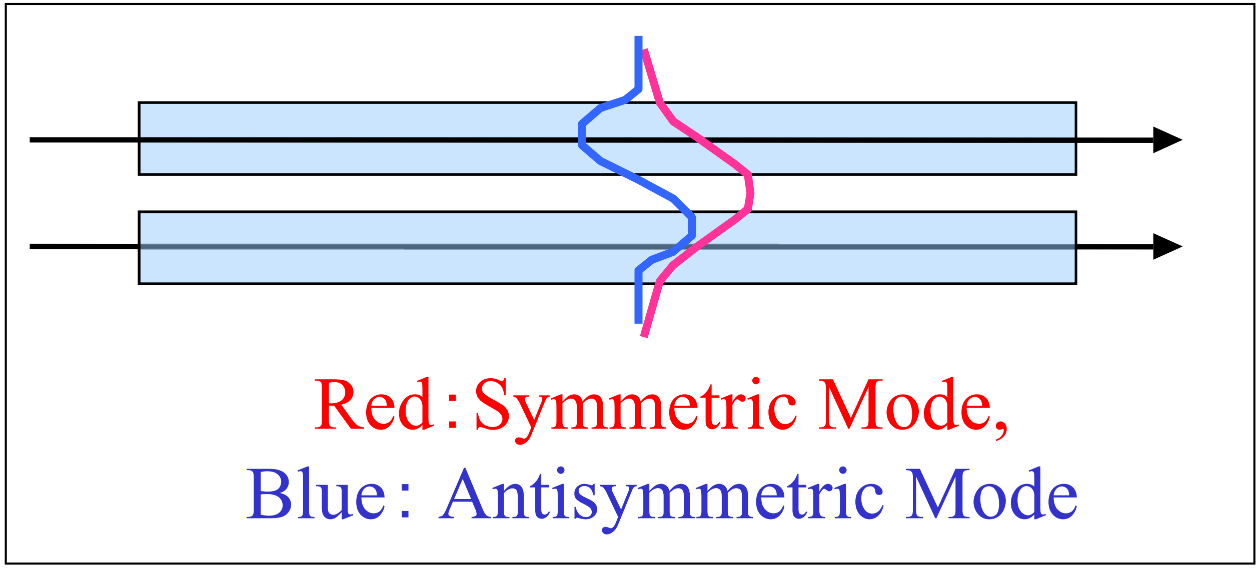
General Solution

$$u_1 = A_+ \exp(-j\beta_+ X) + A_- \exp(-j\beta_- X)$$

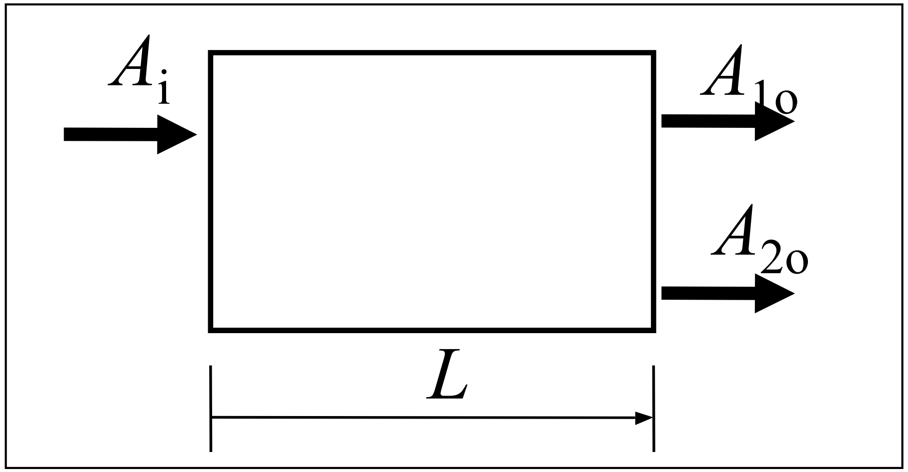
$$u_2 = rA_+ \exp(-j\beta_+ X) - rA_- \exp(-j\beta_- X)$$

Where $\beta_\pm = \beta_u \pm |\kappa| \qquad r = |\kappa| / \kappa$

When κ is Real, Two Partial Waves Correspond to



Application of Boundary Condition



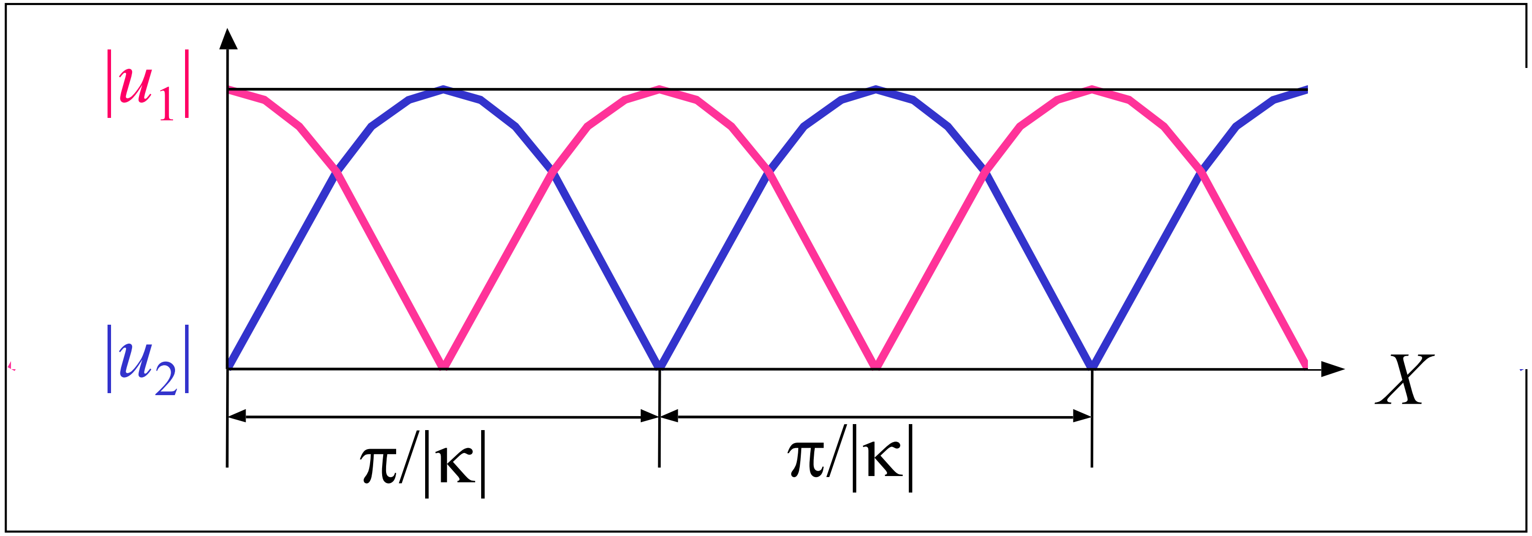
Boundary Condition

$u_1(0)=A_i$ & $u_2(0)=0$

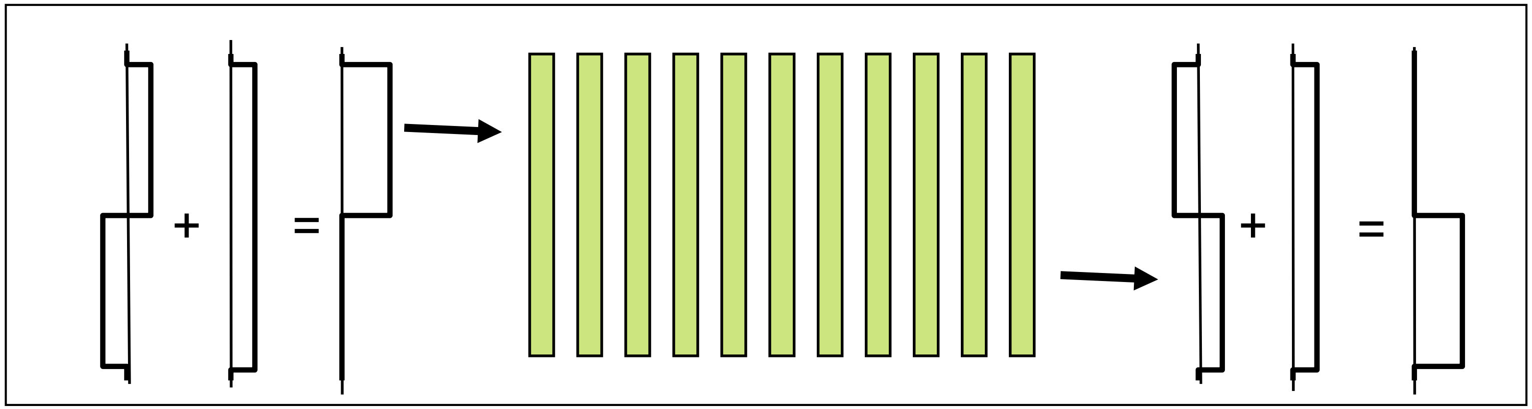
$\Rightarrow A_+=A_-=A_i/2$

$$u_1 = A_i \exp(-j\beta_u X)\cos(|\kappa| X)$$

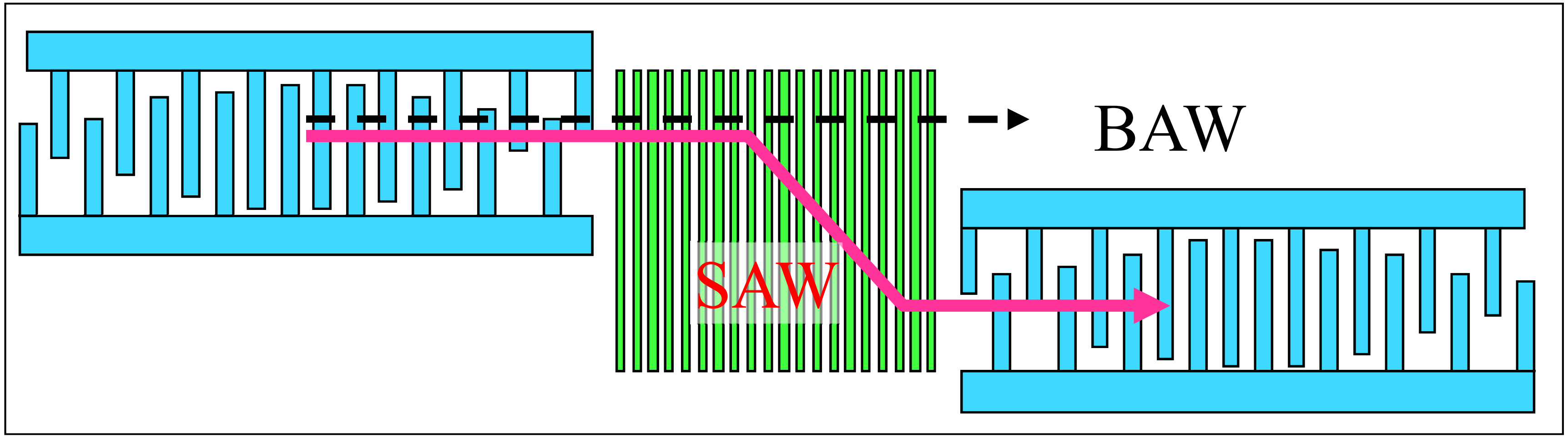
$$u_2 = -jrA_i \exp(-j\beta_u X)\sin(|\kappa| X)$$



Multi-Strip-Coupler (MSC)



Velocity Difference in Short- & Open-Circuited Gratings



Transversal Filter Using MSC

When two waveguides are not equivalent

$$\frac{\partial u_1}{\partial X} = -j(\beta_u + \delta)u_1 - j\kappa u_2$$

κ: real value

$$\frac{\partial u_2}{\partial X} = -j\kappa u_1 - j\beta_u u_2$$

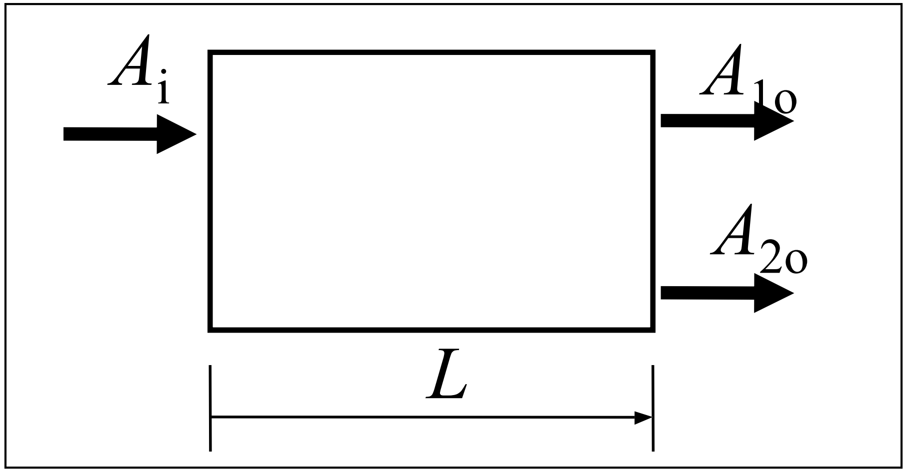
General Solution

$$u_1 = A_+ \exp(-j\beta_+ X) + A_- \exp(-j\beta_- X)$$

$$u_2 = r_+ A_+ \exp(-j\beta_+ X) + r_- A_- \exp(-j\beta_- X)$$

where $\beta_\pm = \beta_u + \delta / 2 \pm \Delta$ $\qquad r_\pm = (\delta / 2 \mp \Delta) / \kappa$

$$\Delta = \sqrt{(\delta / 2)^2 + \kappa^2}$$



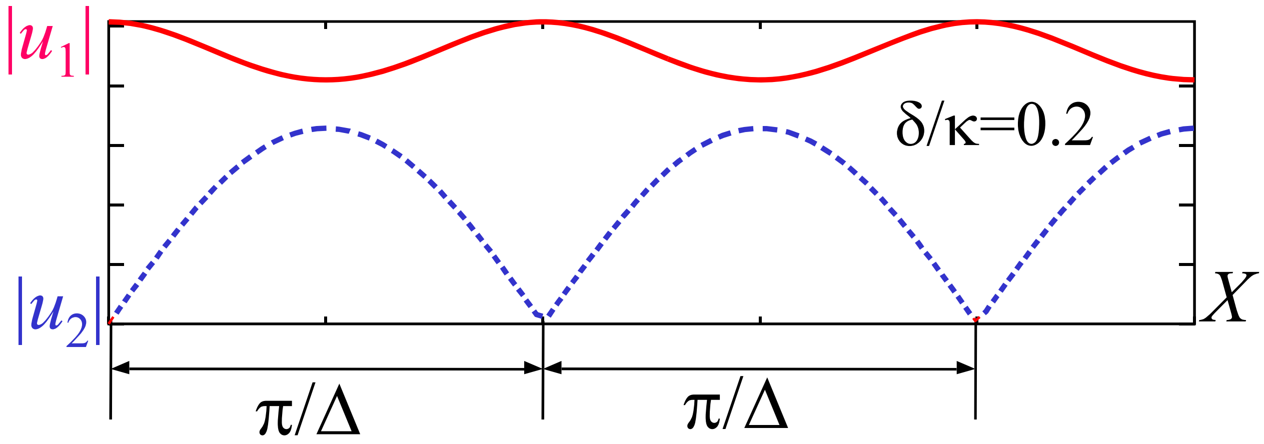
Boundary Condition

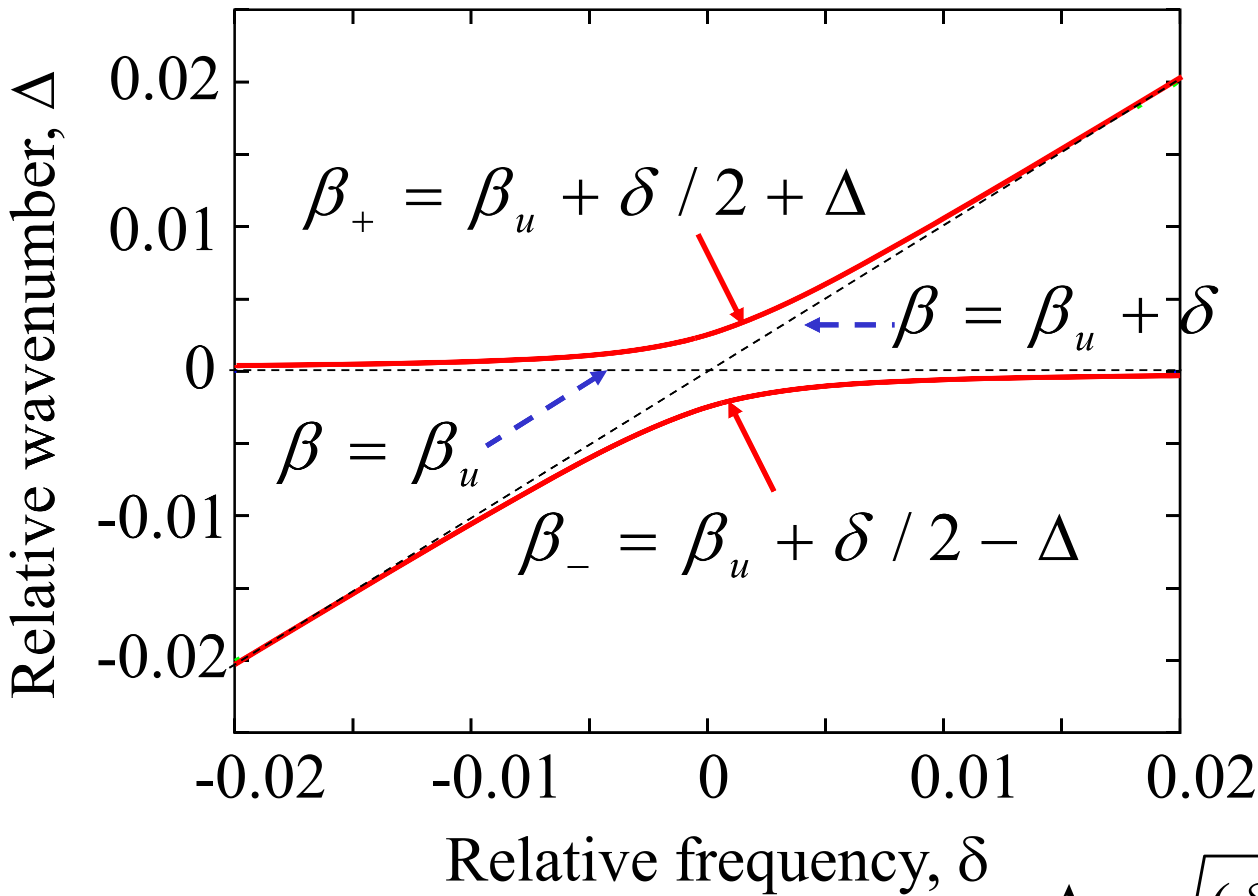
$u_1(0)=A_i$, $u_2(0)=0$

$$u_1 = A_i \exp\{-j(\beta_u + \delta/2)X\}\{\cos(\Delta X) - j(\delta/2\Delta)\sin(\Delta X)\}$$

$$u_2 = j(\kappa/\Delta)A_i \exp\{-j(\beta_u + \delta/2)X\}\sin(\Delta X)$$

$$\Delta = \sqrt{(\delta/2)^2 + \kappa^2}$$





$$\Delta = \sqrt{(\delta / 2)^2 + \kappa^2}$$

Influence of Coupling Obvious Only When δ is Small

Split Width ∝ Coupling Strength

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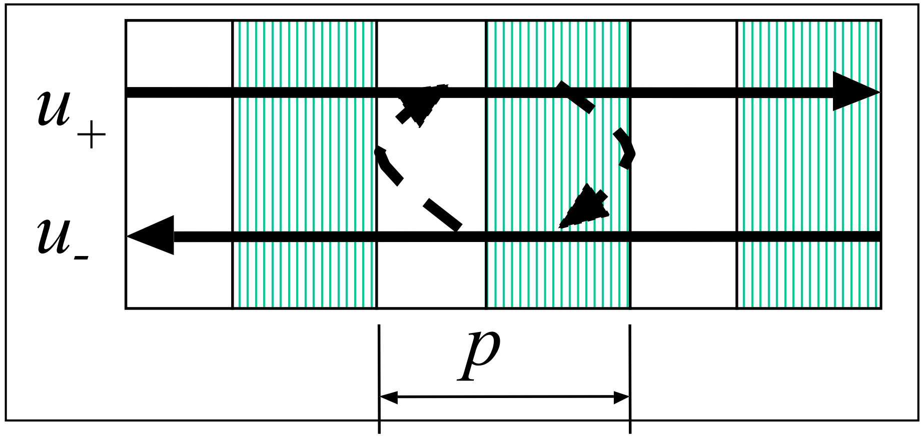
- Reflective Coupling

Floquet Theorem



Due to Periodicity, Eigen Modes in Infinite Periodic Gratings Satisfy

$$u_{\pm}(X+p) = u_{\pm}(X)\exp(\mp j\beta_0 p)$$

Where β_0 is Wavenumber of Grating Mode

Define $u_{\pm}(X) = U_{\pm}(X)\exp(\mp j\beta_0 X)$

Then We Obtain

$$U_{\pm}(X+p) = U_{\pm}(X)$$: Periodic Function

Since $U_{\pm}(X)$ is Periodic Function

$$U_{\pm}(X) = \sum_{n=-\infty}^{+\infty} A_{\pm}^{(n)} \exp(\mp nj\beta_G X)$$

Where $\beta_G=2\pi/p$: Grating Vector

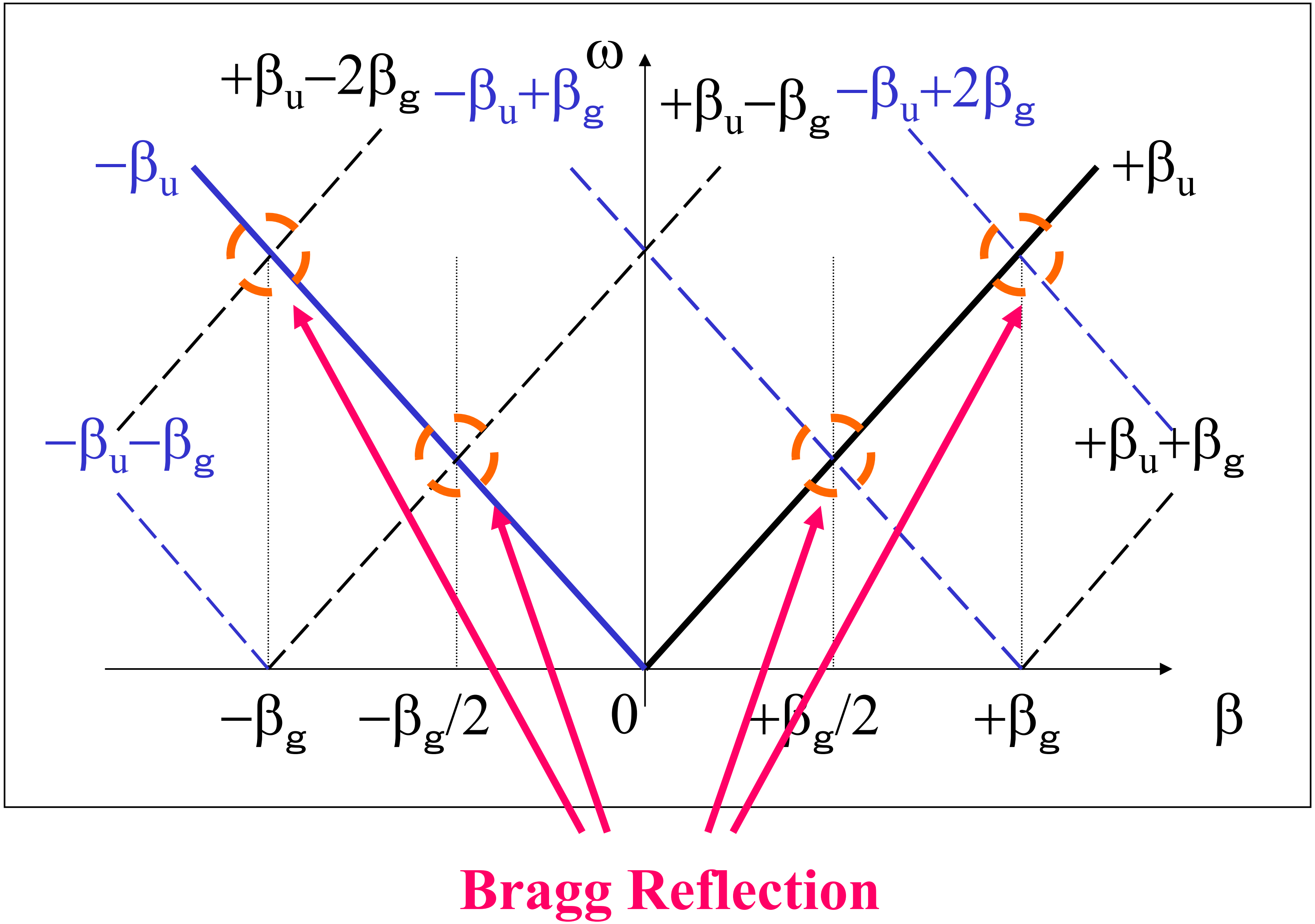
$A_{\pm}^{(n)}$: Amplitude of n-th Partial Wave

$$u_{\pm}(X) = \sum_{n=-\infty}^{+\infty} A_{\pm}^{(n)} \exp(\mp j\beta_n X)$$

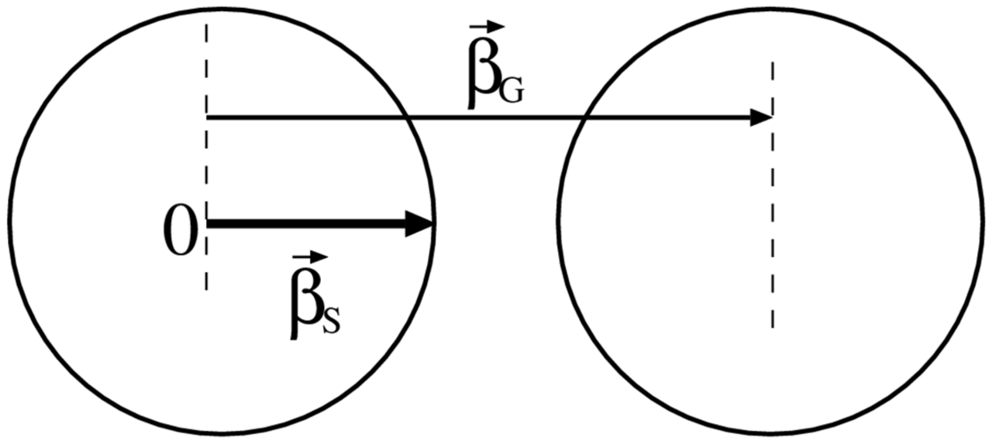
Where $\beta_n=\beta_0+n\beta_G$

Incident Wave with β is Spatially Modulated, and Components with $\beta+n\beta_G$ are Generated.

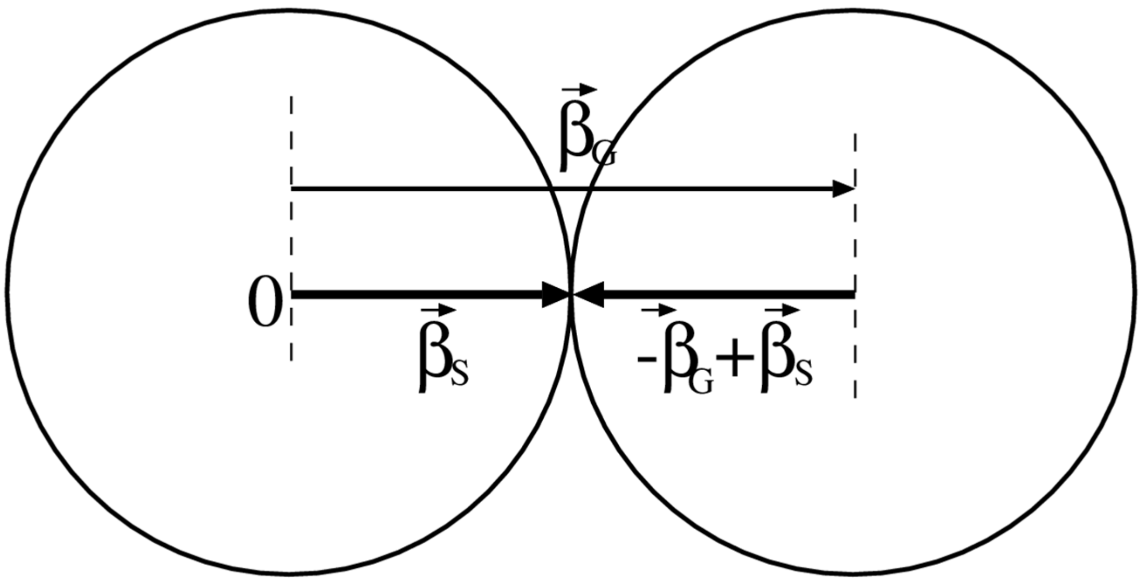
SAW Dispersion in Periodic Structures



2D Expression of Bragg Reflection

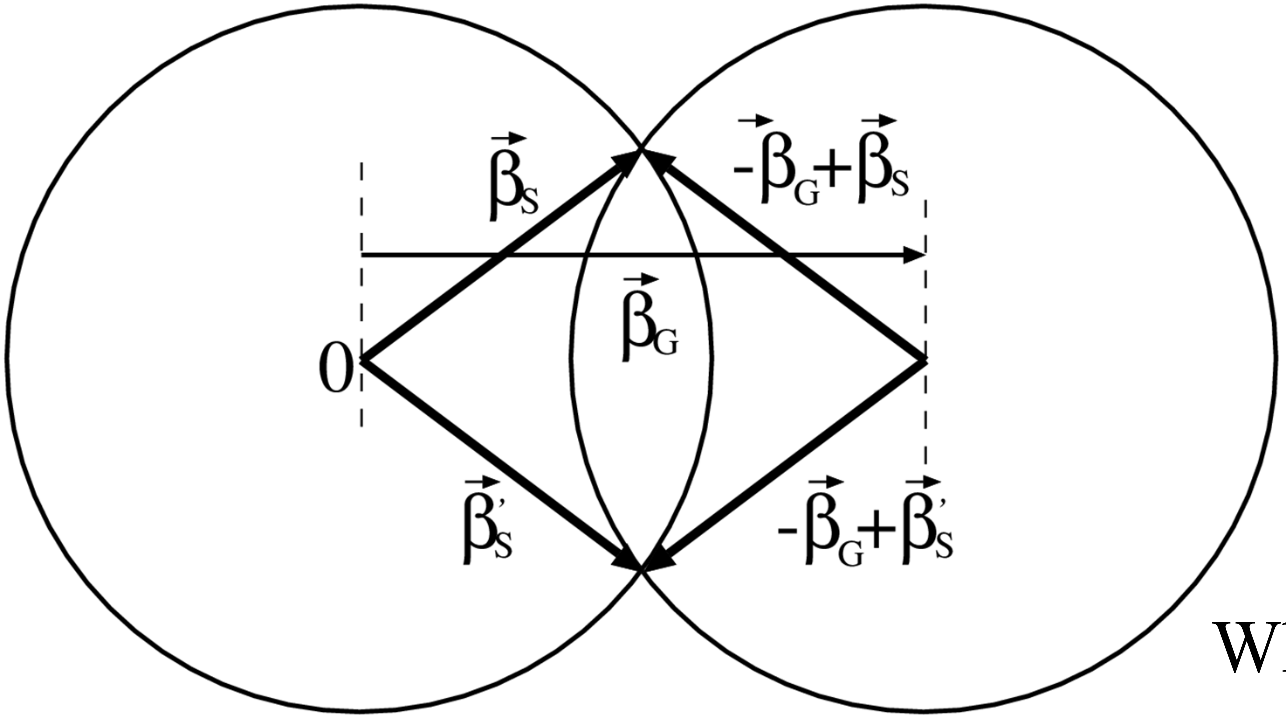


When $\left|\vec{\beta}_S - \vec{\beta}_G\right| > \left|\vec{\beta}_S\right|$



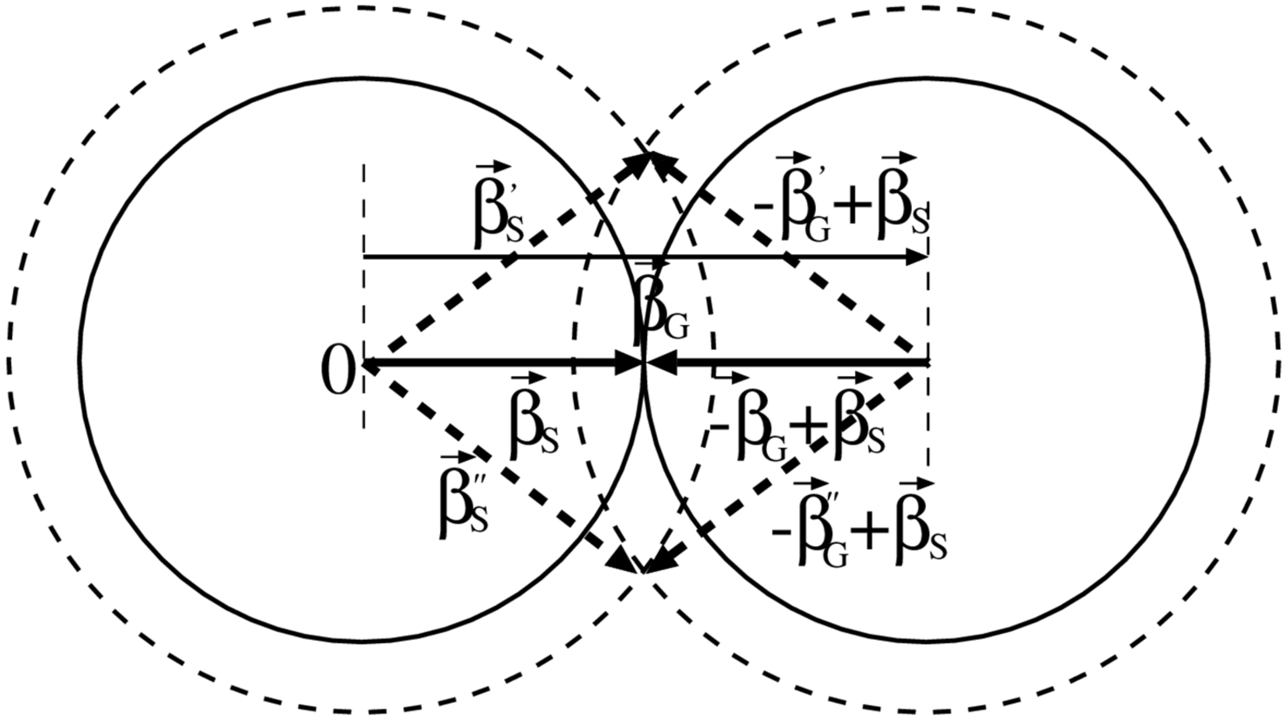
When $\left|\vec{\beta}_S - \vec{\beta}_G\right| > \left|\vec{\beta}_S\right|$

Lateral Propagation with Bragg Reflection



When $\left|\vec{\beta}_S - \vec{\beta}_G\right| < \left|\vec{\beta}_S\right|$

When Two SAWs Coupled through Bragg Reflection



COM Analysis for Periodic Structures

Eigen Mode Equations **[General Solution:** $u_{\pm} \propto \exp(\mp j\beta_u X)$ **]**

$$\frac{\partial u_+}{\partial X} = -j\beta_u u_+ \quad -j\kappa_{12} u_- \exp(-j\beta_G X)$$

$$\frac{\partial u_-}{\partial X} = +j\beta_u u_- \quad +j\kappa_{12}^* u_+ \exp(+j\beta_G X)$$

COM Equations for Forward & Backward Waves

β_u: Wavenumber of Uncoupled Wave

β_G: Grating Vector ($2\pi/p$), p: Periodicity

κ_{12} : Mutual Coupling Coefficient
= Reflectivity per Unit Length

For Derivation, Loss Less Condition was Applied

Define $U_{\pm}(X)=u_{\pm}(X)\exp(\pm j\beta_G X/2)$.

Since $u_{\pm}(X)=U_{\pm}(X)\exp(\mp j\beta_G X/2)$,

$$\frac{\partial U_+}{\partial X} = -j\theta_u U_+ - j\kappa_{12} U_-$$

$$\frac{\partial U_-}{\partial X} = +j\kappa_{12}^* U_+ + j\theta_u U_-$$

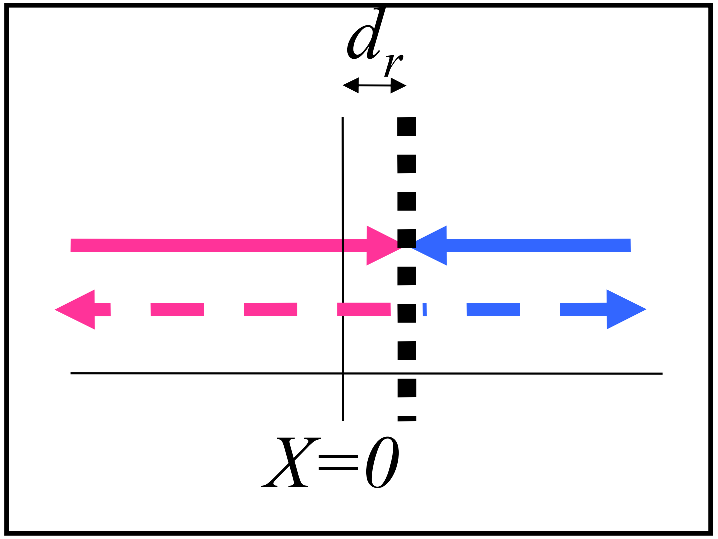
where $\theta_u=\beta_u-\beta_G/2$: Detuning Factor

($\theta_u=0$ corresponds to Bragg Condition)

Origin of Phase in κ_{12}

Displacement of Reflection Center from Origin

$d_r/p_I=\angle(\kappa_{12})/4\pi$



General Solution

$$U_+(X) = A_+ \exp(-j\theta_p X) + \Gamma_- A_- \exp(+j\theta_p X)$$

$$U_-(X) = \Gamma_+ A_+ \exp(-j\theta_p X) + A_- \exp(+j\theta_p X)$$

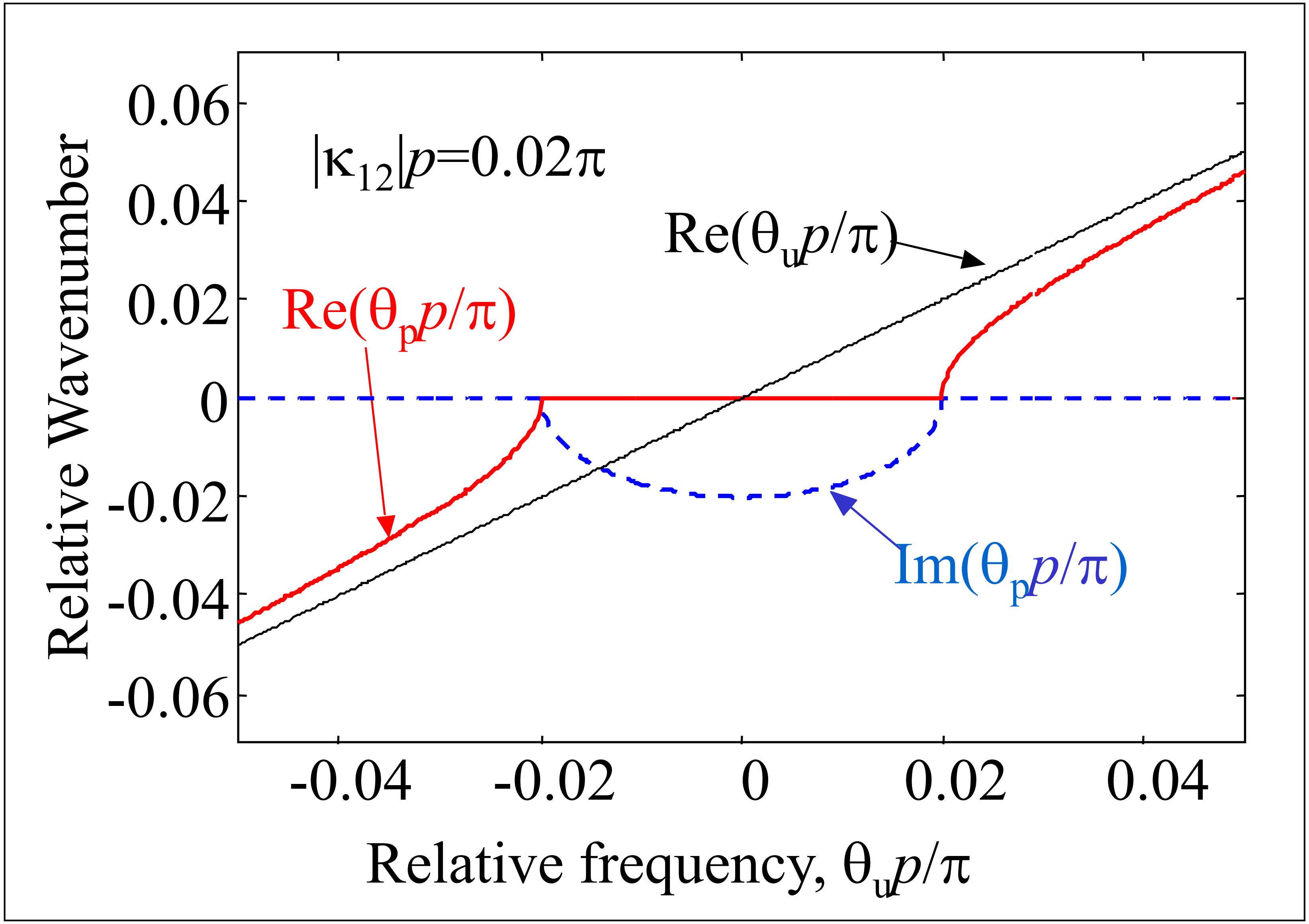
$\beta_p = \theta_p + \pi/p$: Wavenumber of ***Perturbed*** Wave

$$\theta_p = \sqrt{\theta_u^2 - |\kappa_{12}|^2}$$

$\Gamma_+ = (\theta_p - \theta_u)/\kappa_{12}$ & $\Gamma_- = (\theta_p - \theta_u)/\kappa_{12}^*$: Reflection Coefficient of Semi-Infinite Grating Looking toward $\pm X$ direction

⇒ κ_{12} is Real When Grating is Symmetric

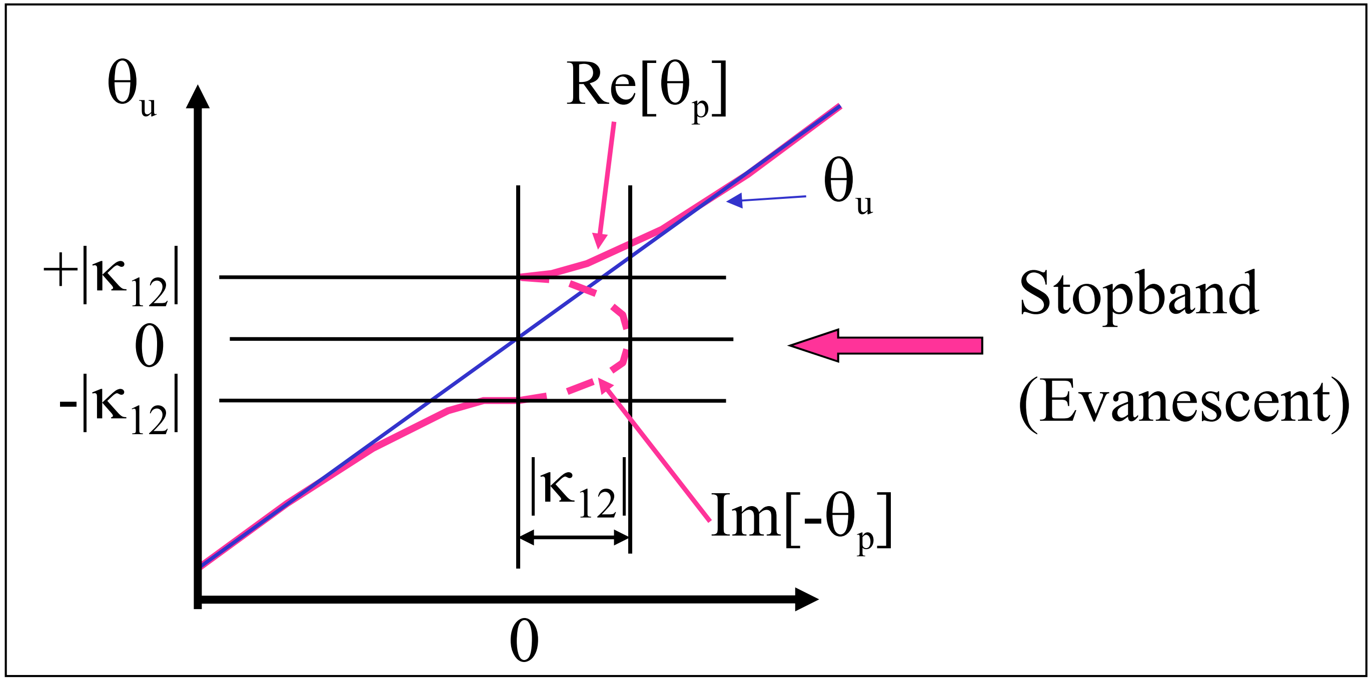
$A_\pm$: Amplitude of Partial Wave



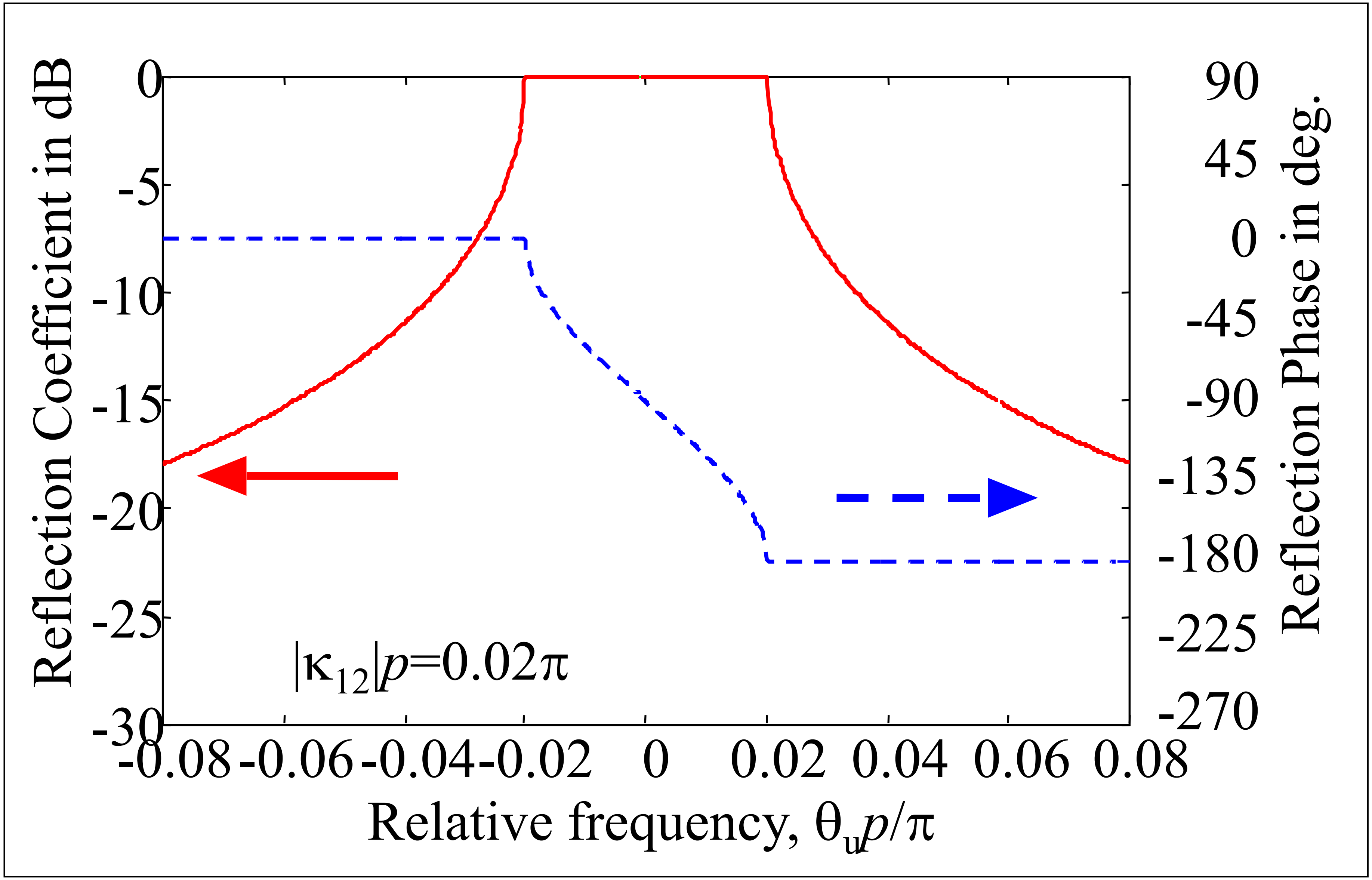
$|\kappa_{12}|p$=0.02π
Re($\theta_u p/\pi$)
Re($\theta_p p/\pi$)
Im($\theta_p p/\pi$)
Relative Wavenumber
Relative frequency, $\theta_u p/\pi$
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.04
-0.02
0
0.02
0.04

Behavior Near Bragg Frequency

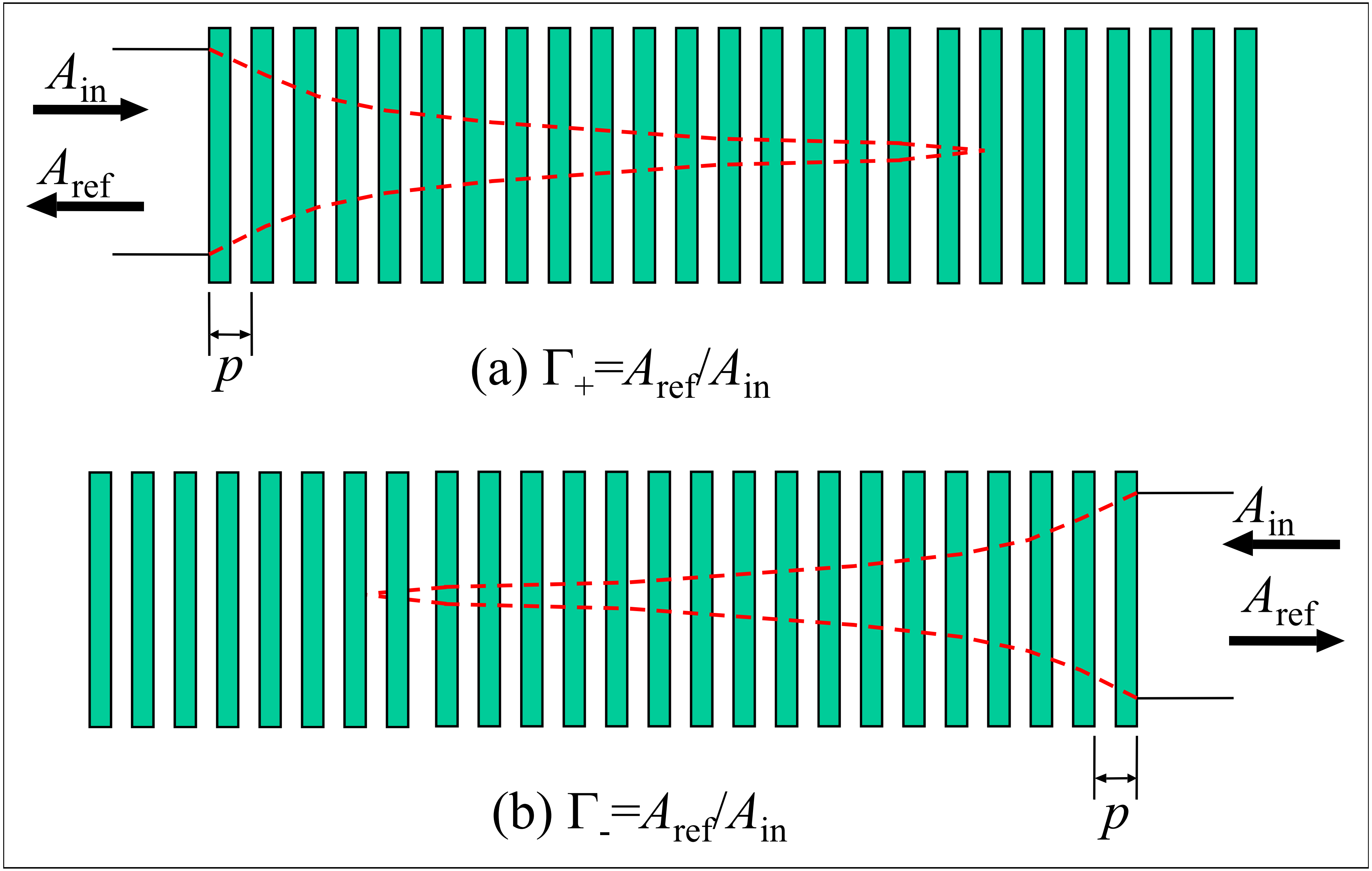
$$\theta_p = \sqrt{\theta_u^{\,2} - |\kappa_{12}|^2}$$



$|\kappa_{12}|$ determines Both Stopband Width & Attenuation Constant

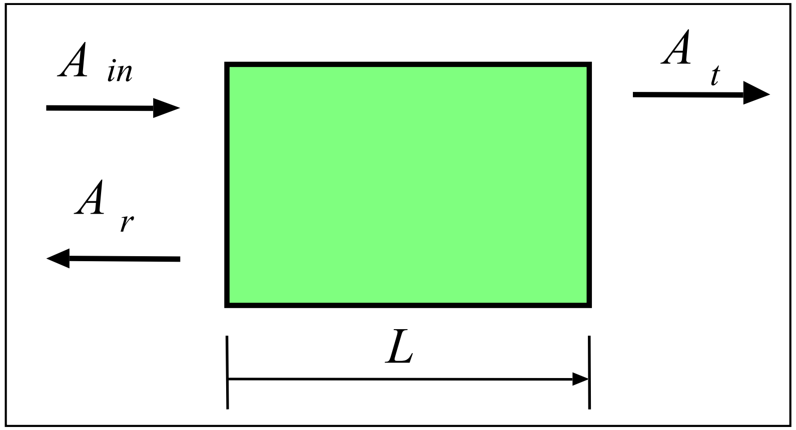


Reflection Coefficient in dB
0
-5
-10
-15
-20
-25
-30
90
45
0
-45
-90
-135
-180
-225
-270
Reflection Phase in deg.
|κ12|p=0.02π
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
Relative frequency, θup/π



A_{in}
A_{ref}
p
(a) $\Gamma_{+}=A_{ref}/A_{in}$
A_{in}
A_{ref}
p
(b) $\Gamma_{-}=A_{ref}/A_{in}$

Application of Boundary Condition



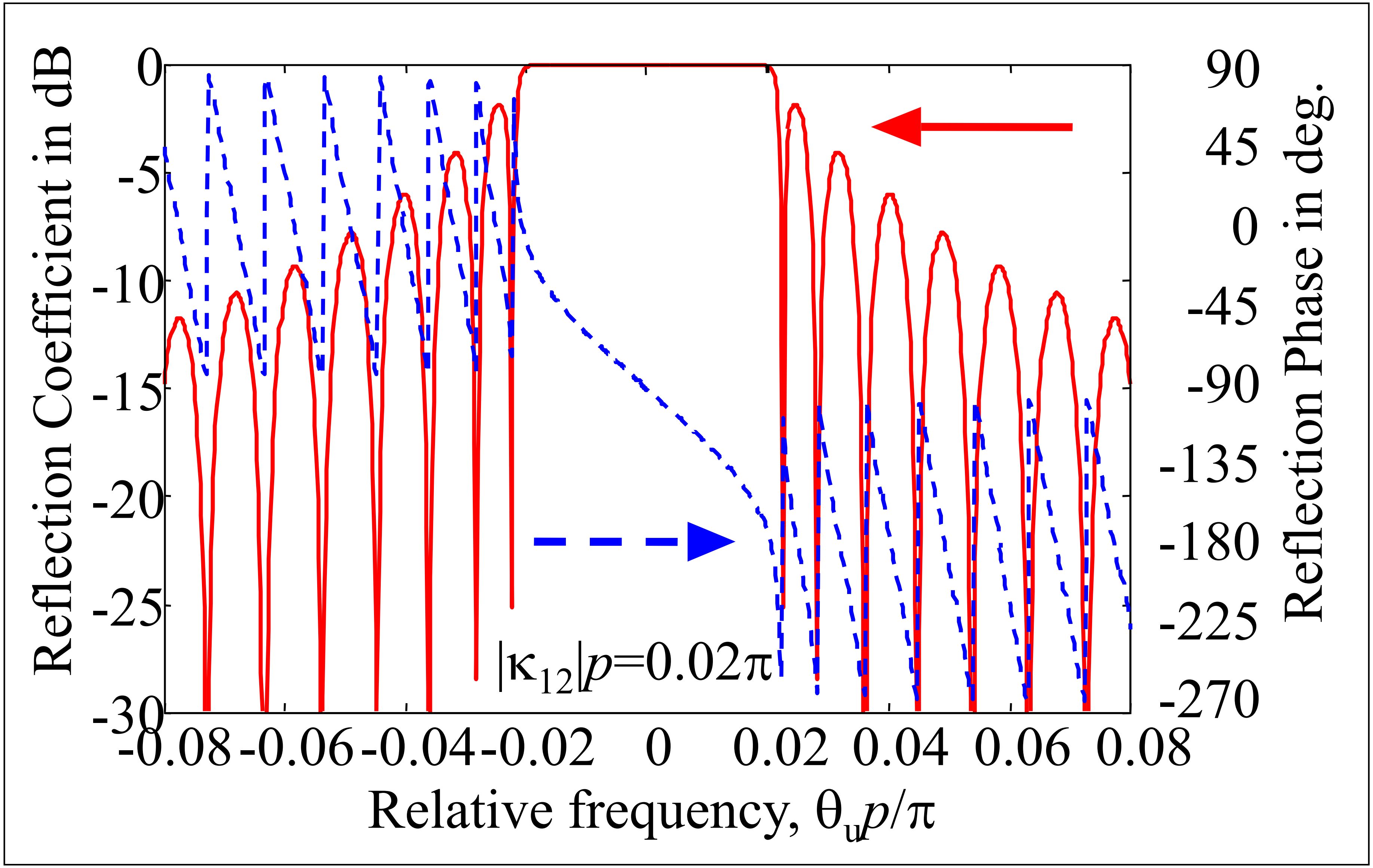
$$U_+(X) = A_+ \exp(-j\theta_p X) + \Gamma_- A_- \exp(+j\theta_p X)$$

$$U_-(X) = \Gamma_+ A_+ \exp(-j\theta_p X) + A_- \exp(+j\theta_p X)$$

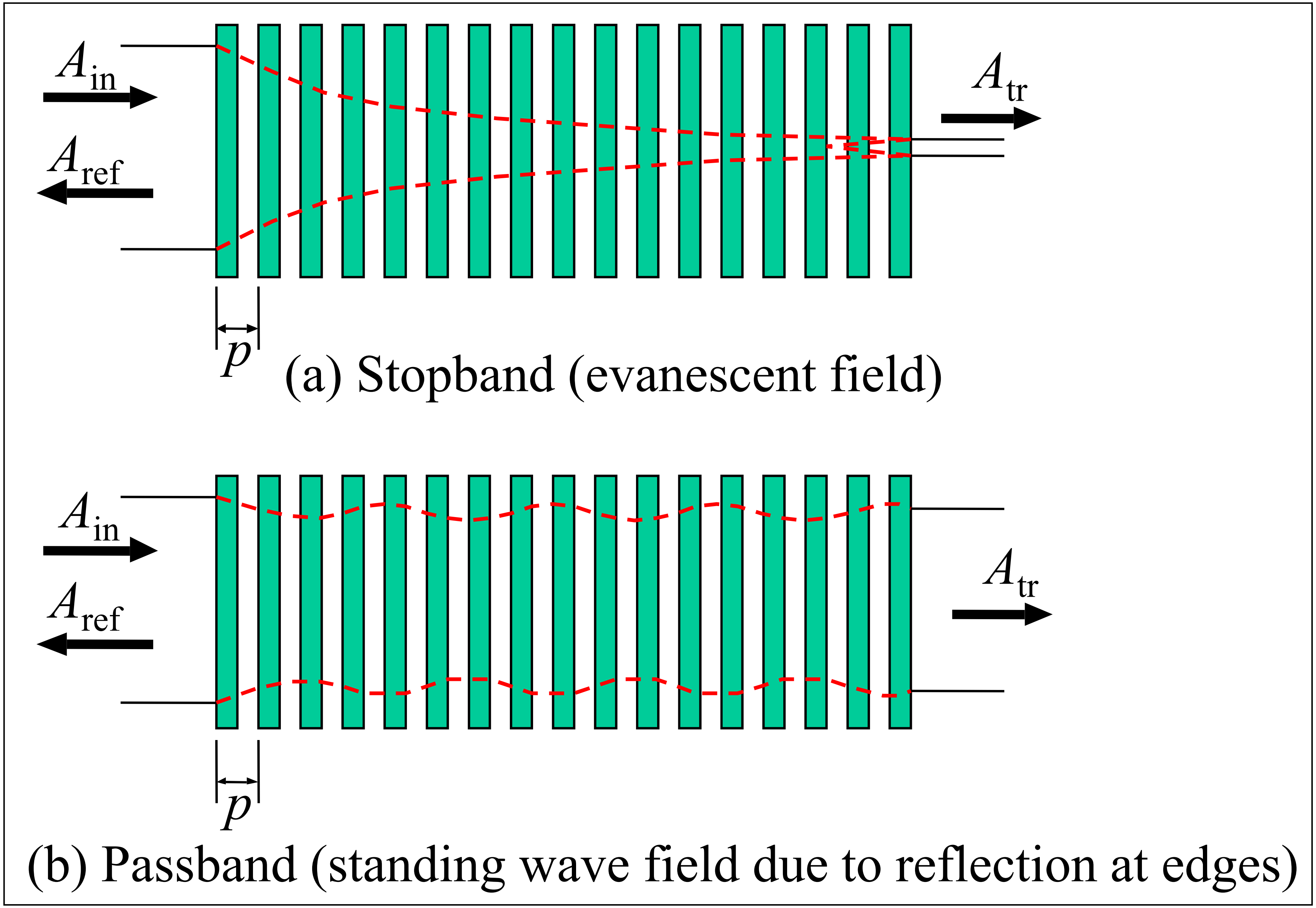
Since $U_+(0)=A_{in}$ & $U_-(L)=0$,

$$\Gamma = \frac{A_r}{A_{in}} = \frac{\Gamma_+[1-\exp(-2j\theta_p L)]}{1-\Gamma_+\Gamma_- \exp(-2j\theta_p L)}$$

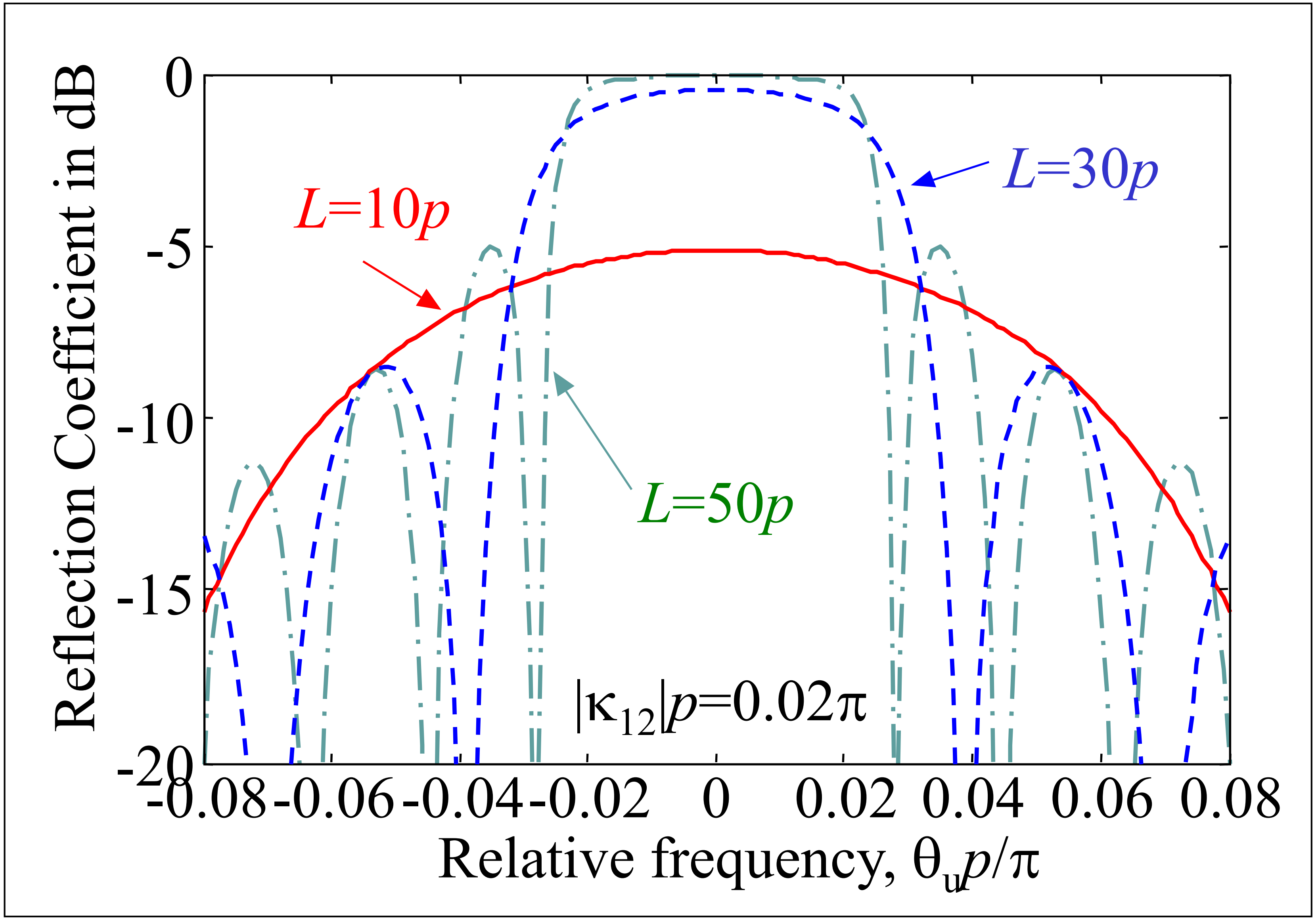
$$T = \frac{A_t}{A_{in}} = \frac{\exp(-j\theta_p L)(1-\Gamma_+\Gamma_-)}{1-\Gamma_+\Gamma_- \exp(-2j\theta_p L)}$$



Reflection Coefficient in dB
0
-5
-10
-15
-20
-25
-30
Reflection Phase in deg.
90
45
0
-45
-90
-135
-180
-225
-270
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
Relative frequency, θ_u p/π
|κ_12|p=0.02π



A_{in}
A_{ref}
A_{tr}
p
(a) Stopband (evanescent field)
A_{in}
A_{ref}
A_{tr}
p
(b) Passband (standing wave field due to reflection at edges)

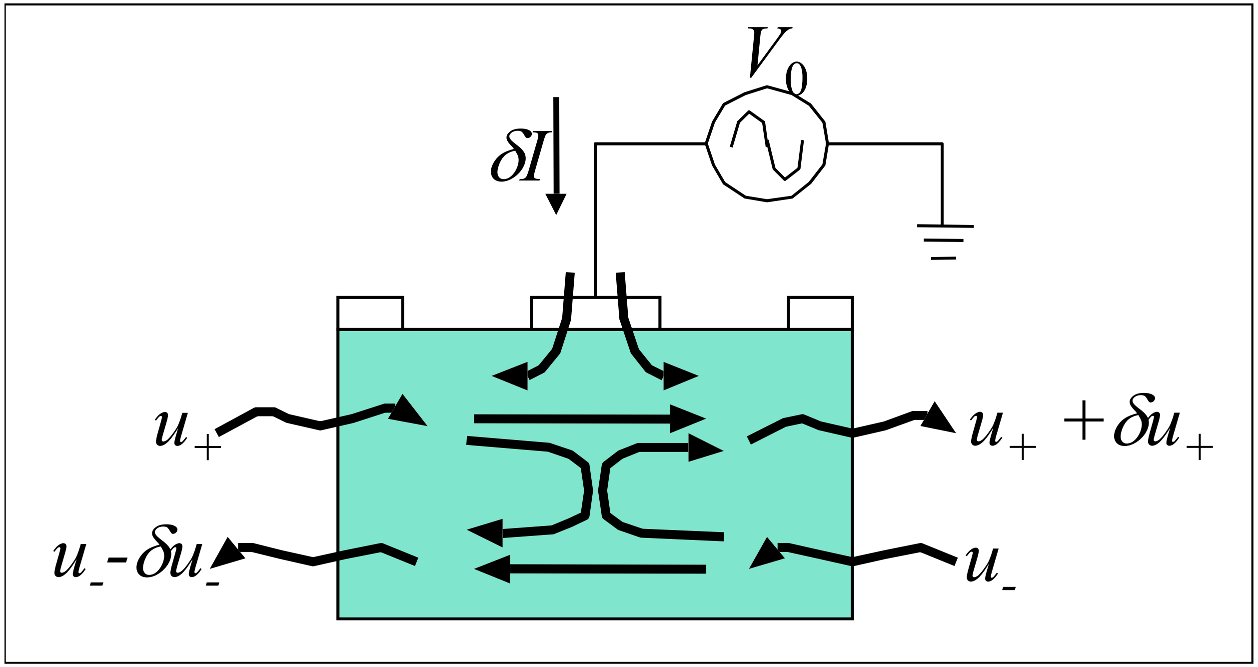


Reflection Coefficient in dB
0
-5
-10
-15
-20
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
Relative frequency, $\theta_u p/\pi$
L=10p
L=30p
L=50p
$|\kappa_{12}|p$=0.02π

Contents

- IDT Modeling

COM Equation for SAW Devices



ζ : Transduction Coefficient

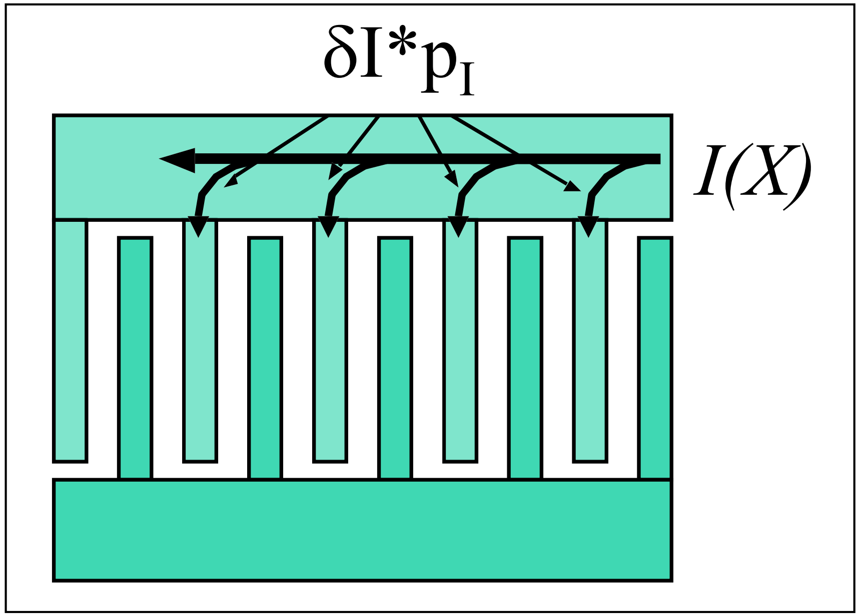
$p_I(=2p)$: IDT Periodicity

$$\frac{\partial u_+}{\partial X} = -j\beta_u u_+ - j\kappa_{12} u_- \exp(-j\beta_G X) + j\zeta V_0 \exp(-j\beta_G X/2)$$

$$\frac{\partial u_-}{\partial X} = +j\kappa_{12}^* u_+ \exp(+j\beta_G X) + j\beta_u u_- - j\zeta^* V_0 \exp(+j\beta_G X/2)$$

Spatial Components with $\pm\beta_G/2(=\pm 2\pi/p_I)$ are Considered

Equation for Current on Bus-Bar



C: Static Capacitance per Unit Length

χ=2 for RMS *I*,*V* & *u*

χ=4 for peak *I*,*V* & RMS *u*

$$\frac{\partial I}{\partial X} = -j\chi\zeta^* u_+ \exp(+j\beta_G X/2) - j\chi\zeta u_- \exp(-j\beta_G X/2) + j\omega C V_0$$

Spatial Components with $\pm\beta_G/2(=\pm 2\pi/p_I)$ are Considered

For Derivation, Loss Less Condition & Bidirectionality (When Mechanical Reflection is Zero) are Applied

Final COM Equations

$$\frac{\partial u_+}{\partial X} = -j\theta_u u_+ - j\kappa_{12} u_- \exp(-j\beta_G X) + j\zeta V_0 \exp(-j\beta_G X/2)$$

$$\frac{\partial u_-}{\partial X} = j\kappa_{12}^* u_+ \exp(+j\beta_G X) + j\theta_u u_- - j\zeta^* V_0 \exp(+j\beta_G X/2)$$

$$\frac{\partial I}{\partial X} = -j\chi\zeta^* u_+ \exp(+j\beta_G X/2) - j\chi\zeta u_- \exp(-j\beta_G X/2) + j\omega C V_0$$

Define $U_\pm(X)=u_\pm(X)\exp(\pm j\beta_G X/2)$. Then

Since $u_\pm(X)=U_\pm(X)\exp(\mp j\beta_G X/2)$,

$$\frac{\partial U_+}{\partial X} = -j\theta_u U_+ - j\kappa_{12} U_- + j\zeta V_0$$

$$\frac{\partial U_-}{\partial X} = +j\kappa_{12}^* U_+ + j\theta_u U_- - j\zeta^* V_0$$

$$\frac{\partial I}{\partial X} = -j\chi\zeta^* U_+ - j\chi\zeta U_- + j\omega C V_0$$

General Solution

$$U_+(X) = A_+ \exp(-j\theta_p X) + \Gamma_- A_- \exp(+j\theta_p X) + \xi_+ V_0$$

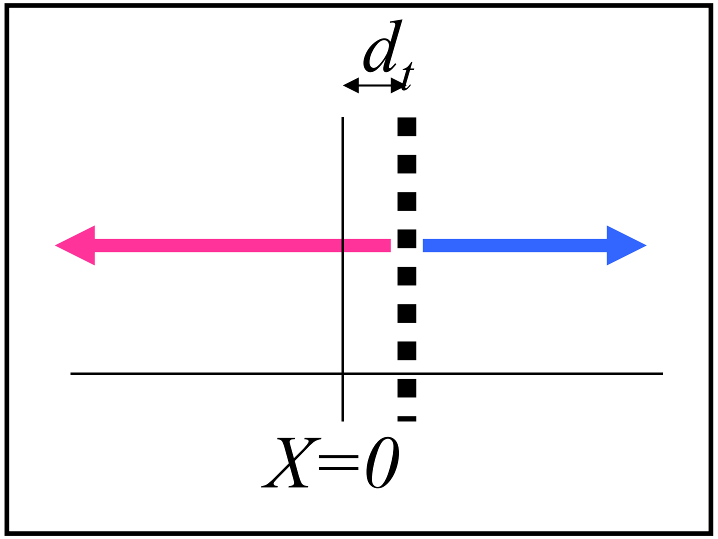
$$U_-(X) = \Gamma_+ A_+ \exp(-j\theta_p X) + A_- \exp(+j\theta_p X) + \xi_- V_0$$

Where $\xi_+ = (\zeta\theta_u - \zeta^*\kappa_{12})/\theta_p^2$ & $\xi_- = (\zeta^*\theta_u - \zeta\kappa_{12}^*)/\theta_p^2$: Excitation Efficiency toward $\pm X$ Direction

Origin of Phase in ζ

Displacement of Excitation Center from Origin

$d_t/p_I = \angle(\zeta)/2\pi$



Short Circuited (SC) Grating

Since V_0=0,

$$\frac{\partial U_+}{\partial X} = -j\theta_u U_+ - j\kappa_{12} U_-$$

$$\frac{\partial U_-}{\partial X} = +j\kappa_{12}^* U_+ + j\theta_u U_-$$

$$\Rightarrow \quad \theta_p = \sqrt{\theta_u^2 - |\kappa_{12}|^2}$$

Open Circuited (OC) Grating

Since δ*I*=0,

$$\frac{\partial U_+}{\partial X} = -j\hat{\theta}_u U_+ - j\hat{\kappa}_{12} U_-$$

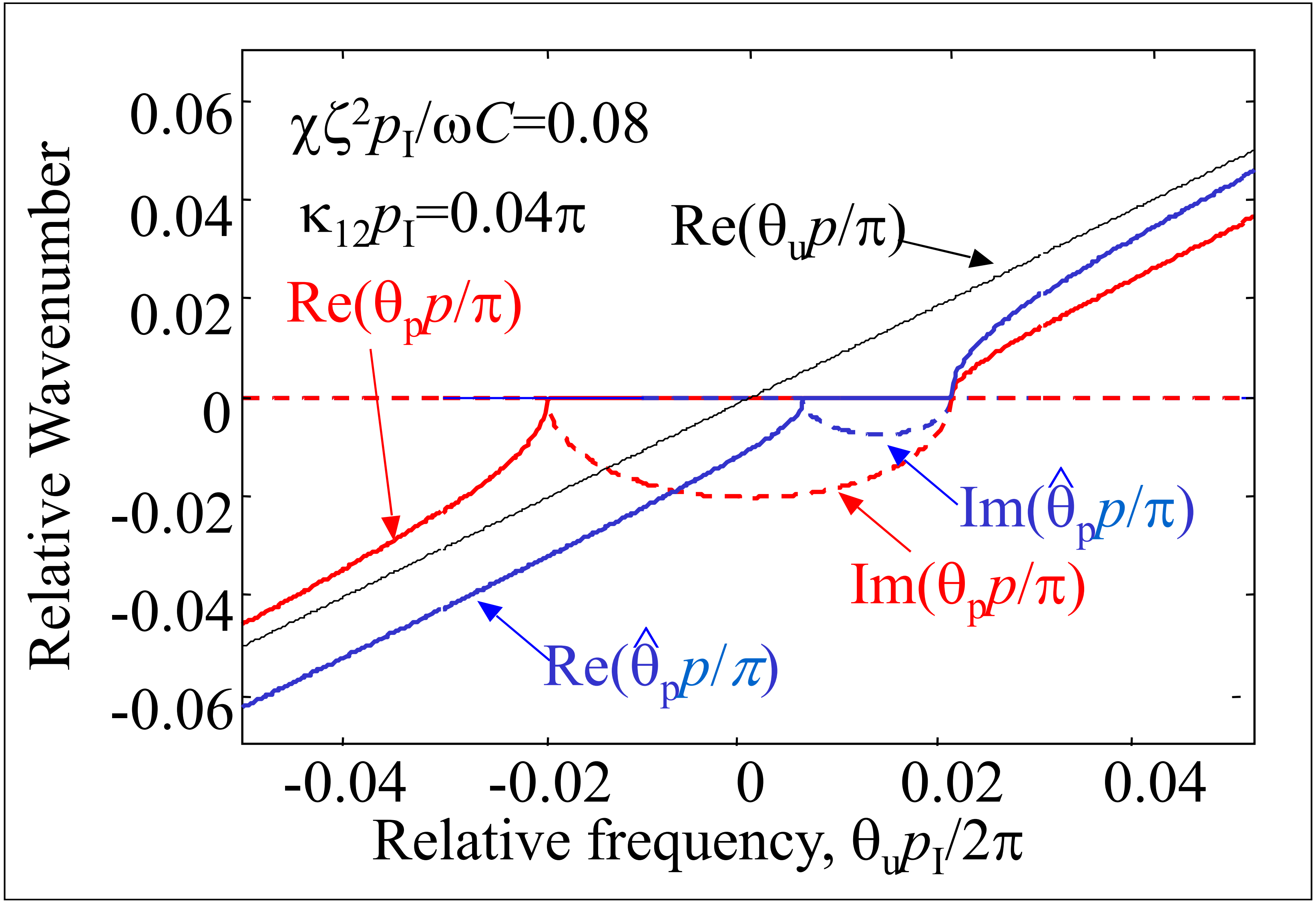
$$\frac{\partial U_-}{\partial X} = +j\hat{\kappa}_{12}^* U_+ + j\hat{\theta}_u U_-$$

where

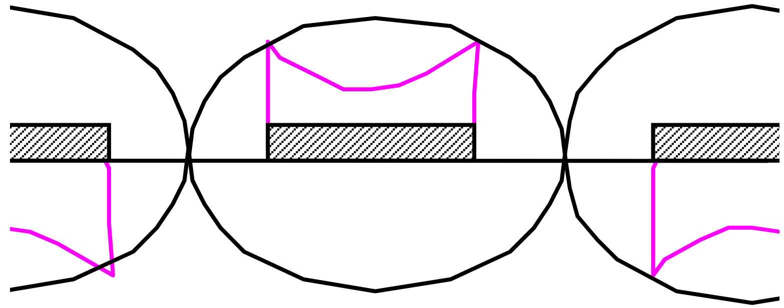
$$\hat{\theta}_u = \theta_u - \chi|\zeta|^2/\omega C$$

$$\hat{\kappa}_{12} = \kappa_{12} - \chi\zeta^2/\omega C$$

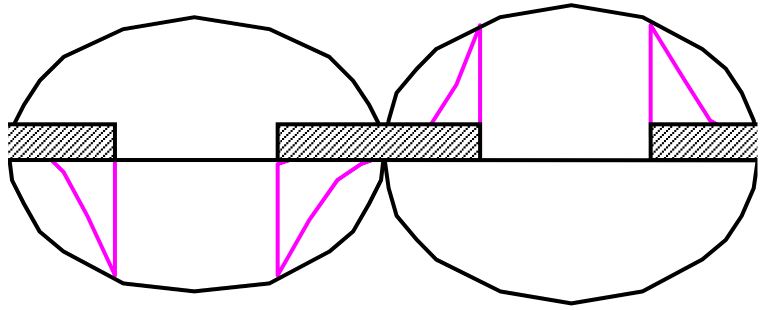
$$\Rightarrow \quad \hat{\theta}_p = \sqrt{\hat{\theta}_u^2 - |\hat{\kappa}_{12}|^2}$$


χζ²p_I/ωC=0.08
κ12p_I=0.04π
Re(θ_u p/π)
Re(θ_p p/π)
Im(θ̂_p p/π)
Im(θ_p p/π)
Re(θ̂_p p/π)
Relative Wavenumber
Relative frequency, θ_u p_I/2π
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.04
-0.02
0
0.02
0.04

Field Distribution at Stopband with $\beta p = \pi$ (Bidirectional Case)



(a) Symmetric Mode



(b) Antisymmetric Mode

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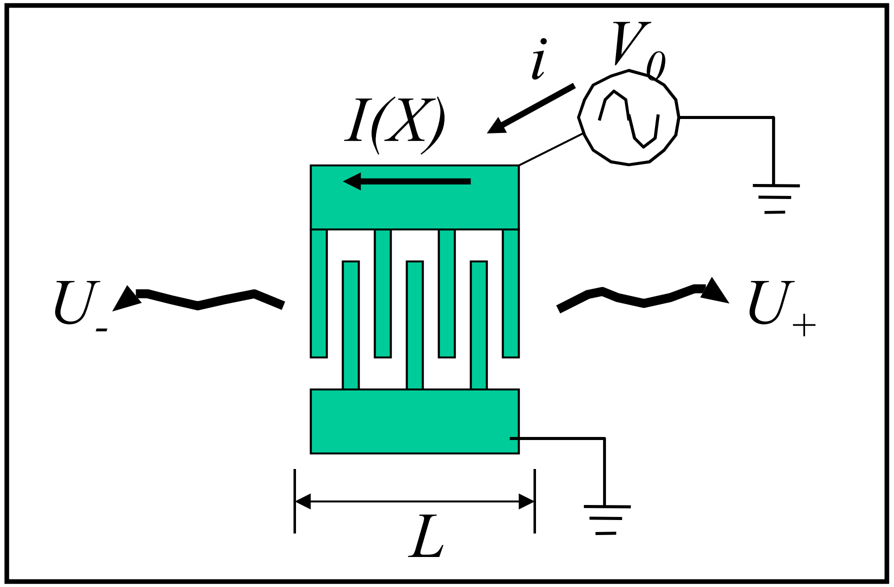
- IDT Properties

SAW Excitation by IDT
(When ζ and κ_{12} are Real)



$\Gamma_{\pm}=(\theta_p-\theta_u)/\kappa_{12}\equiv\Gamma_0$

$\xi_{\pm}=\zeta/(\theta_u+\kappa_{12})\equiv\xi_0$

Boundary Conditions: $U_+(-L/2)=0,\ U_-(+L/2)=0,\ I(-L/2)=0$

$$A_+ = A_- = \frac{-\xi_0 V_0}{\exp(+j\theta_p L/2)+\Gamma_0\exp(-j\theta_p L/2)}$$

$$Y = V_0^{-1}\int_{-L/2}^{+L/2}\frac{\partial I(X)}{\partial X}dX = \int_{-L/2}^{+L/2}[-j\chi\zeta V_0^{-1}(U_+ + U_-) + j\omega C]dX$$

$$Y = \int_{-L/2}^{+L/2} [-2j\chi\zeta V_0^{-1} A_+ (1+\Gamma_0)\cos(\theta_p X) - j(2\chi\xi_0\zeta - \omega C)]dX$$

$$= \frac{2j\chi\xi_0\zeta(1+\Gamma_0)L\,\mathrm{sinc}(\theta_p L/2)}{\exp(+j\theta_p L/2) + \Gamma_0 \exp(-j\theta_p L/2)} - j(2\chi\xi_0\zeta - \omega C)L$$

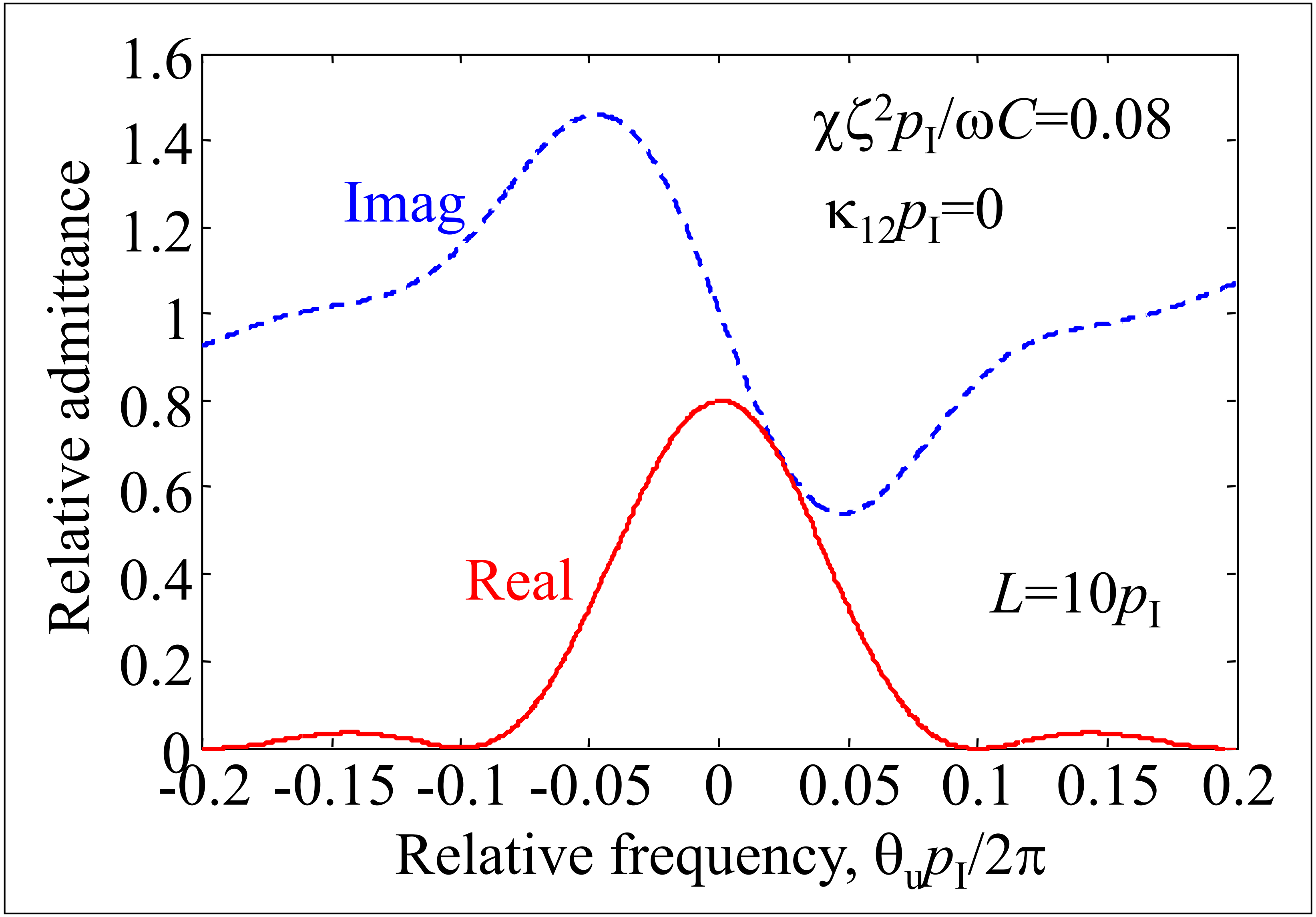
When $\kappa_{12}=0$, $\theta_p=\theta_u$, $\Gamma_0=0$ & $\xi_0=\zeta/\theta_u$. Then

$$Y = \frac{2j\chi\zeta^2 L}{\theta_u}[\mathrm{sinc}(\theta_u L) - j\,\mathrm{sinc}(\theta_u L/2)\sin(\theta_u L/2) - 1] + j\omega CL$$

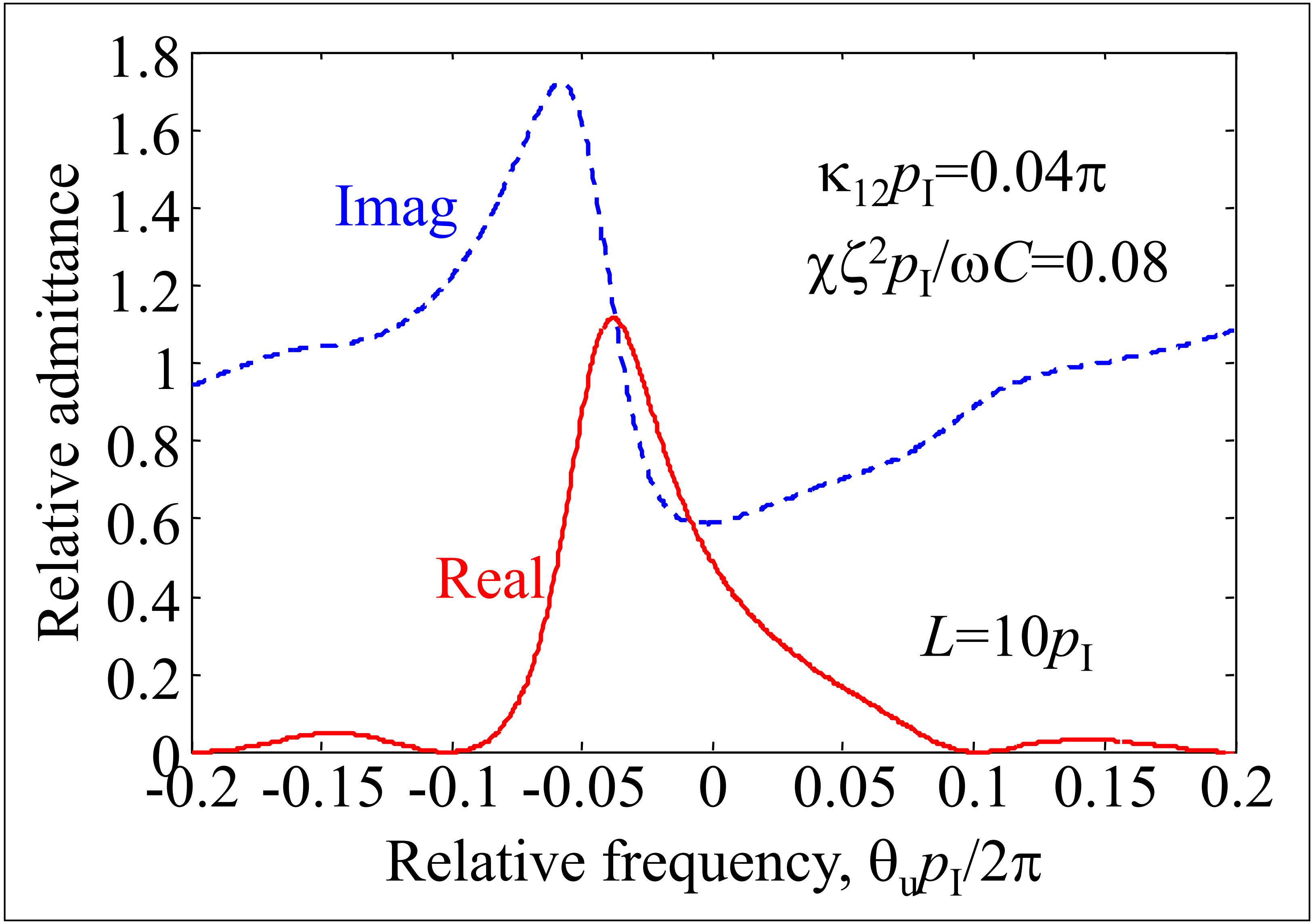
$$= \chi\zeta^2 L^2 \mathrm{sinc}^2(\theta_u L/2) + \frac{2j\chi\zeta^2 L}{\theta_u}[\mathrm{sinc}(\theta_u L) - 1] + j\omega CL$$

Comparison : Delta Function Model Gives

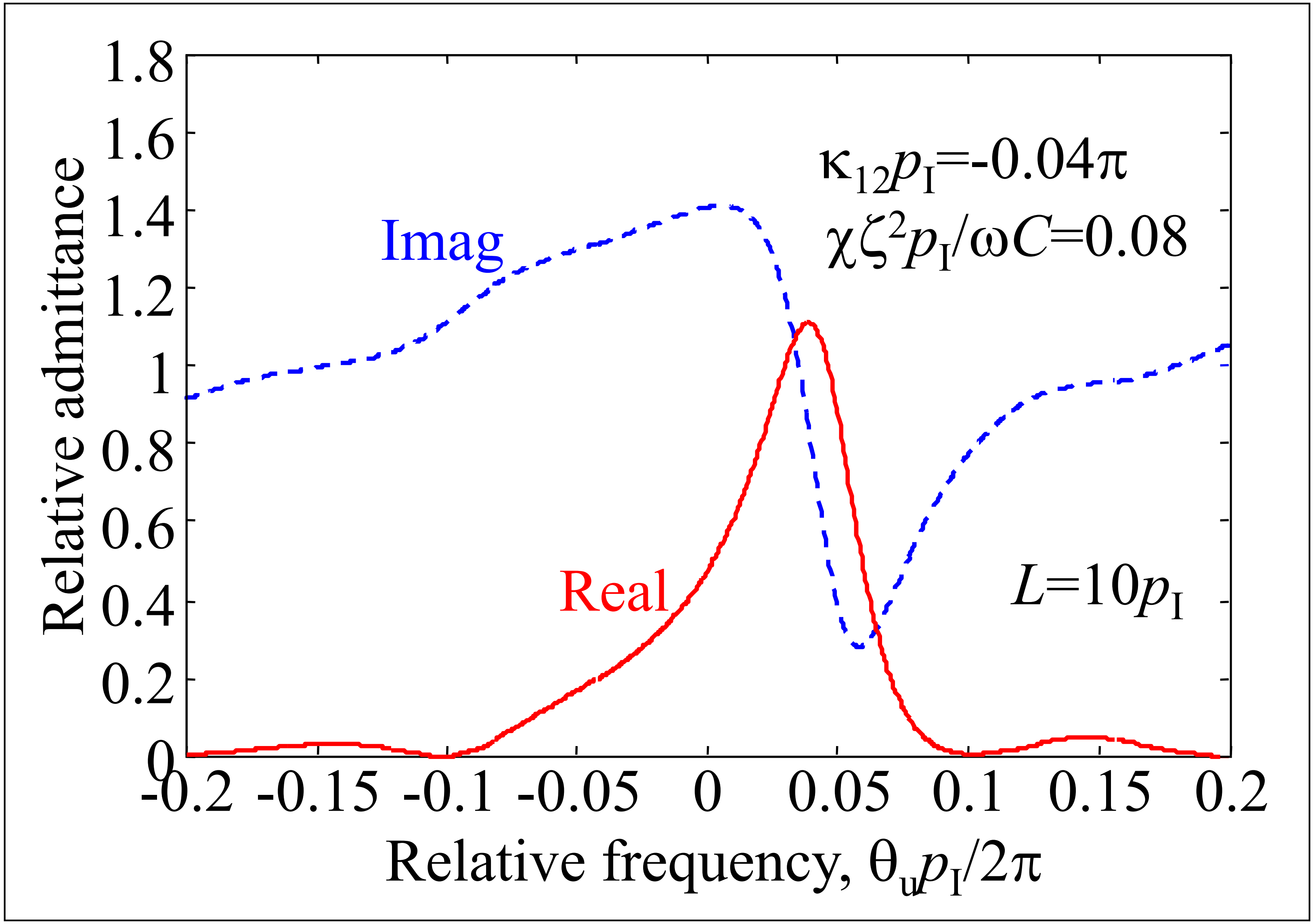
$$Y = \chi(\zeta p_I)^2 \frac{\sin^2(\theta_u L/2) + 2^{-1} j\sin(\theta_u L) - jL/p_I \sin(\theta_u p_I/2)}{\sin^2(\theta_u p_I/2)} + j\omega CL$$



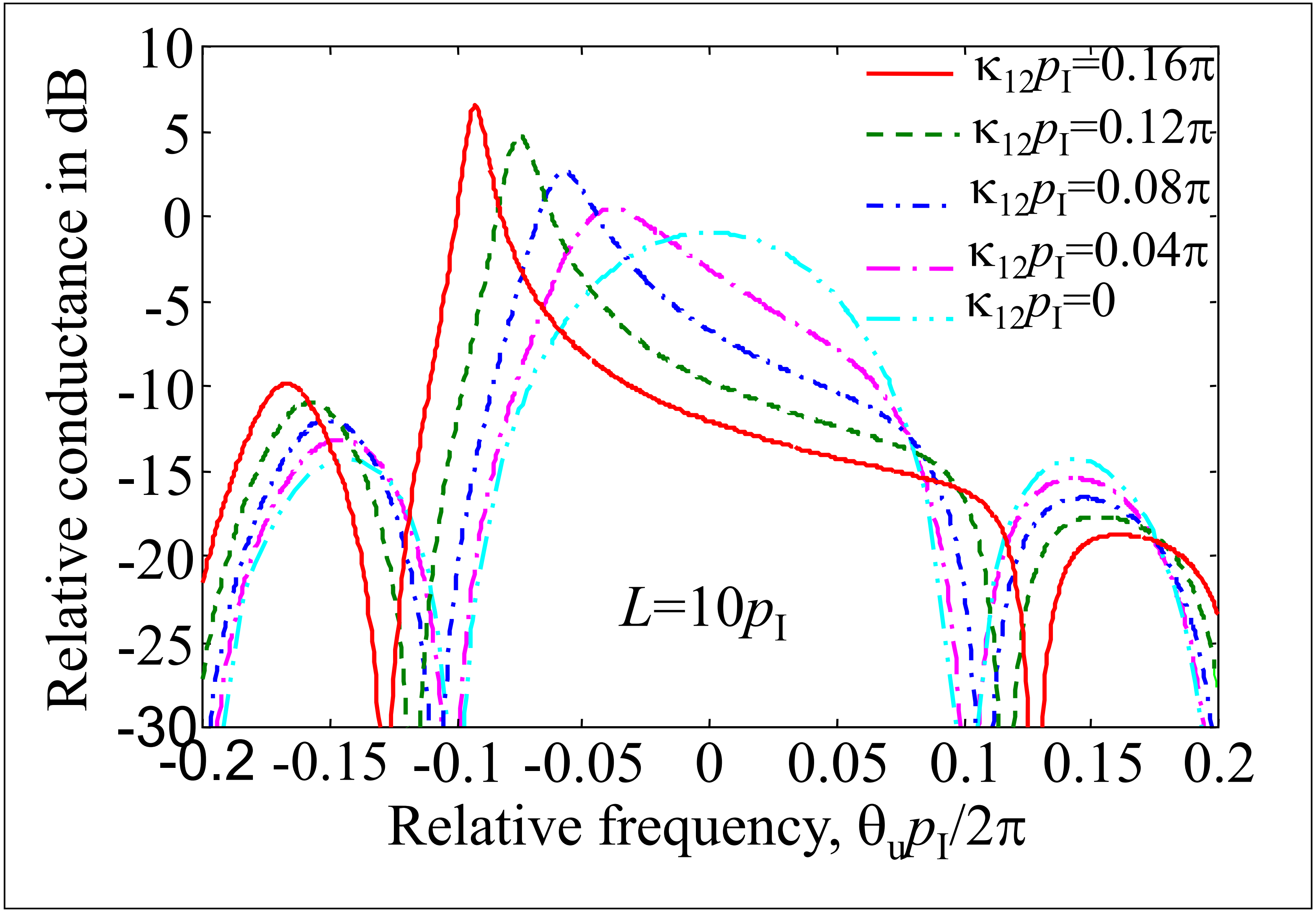
$\chi\zeta^2 p_I/\omega C$=0.08
$\kappa_{12}p_I$=0
Imag
Real
L=10p_I
Relative admittance
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
Relative frequency, $\theta_u p_I/2\pi$



Relative admitance
Relative frequency, θupI/2π
Imag
Real
κ12pI=0.04π
χζ2pI/ωC=0.08
L=10pI
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2



Relative admittance
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Imag
Real
κ12pI=-0.04π
χζ2pI/ωC=0.08
L=10pI
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
Relative frequency, θupI/2π



Relative conductance in dB
10
5
0
-5
-10
-15
-20
-25
-30
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
Relative frequency, θ_u p_I/2π
κ_12 p_I=0.16π
κ_12 p_I=0.12π
κ_12 p_I=0.08π
κ_12 p_I=0.04π
κ_12 p_I=0
L=10p_I

Input Admittance for Infinite IDT

Since $\partial U_{\pm}/\partial X=0$ & $i=p_I\partial I/\partial X$,

$$0 = -j\theta_u U_+ - j\kappa_{12} U_- + j\zeta V_0$$

$$0 = +j\kappa_{12}^* U_+ + j\theta_u U_- - j\zeta^* V_0$$

$$i = -j\chi\zeta^* p_I U_+ - j\chi\zeta p_I U_- + j\omega C p_I V_0$$

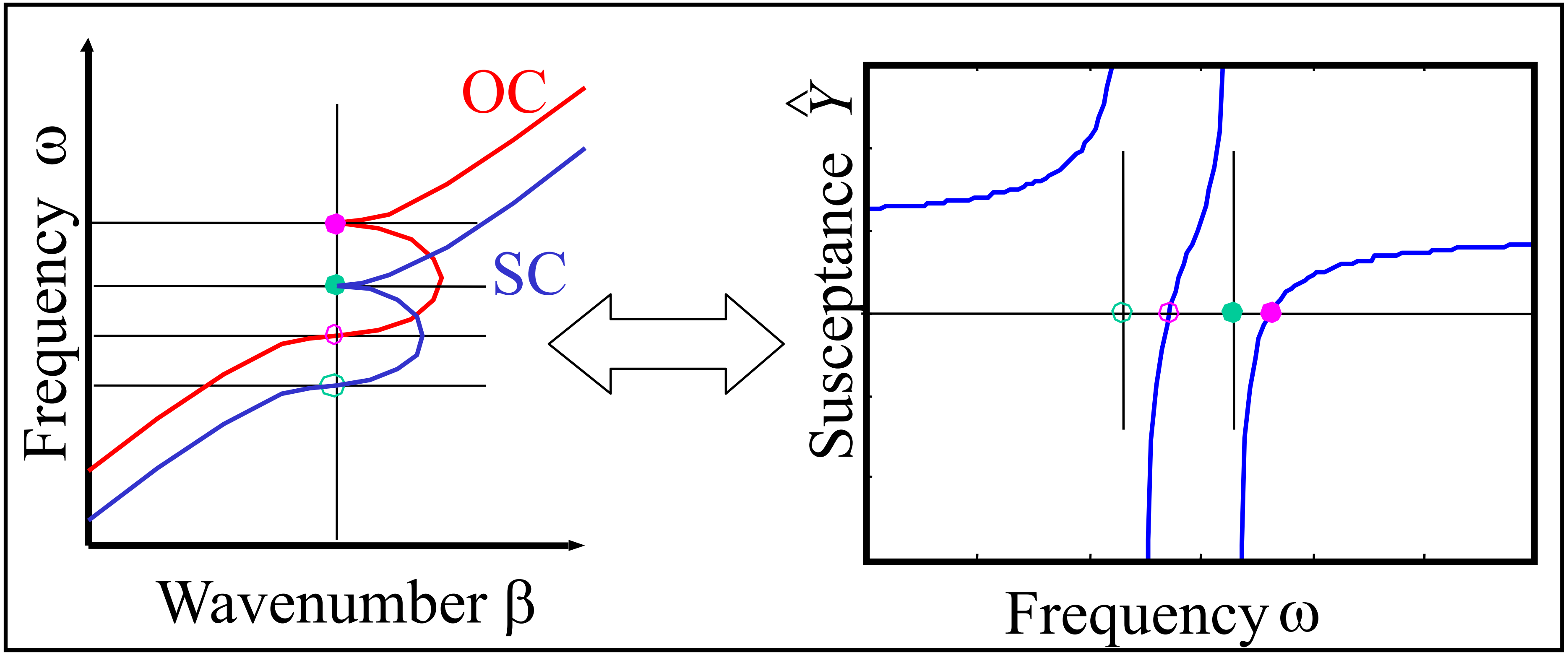
Then

$$\hat{Y} = \frac{i}{V_0} = -j\chi p_I \frac{2\theta_u|\zeta|^2 - \kappa_{12}\zeta^{*2} - \kappa_{12}^*\zeta^2}{\theta_u^2 - |\kappa_{12}|^2} + j\omega C p_I$$

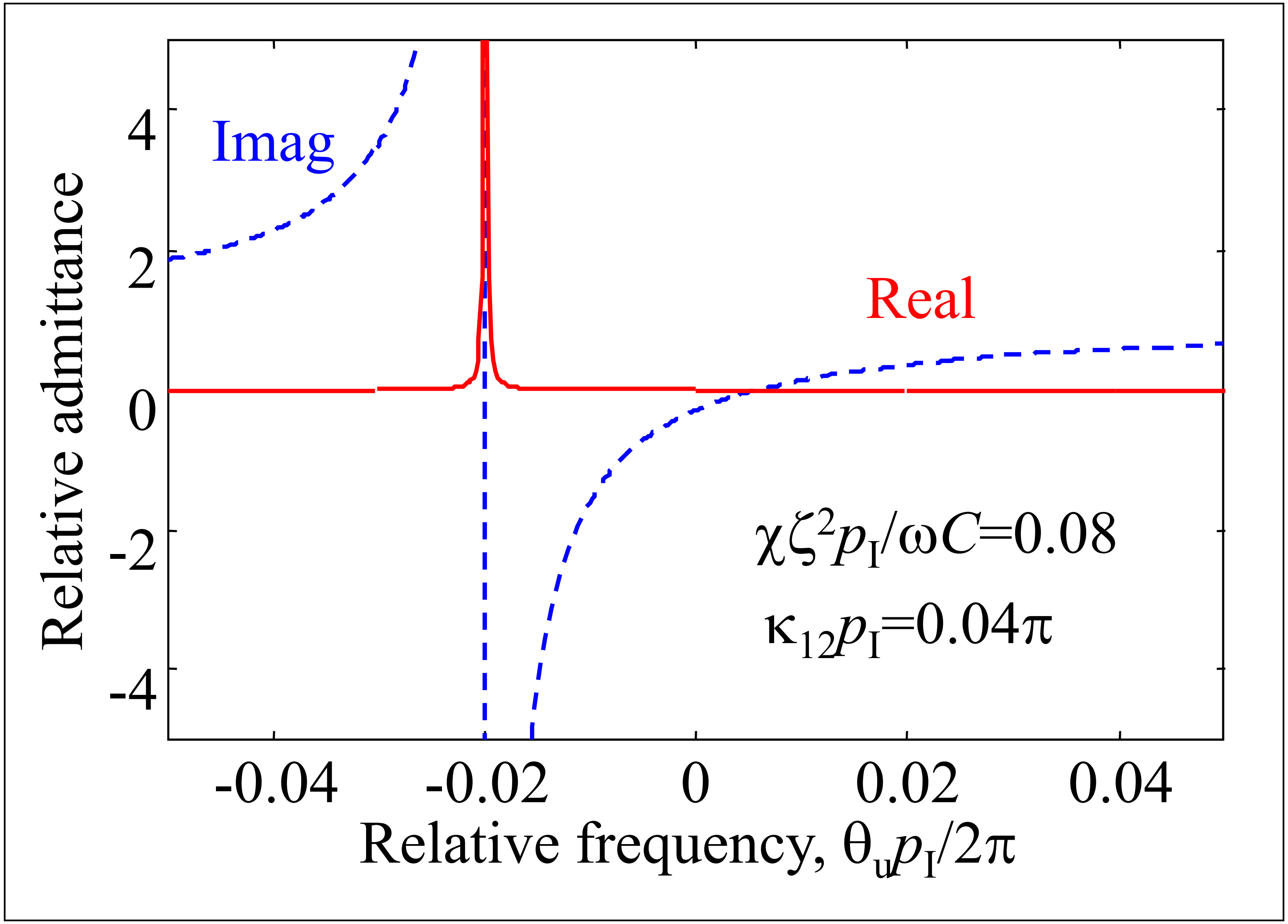
$$= j\omega C p_I \frac{(\theta_u - \theta_{oc}^+)(\theta_u - \theta_{oc}^-)}{(\theta_u - \theta_{sc}^+)(\theta_u - \theta_{sc}^-)}$$

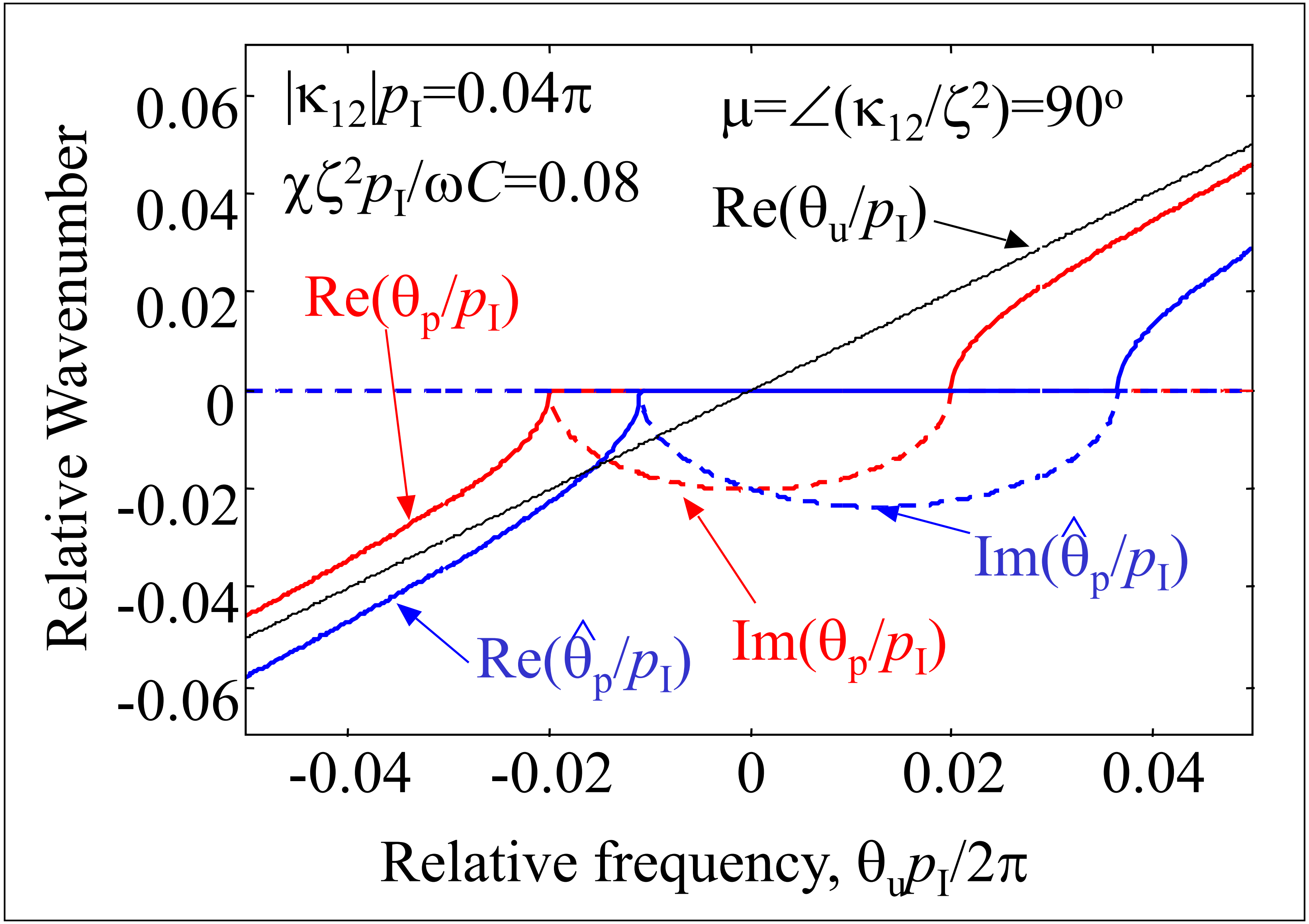
Where $\theta_{oc}^{\pm}=\chi|\zeta|^2/\omega C\pm|\kappa_{12}-\chi\zeta^2/\omega C|$, $\theta_{sc}^{\pm}=\pm|\kappa_{12}|$

COM Parameter Determination by Input Admittance of Infinite IDT

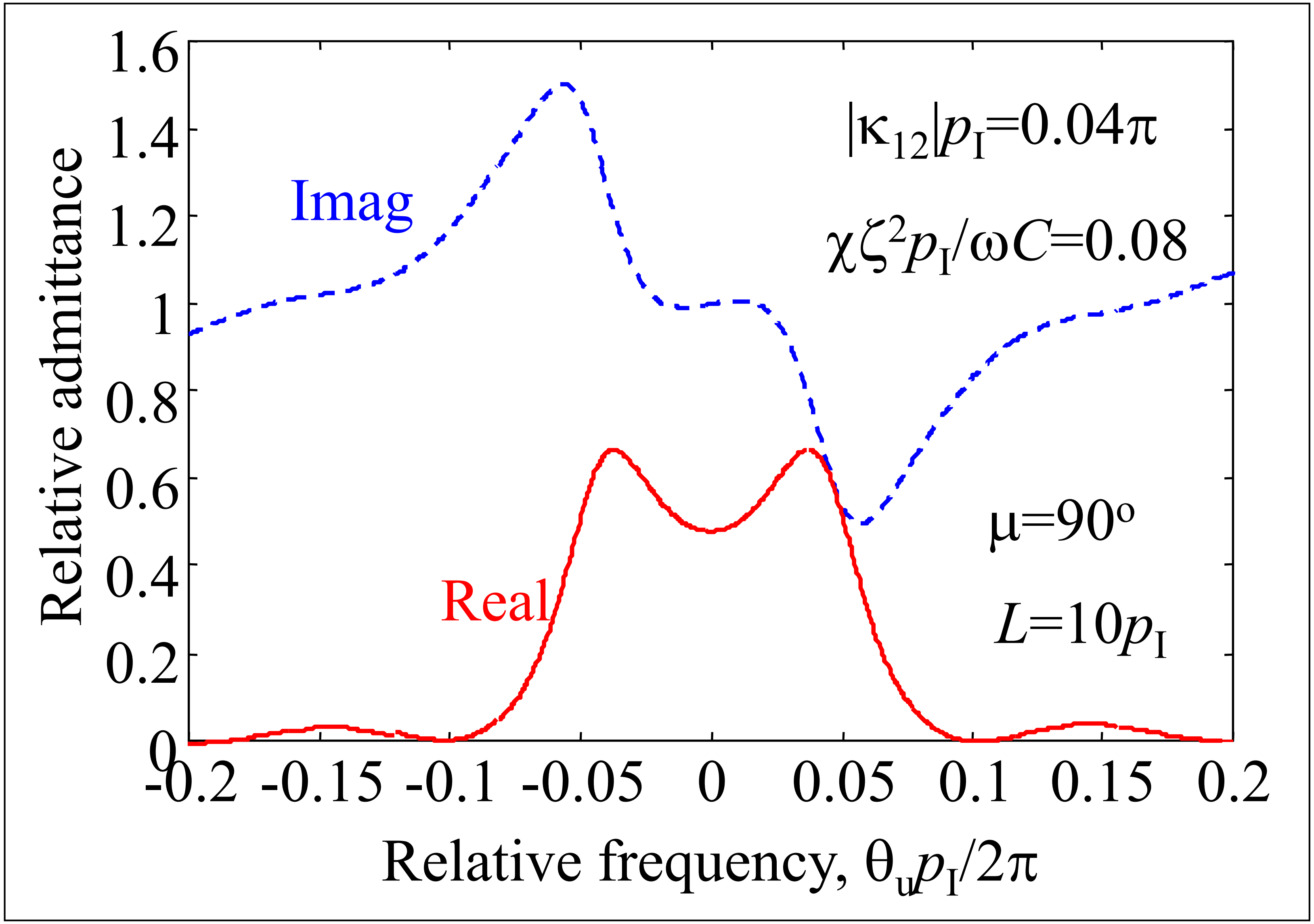


$$\hat{Y}(\omega) = j\omega Cp_{\mathrm{I}} \frac{(\omega-\omega_{\mathrm{oc}}^{+})(\omega-\omega_{\mathrm{oc}}^{-})}{(\omega-\omega_{\mathrm{sc}}^{+})(\omega-\omega_{\mathrm{sc}}^{-})}$$

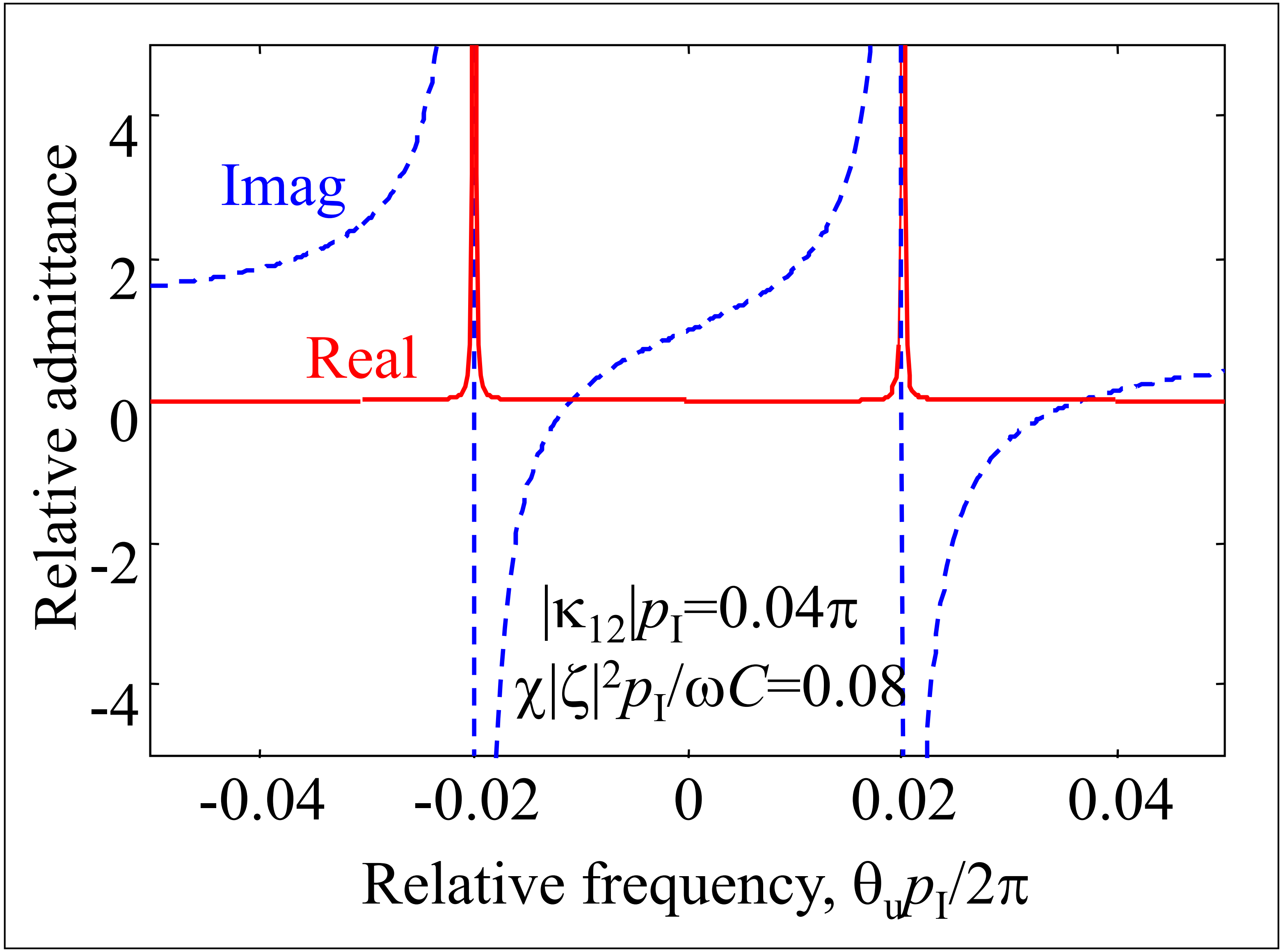

Imag
Real
$\chi\zeta^2 p_I/\omega C$=0.08
$\kappa_{12} p_I$=0.04π
Relative admitance
4
2
0
-2
-4
-0.04
-0.02
0
0.02
0.04
Relative frequency, $\theta_u p_I/2\pi$



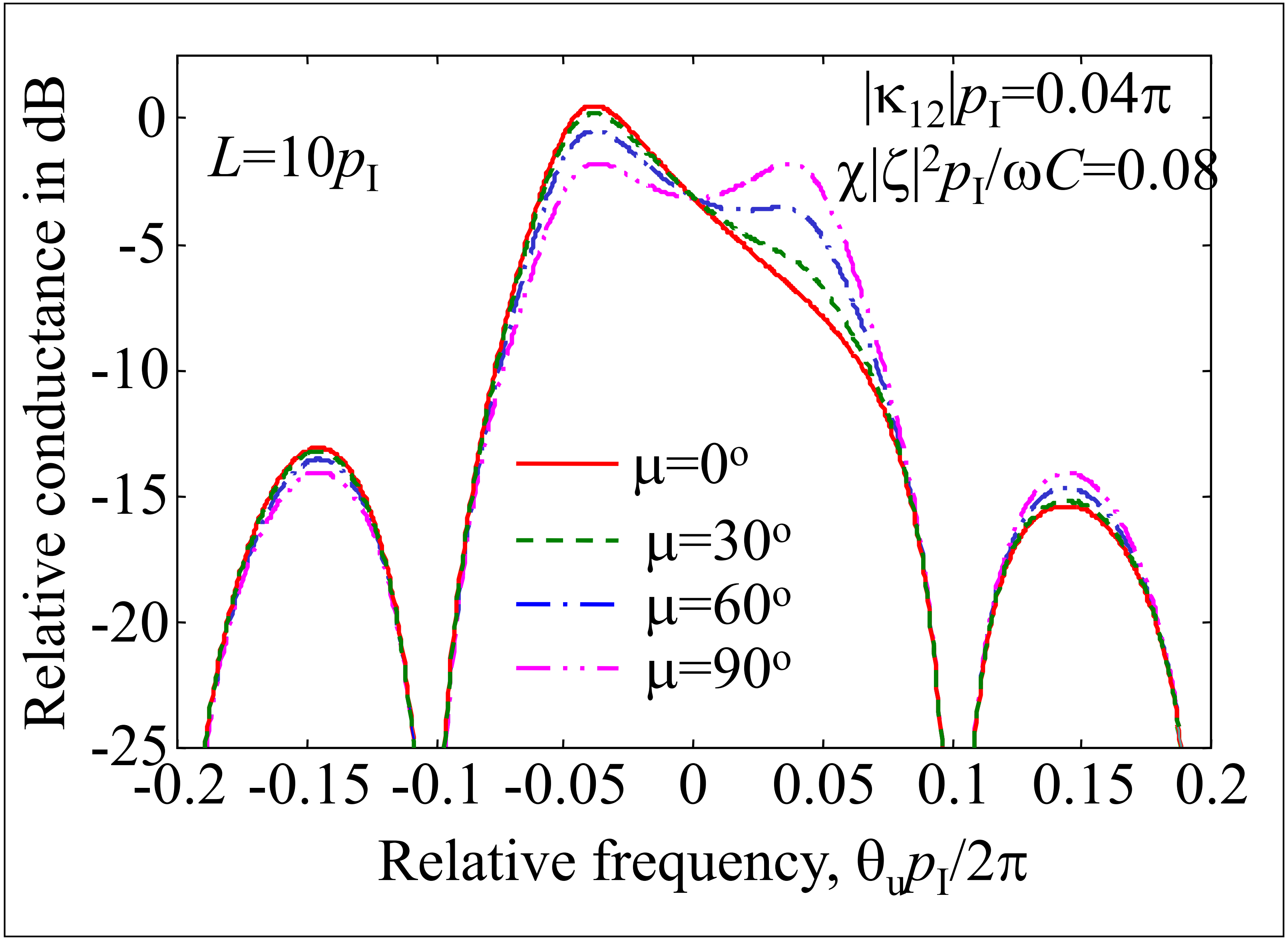
|κ12|pI=0.04π
χζ2pI/ωC=0.08
μ=∠(κ12/ζ2)=90o
Re(θu/pI)
Re(θp/pI)
Im(θp/pI)
Re(θ̂p/pI)
Im(θ̂p/pI)
Relative Wavenumber
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.04
-0.02
0
0.02
0.04
Relative frequency, θupI/2π



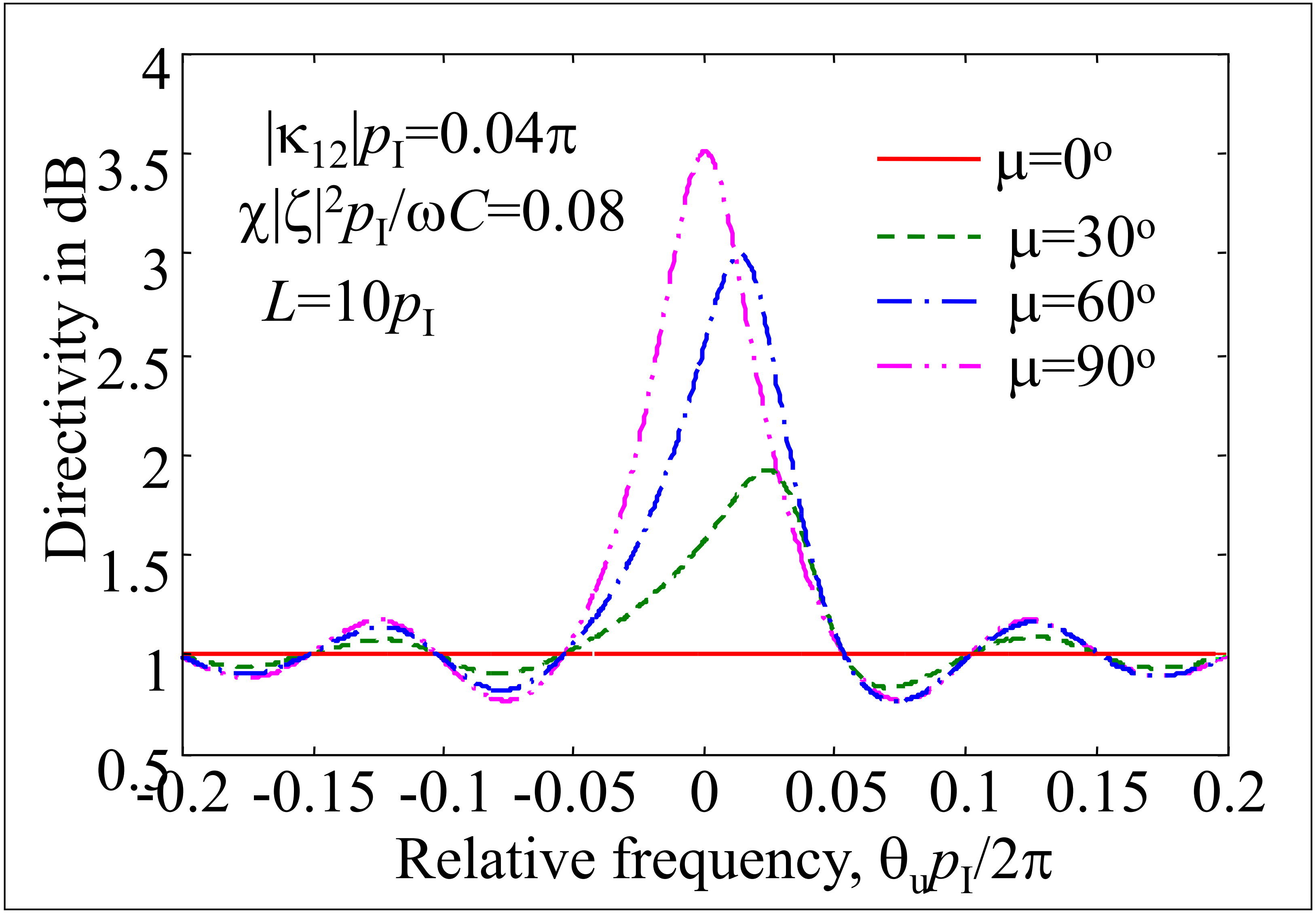
Relative admittance
Relative frequency, $\theta_u p_I/2\pi$
Imag
Real
$|\kappa_{12}|p_I$=0.04π
$\chi\zeta^2 p_I/\omega C$=0.08
μ=90°
L=10p_I
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2



Imag
Real
Relative admittance
4
2
0
-2
-4
|κ12|pI=0.04π
χ|ζ|2pI/ωC=0.08
-0.04
-0.02
0
0.02
0.04
Relative frequency, θupI/2π



$\mu = \angle(\kappa_{12}/\zeta^2)$

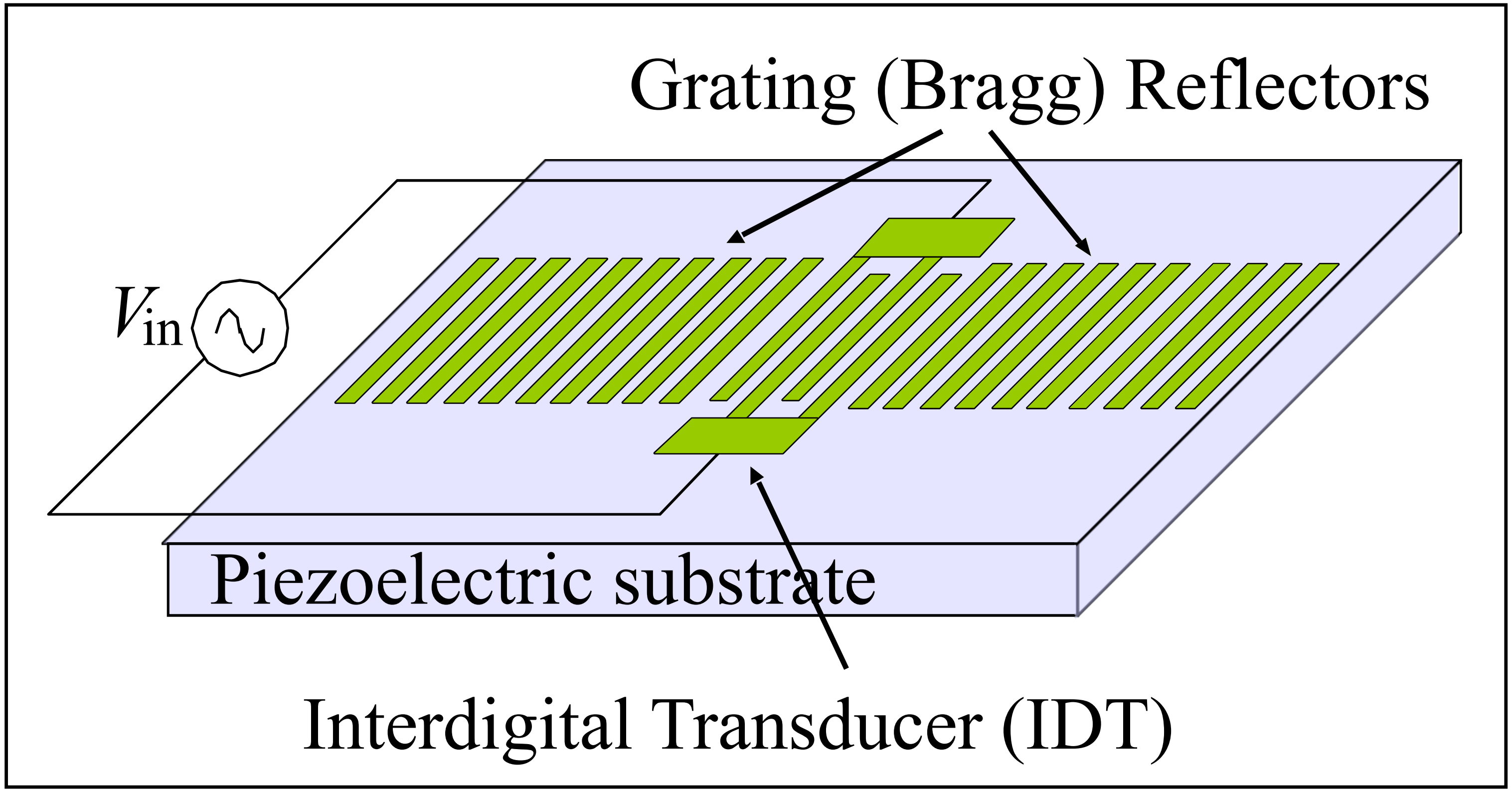


$|\kappa_{12}|p_I=0.04\pi$
$\chi|\zeta|^2 p_I/\omega C=0.08$
$L=10p_I$
μ=0°
μ=30°
μ=60°
μ=90°
Directivity in dB
Relative frequency, $\theta_u p_I/2\pi$
4
3.5
3
2.5
2
1.5
1
0.5
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2

Contents

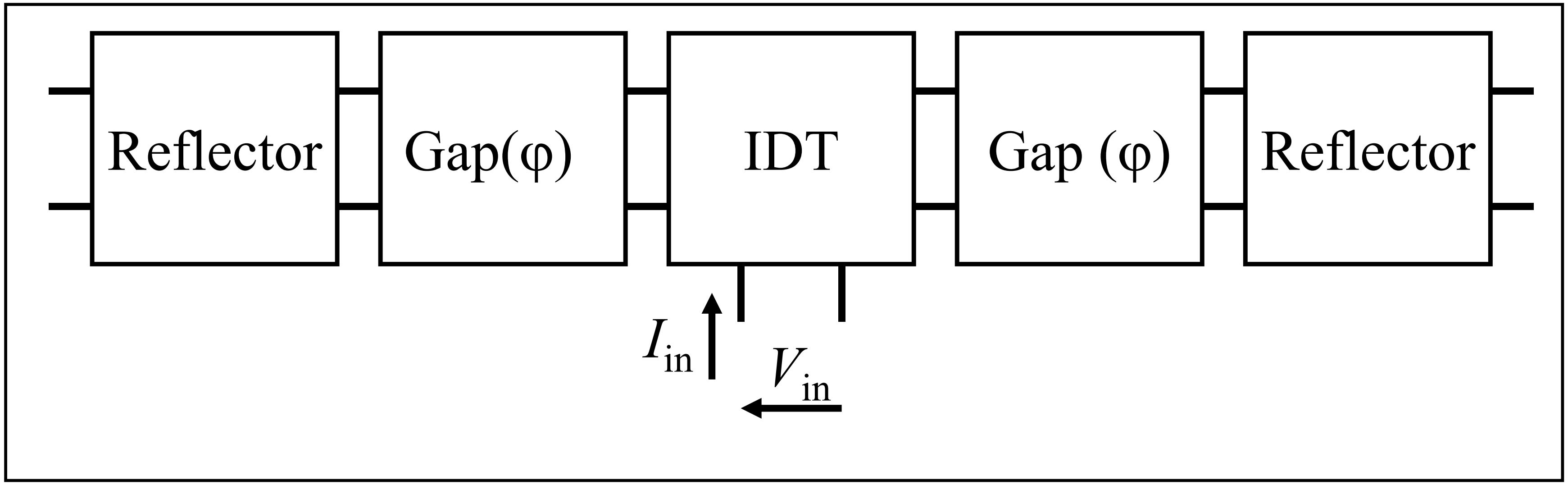
- SAW Device Simulation

Simulation of Complex Structures



- ***Combination of Periodic Structures***

Cascade-Connection of Elements



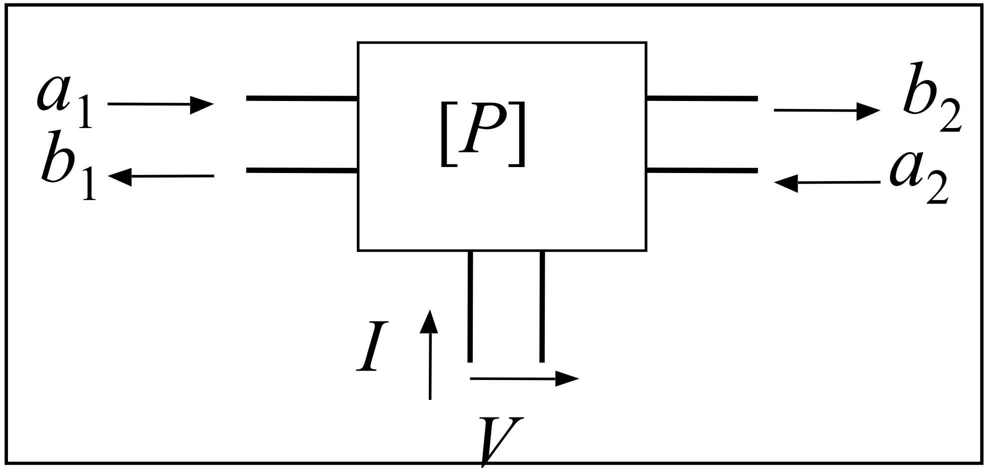
SC Grating = Short-Circuited IDT

OC Grating = IDT with Isolated Fingers

Gap = Reflection-less, Excitation-less IDT

IDT Modeling ⇨ ***Device Modeling***

P-Matrix Expression



$$\begin{pmatrix} b_1 \\ b_2 \\ I \end{pmatrix} = \begin{pmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ -\chi P_{13} & -\chi P_{23} & P_{33} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ V \end{pmatrix}$$

Unitary Condition:

$$|P_{11}|^2 + |P_{12}|^2 = 1, |P_{22}|^2 + |P_{12}|^2 = 1$$

$$p_{11} p_{13}^* + p_{12} p_{23}^* + p_{13} = 0$$

$$p_{12} p_{13}^* + p_{22} p_{23}^* + p_{23} = 0$$

$$\frac{\chi}{2}\left[|P_{11}|^2 + |P_{12}|^2\right] = \Re(p_{33})$$

Use of COM Model Gives

$$P_{11} = \frac{\Gamma_-(1-E^2)}{1-\Gamma_+\Gamma_- E^2},\ P_{22} = \frac{\Gamma_+(1-E^2)}{1-\Gamma_+\Gamma_- E^2}, P_{12} = \frac{E(1-\Gamma_+\Gamma_-)}{1-\Gamma_+\Gamma_- E^2}$$

$$P_{13} = \frac{(1-E)\{\xi_-(1+\Gamma_+\Gamma_- E) - \xi_+\Gamma_+(1+E)}{1-\Gamma_+\Gamma_- E^2}$$

$$P_{23} = \frac{(1-E)\{\xi_+(1+\Gamma_+\Gamma_- E) - \xi_-\Gamma_-(1+E)}{1-\Gamma_+\Gamma_- E^2}$$

$$P_{33} = \frac{\chi(1-E)\{(\xi_+ - \Gamma_-\xi_- E)(\zeta^* + \Gamma_+\zeta) + (\xi_- - \Gamma_+\xi_+ E)(\zeta + \Gamma_-\zeta^*)\}}{1-\Gamma_+\Gamma_- E^2}$$

$$- j\chi L(\zeta^*\xi_+ + \zeta\xi_-) + j\omega CL$$

where $E = \exp(-j\theta_{\mathrm{p}} L)$

When the unit is symmetrical,

$$P_{11} = P_{22} = \frac{\Gamma_0(1-E^2)}{1-\Gamma_0^2E^2},\ \ P_{12} = \frac{E(1-\Gamma_0^2)}{1-\Gamma_0^2E^2}$$

$$P_{13} = P_{23} = \frac{\xi(1-E)(1-\Gamma_0E)}{1+\Gamma_0E}$$

$$P_{33} = 2\chi\xi\zeta\left[\frac{(1-E)(1+\Gamma_0)}{\theta_p(1+\Gamma_0E)} - jL\right] + j\omega CL$$

Recursive Relation for Unit A (left) + B (right)

$$P_{11} = P_{11}^{A} + P_{11}^{B}\frac{P_{21}^{A}P_{12}^{A}}{1 - P_{11}^{B}P_{22}^{A}},\quad P_{22} = P_{22}^{B} + P_{22}^{A}\frac{P_{12}^{B}P_{21}^{B}}{1 - P_{11}^{B}P_{22}^{A}},\quad P_{12} = \frac{P_{12}^{A}P_{12}^{B}}{1 - P_{11}^{B}P_{22}^{A}}$$

$$P_{13} = P_{13}^{A} + P_{12}^{B}\frac{P_{13}^{B} + P_{11}^{B}P_{23}^{A}}{1 - P_{11}^{B}P_{22}^{A}},\ P_{23} = P_{23}^{B} + P_{21}^{B}\frac{P_{23}^{A} + P_{22}^{A}P_{13}^{B}}{1 - P_{11}^{B}P_{22}^{A}}$$

$$P_{33} = P_{33}^{A} + P_{33}^{B} + P_{32}^{A}\frac{P_{13}^{B} + P_{11}^{B}P_{23}^{A}}{1 - P_{11}^{B}P_{22}^{A}} + P_{31}^{B}\frac{P_{23}^{A} + P_{22}^{A}P_{13}^{B}}{1 - P_{11}^{B}P_{22}^{A}}$$

Contents

- Parameter Extraction

Determination of COM Parameters

$$\frac{\partial U_+}{\partial X} = -j\theta_u U_+ - j\kappa_{12} U_- + j\zeta V_0$$

$$\frac{\partial U_-}{\partial X} = +j\kappa_{12}^* U_+ + j\theta_u U_- - j\zeta^* V_0$$

$$\frac{\partial I}{\partial X} = -j\chi\zeta^* U_+ - \chi j\zeta U_- + j\omega C V_0$$

κ_{12}: Mutual Coupling Coefficient (Mostly Constant)

ζ: Transduction Coefficient (Mostly Constant)

C: Capacitance (Mostly Constant)

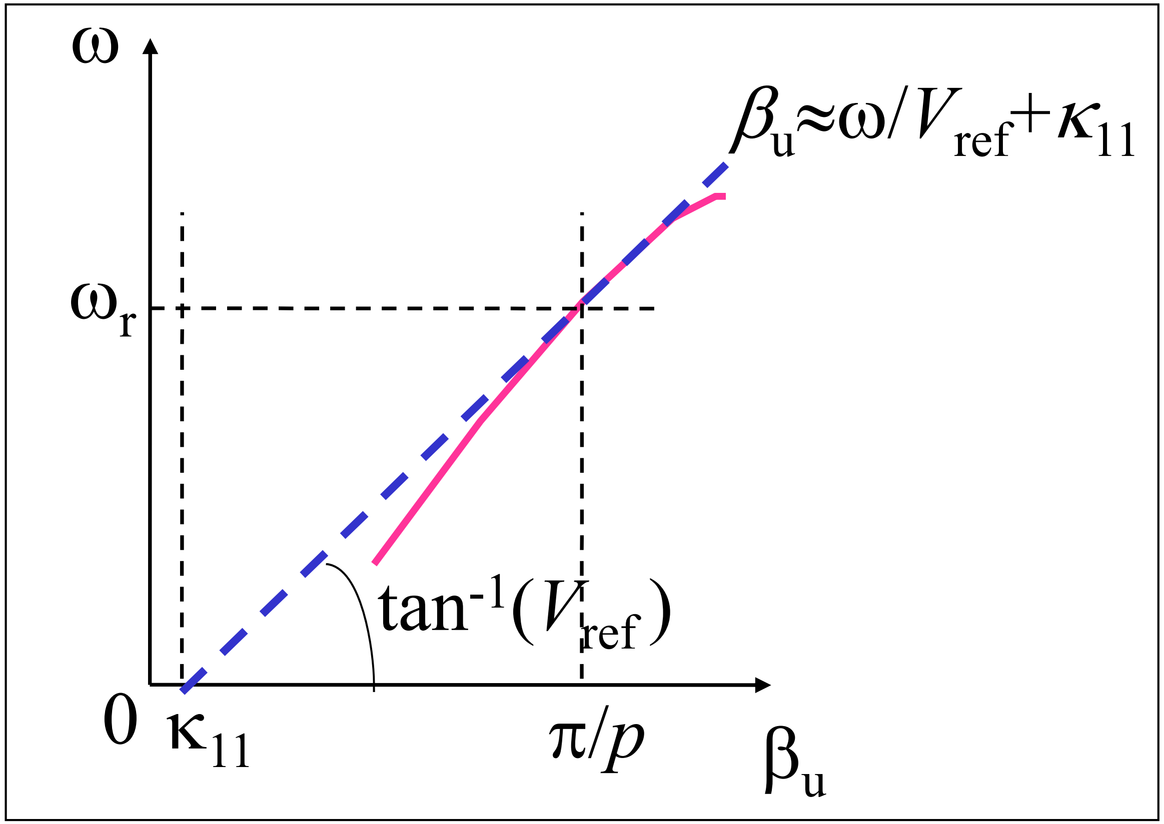
θ_u: detuning factor (Linearly Changes with ω)

$\Rightarrow \quad \theta_u = \omega / V_{ref} - \pi/p + \kappa_{11}$

V_{ref}: Reference SAW Velocity

κ_{11}: Self Coupling Coefficient

Physical Mean of κ_{11} and V_{ref}



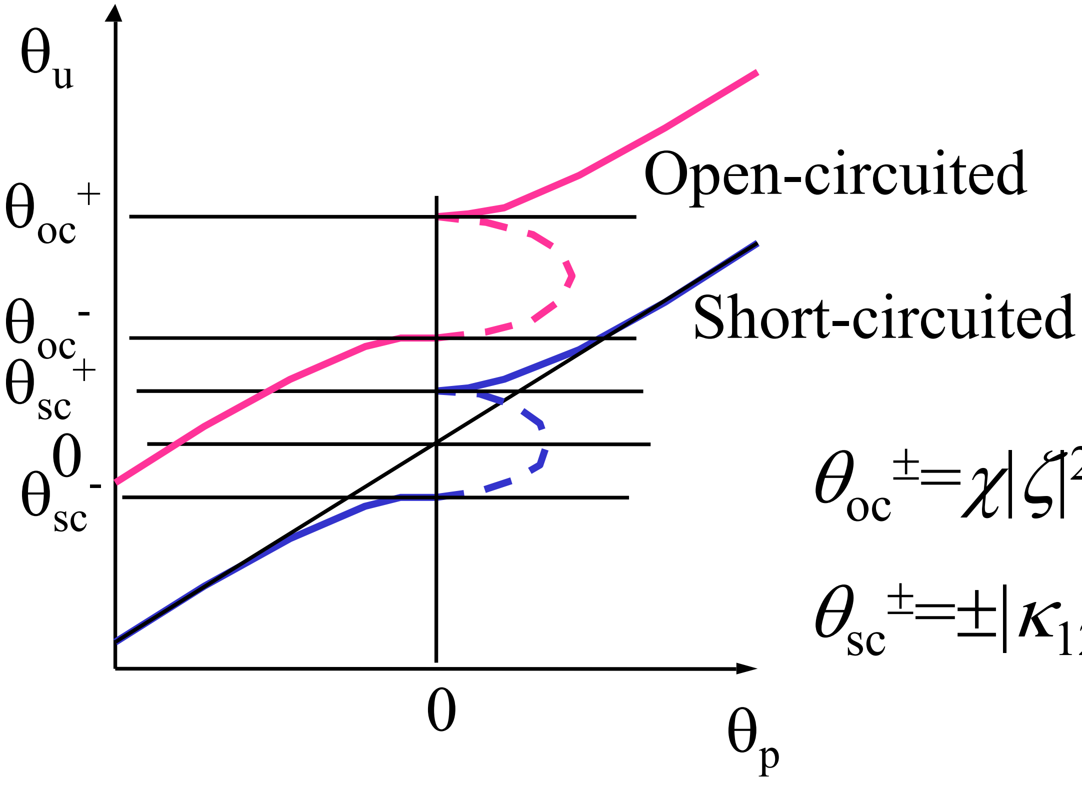
$$\kappa_{11} = \pi/p - \omega_r / V_{ref}$$

For Short-Circuited (SC) Grating, V_0=0

$$\theta_p = \sqrt{\theta_u^2 - |\kappa_{12}|^2}$$

For Open-Circuited (OC) Grating, δI=0

$$\theta_p = \sqrt{(\theta_u - \chi|\zeta|^2/\omega C)^2 - |\kappa_{12} - \chi\zeta^2/\omega C|^2}$$



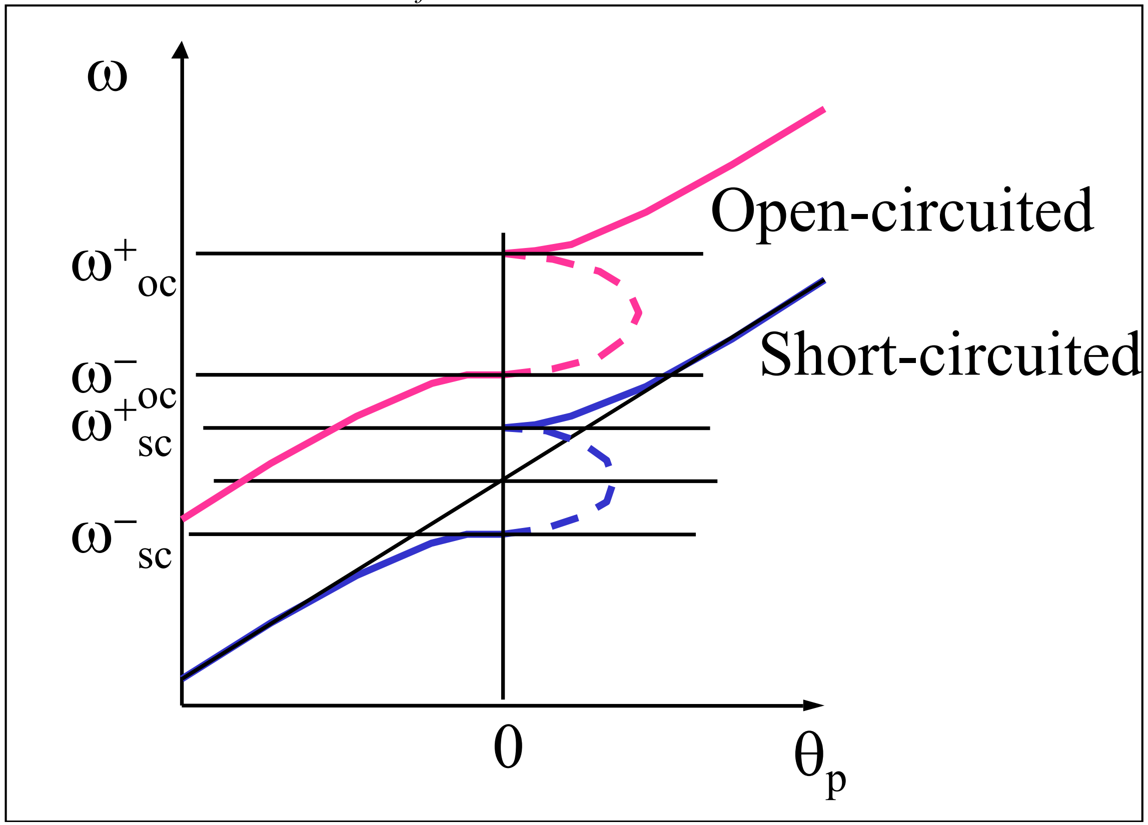
$$\theta_{oc}^{\pm} = \chi|\zeta|^2/\omega C \pm |\kappa_{12} - \chi\zeta^2/\omega C|$$

$$\theta_{sc}^{\pm} = \pm|\kappa_{12}|$$

Since $\chi|\zeta|^2/\omega C \pm|\kappa_{12}-\chi\zeta^2/\omega C|=\omega^{\pm}_{oc}/V_{ref}-\pi/p+\kappa_{11}$
& $\pm|\kappa_{12}|=\omega^{\pm}_{sc}/V_{ref}-\pi/p+\kappa_{11}$,

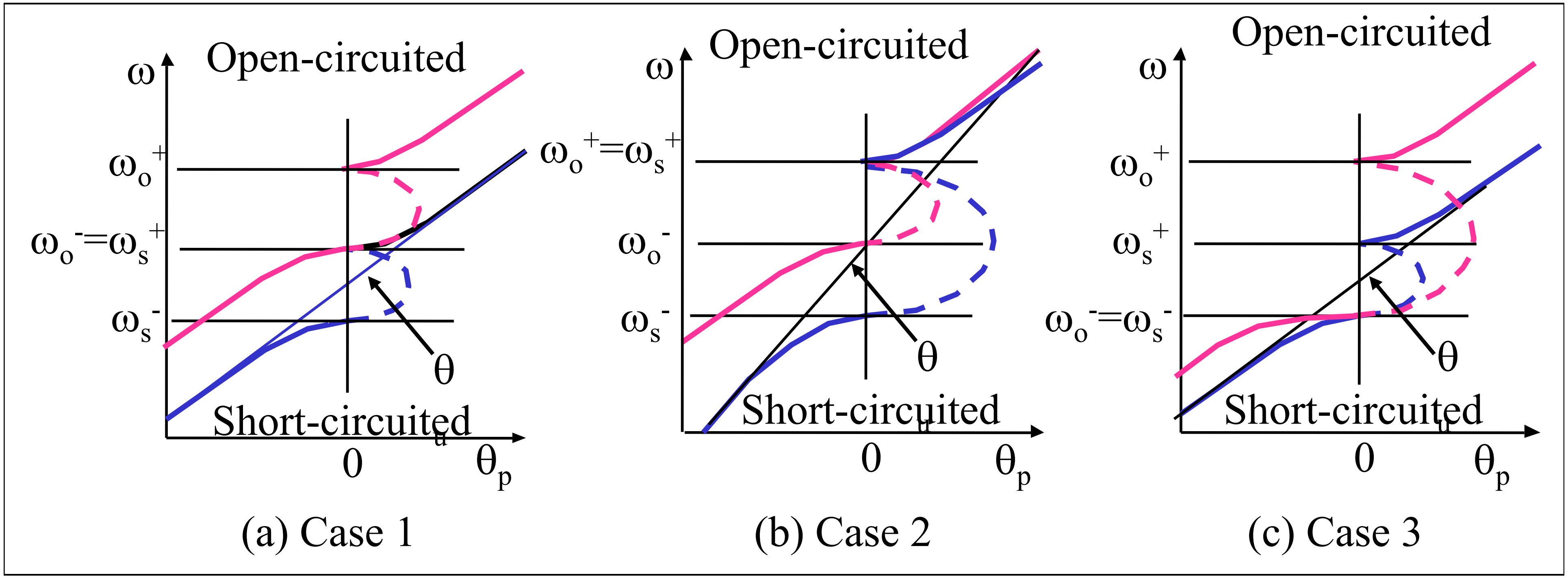
$$\kappa_{11} = \frac{\pi}{p} - \frac{\omega^{+}_{sc} + \omega^{-}_{sc}}{2V_{ref}} \qquad \left|\frac{\chi\zeta^2}{\omega C}\right| = \frac{(\omega^{+}_{oc} + \omega^{-}_{oc}) - (\omega^{+}_{sc} + \omega^{-}_{sc})}{2V_{ref}}$$

$$|\kappa_{12}| = \frac{\omega^{+}_{sc} - \omega^{-}_{sc}}{2V_{ref}} \qquad \left|\kappa_{12} - \frac{\chi\zeta^2}{\omega C}\right| = \frac{\omega^{+}_{oc} - \omega^{-}_{oc}}{2V_{ref}}$$



Sign of $\psi=\angle(\zeta^2/\kappa_{12})$ can not be Determined

When IDT is Bidirectional, ζ^2/κ_{12} is Real



One of Stopband Edge for OC Grating Coincides with that for SC Grating

Relation Between Stopband Edges and COM Parameters

$$\kappa_{11} = \frac{\pi}{p} - \frac{\omega_{sc}^{+} + \omega_{sc}^{-}}{2V_{ref}}$$

$$\kappa_{12} = s\frac{\omega_{sc}^{+} - \omega_{sc}^{-}}{2V_{ref}}$$

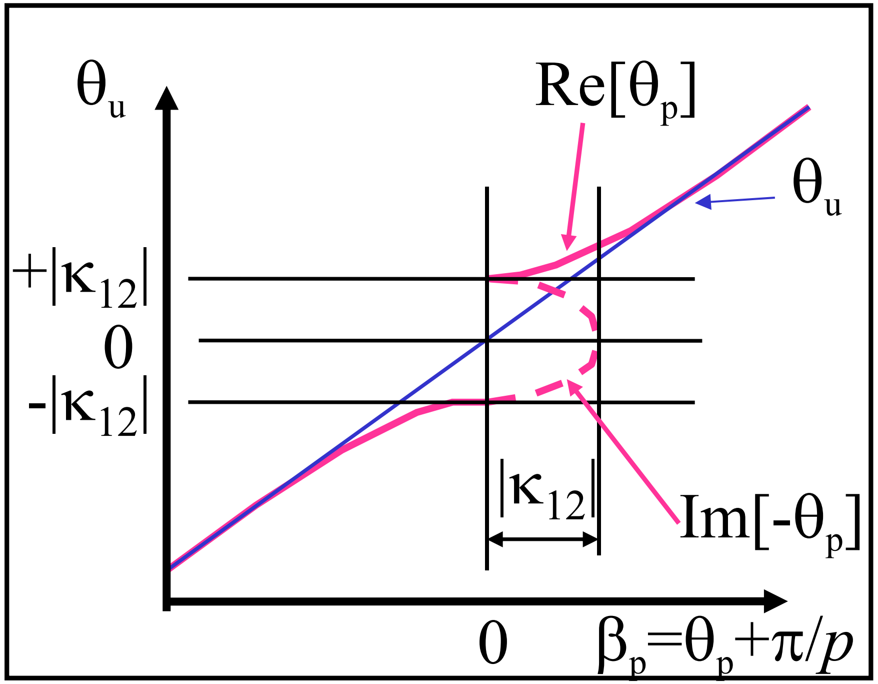
$$\frac{\chi\zeta^{2}}{\omega C} = \frac{(\omega_{oc}^{+} + \omega_{oc}^{-}) - (\omega_{sc}^{+} + \omega_{sc}^{-})}{2V_{ref}}$$

$$s = \begin{cases} 1 & (\omega_{sc}^{+} = \omega_{oc}^{\pm}) \\ -1 & (\omega_{sc}^{-} = \omega_{oc}^{-}) \end{cases}$$

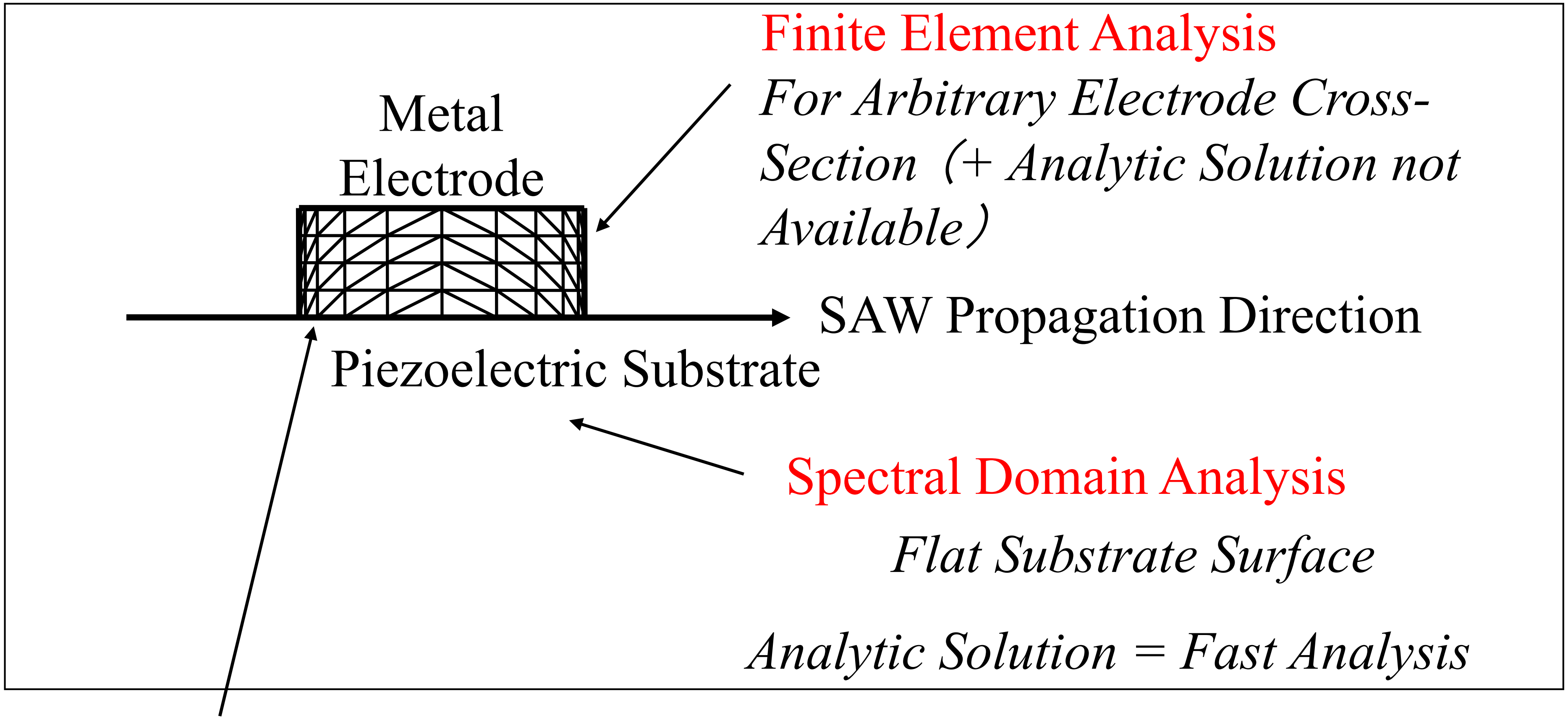
How to Determine V_{ref}?

1. Determination of $|\kappa_{12}|$ by Max[-Im(θ_p)]

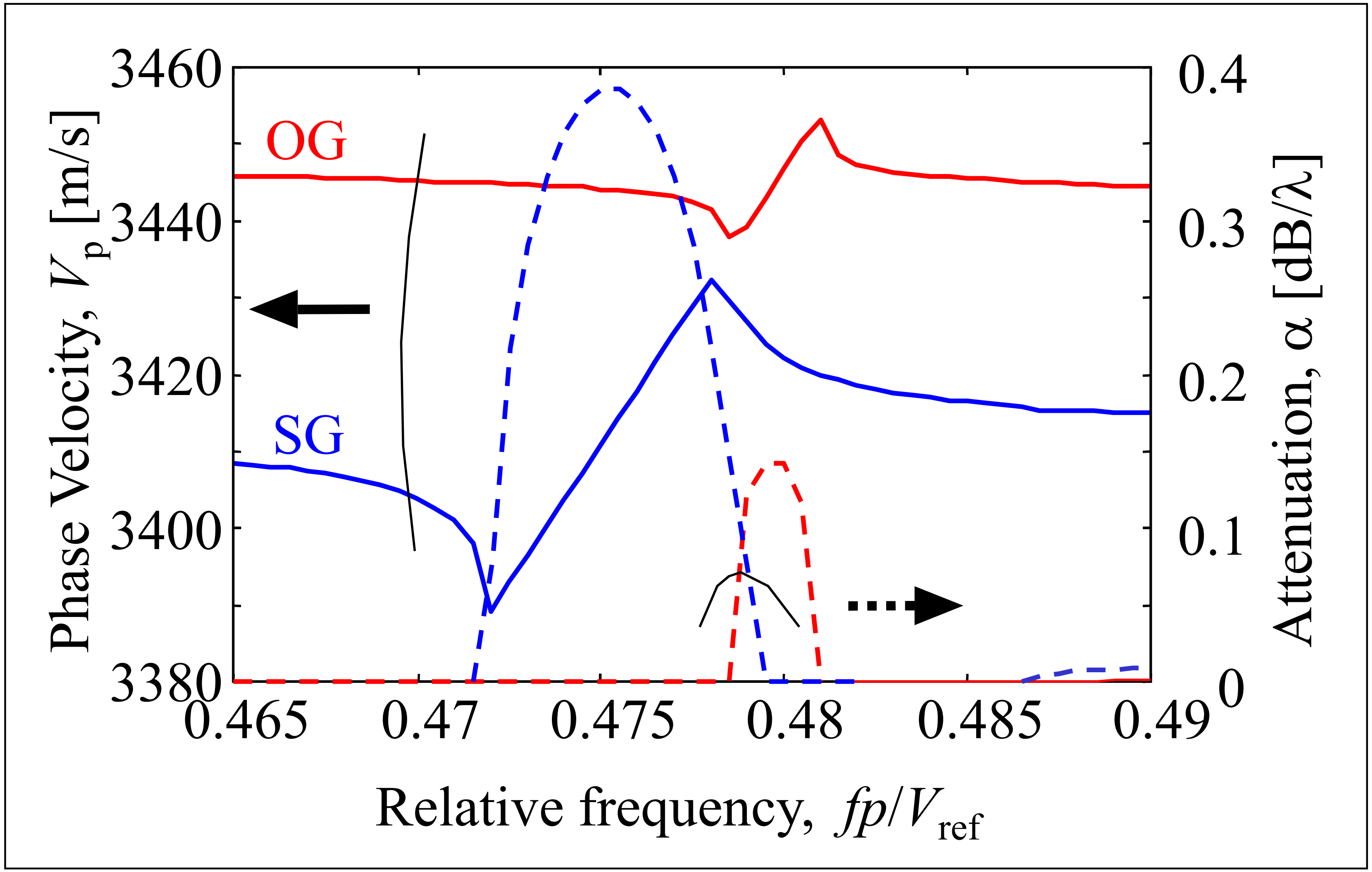
2. Determination of V_{ref} by Stopband Edges



FEMSDA (Full Wave Simulator)



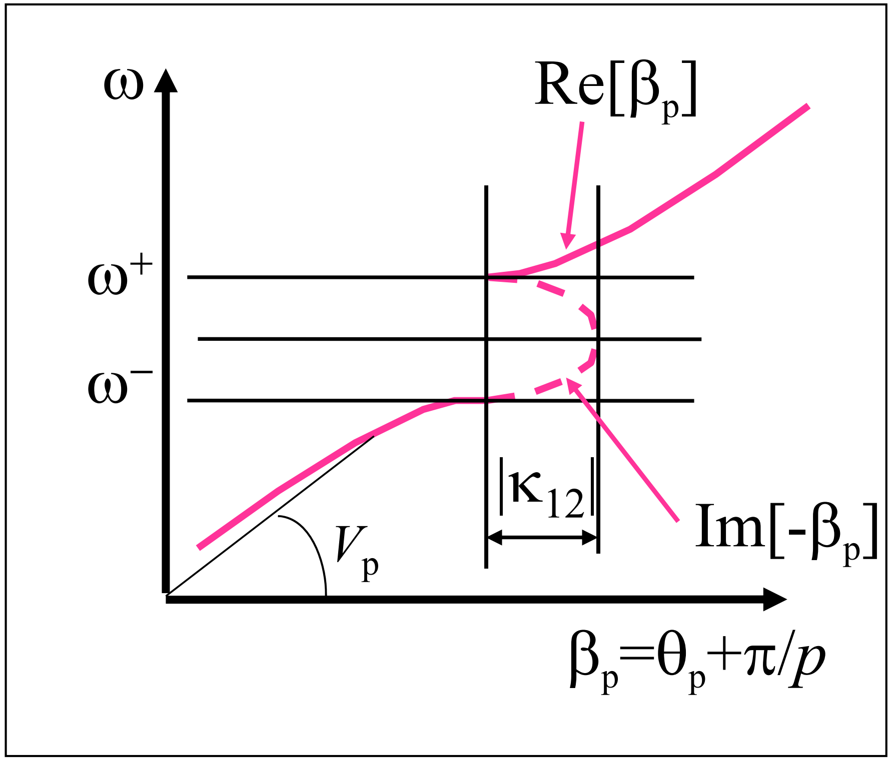
Boundary Condition: Minimization of Radiated Power (Error) from Boundary

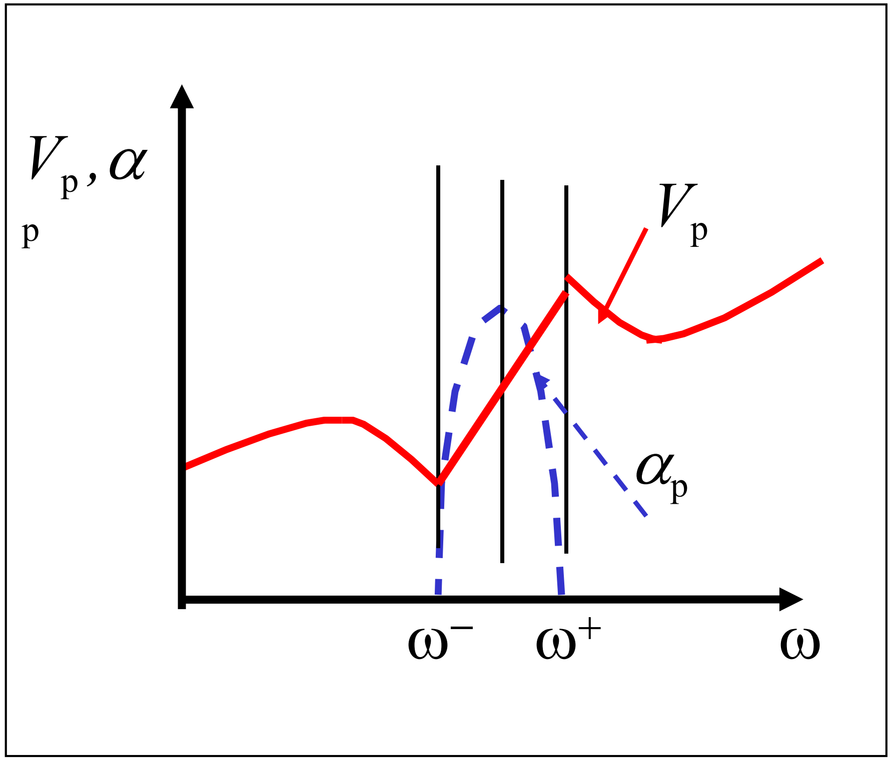


Dispersion of Rayleigh SAW on YZ-LN (h/p=0.07) Calculated by ***FEMSDA***

V_B=3,590.1 m/s (Slow-shear SSBW velocity)

Fitting with Full Wave Analysis

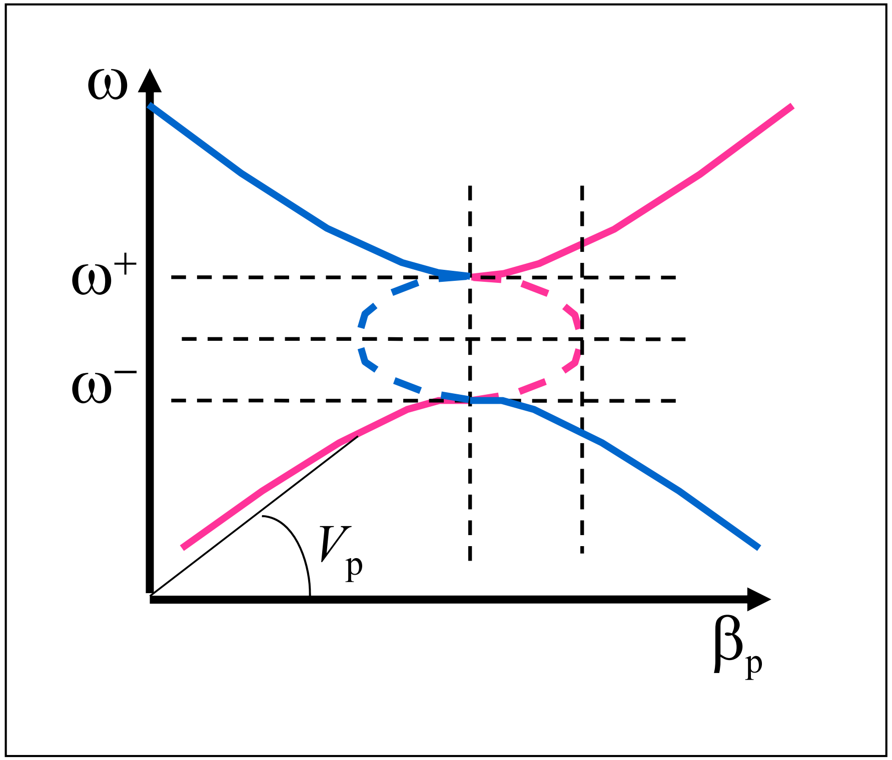


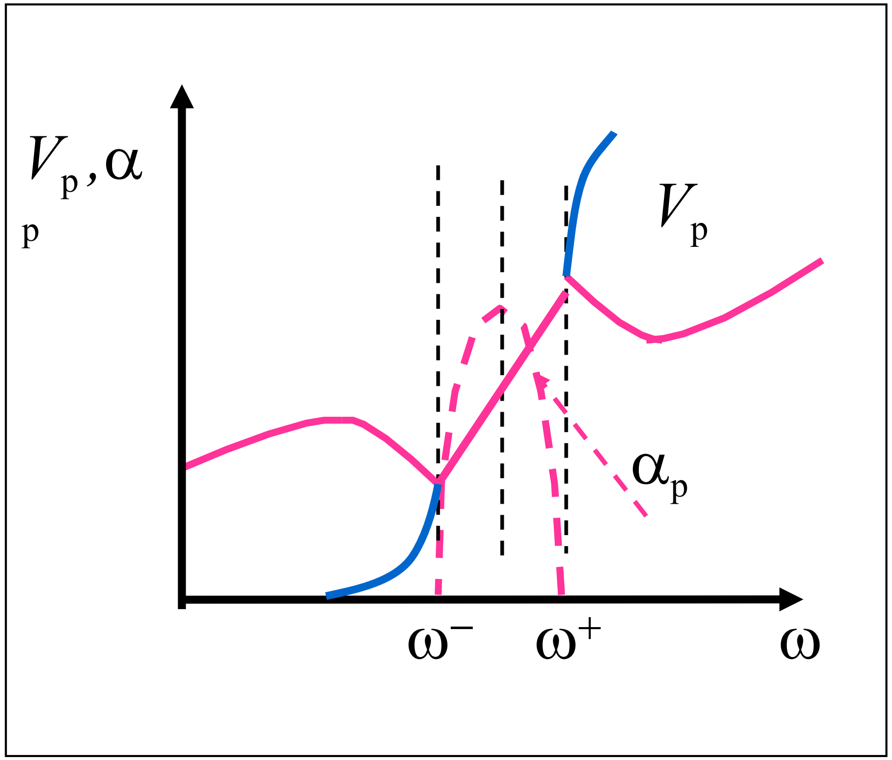


Phase Velocity: $V_p = \omega/\mathrm{Re}(\beta_p)$

Attenuation: $\alpha_p = 40\pi\log_{10}e \times \mathrm{Im}(-\beta_p)\ /\mathrm{Re}(\beta_p)$ [dB/λ]

Existence of Multiple Solutions





Possibility to Jump into Blue Branch

Most Possible near Stopband Edges

Countermeasure: Attacking Upward and/or Downward

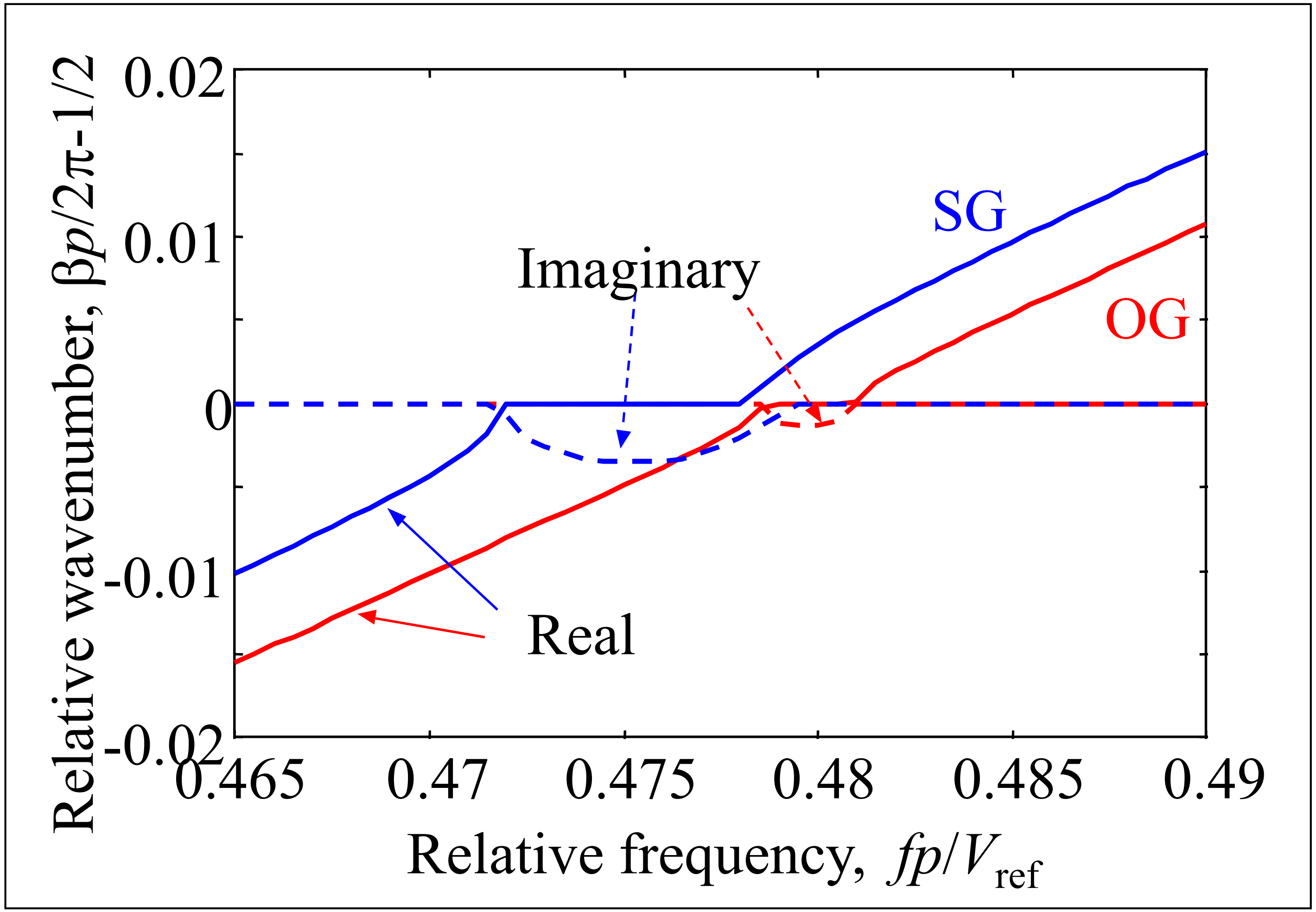
Efficient Calculation by Combining FEMSDA and SYNC

Single-Electrode IDT

1. FEMSDA for determination of β for OC & SC
2. Fitting after Squared
3. SYNC for determination of C

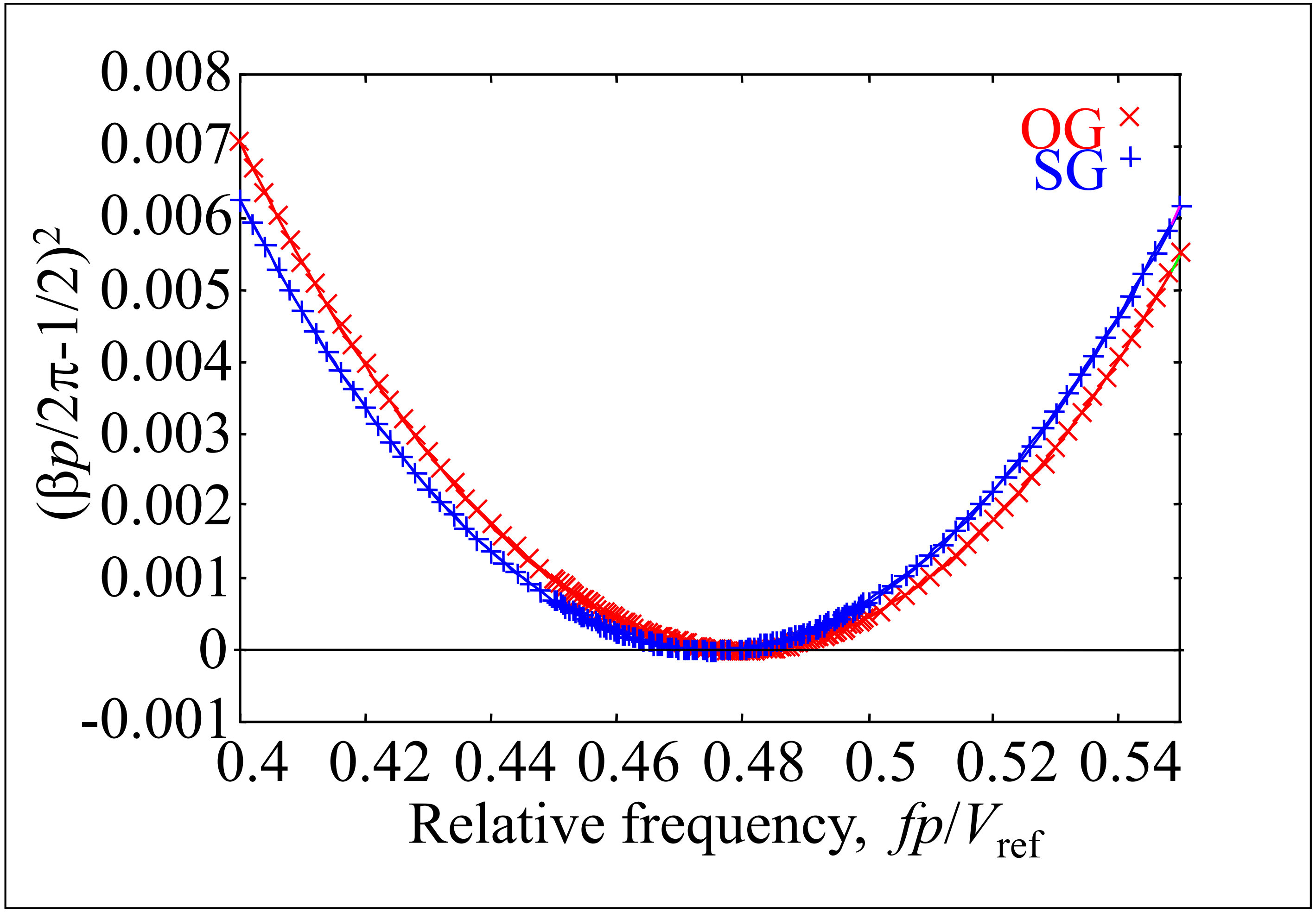
Double-Electrode IDT

1. MSYNC for calculation of input impedance of infinitely long IDT
2. Determination of *C* & frequencies giving stopband edges by fitting
3. MULTI for determination of β for SC



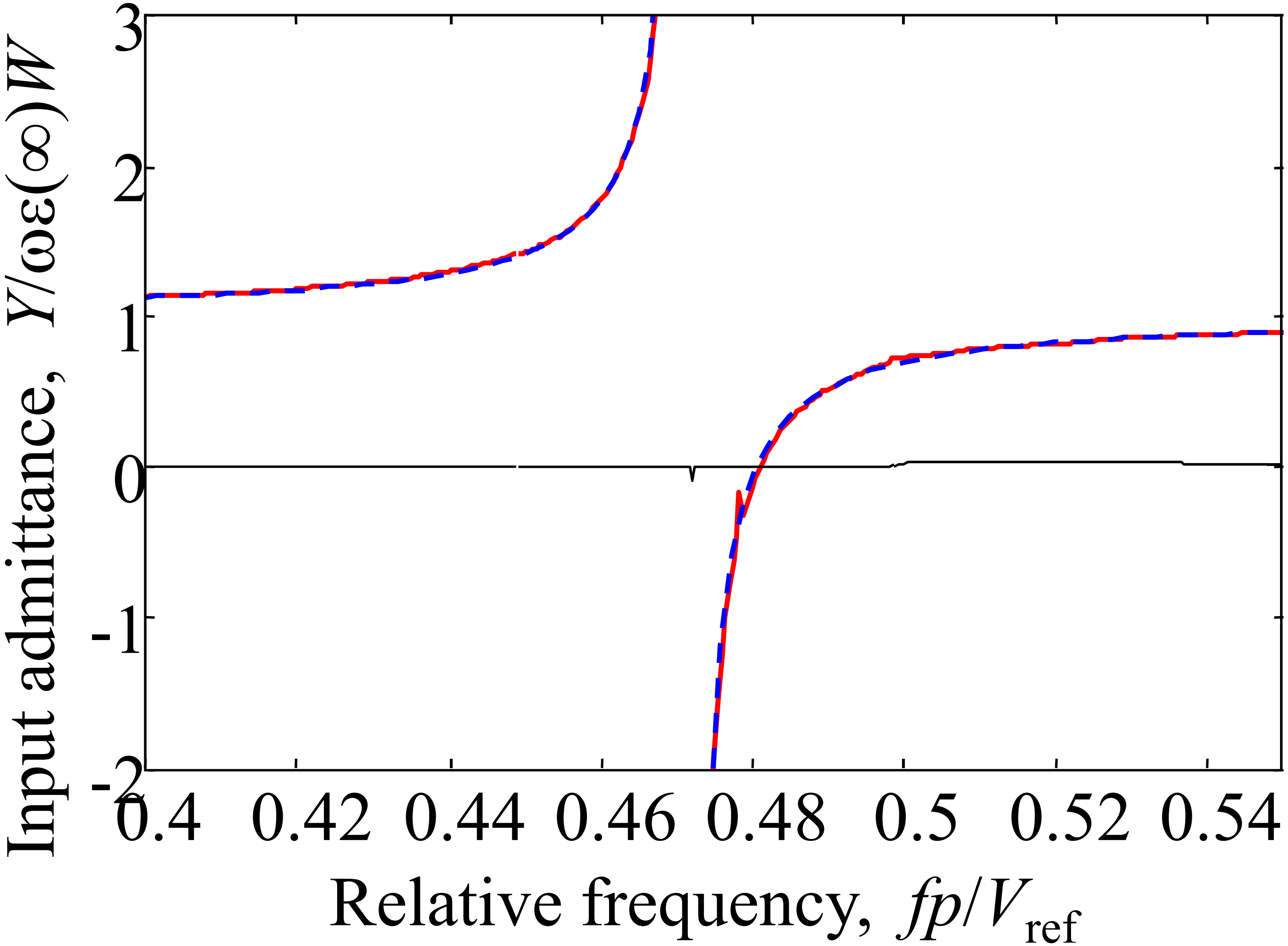
Wavenumber of Rayleigh SAW on YZ-LN (h/p=0.07) Calculated by ***FEMSDA***

V_B=3,590.1 m/s (Slow-shear SSBW velocity)



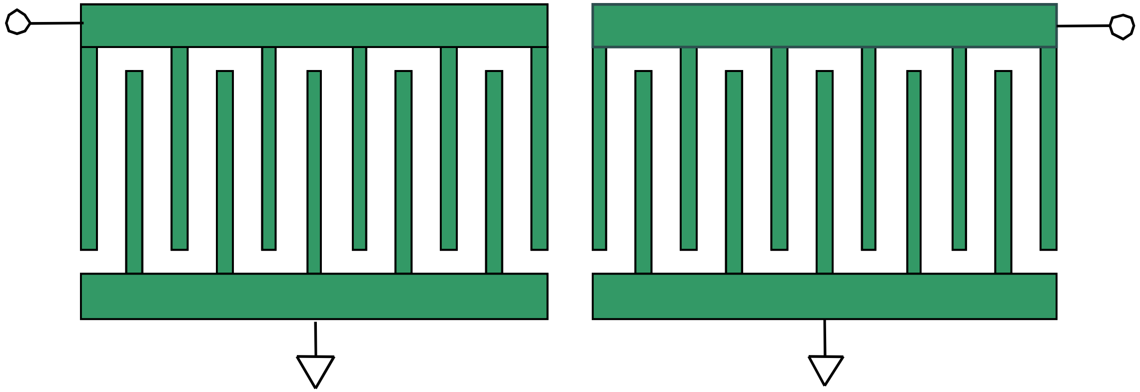
Squared Wavenumber of Rayleigh SAW on YZ-LN (h/p=0.07) Calculated by ***FEMSDA***

V_B=3,590.1 m/s (Slow-shear SSBW velocity)

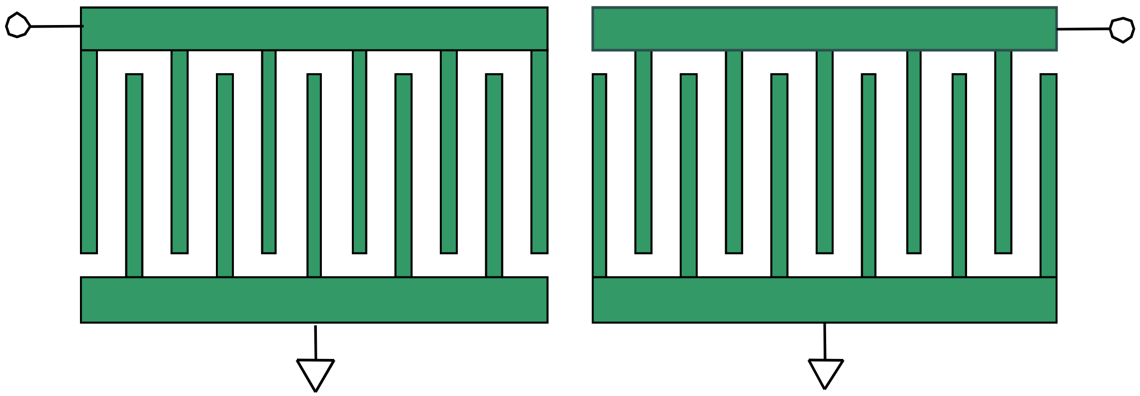


Input Admittance of Infinitely long single-electrode IDT on YZ-LN (h/p=0.07) Calculated by ***SYNC***

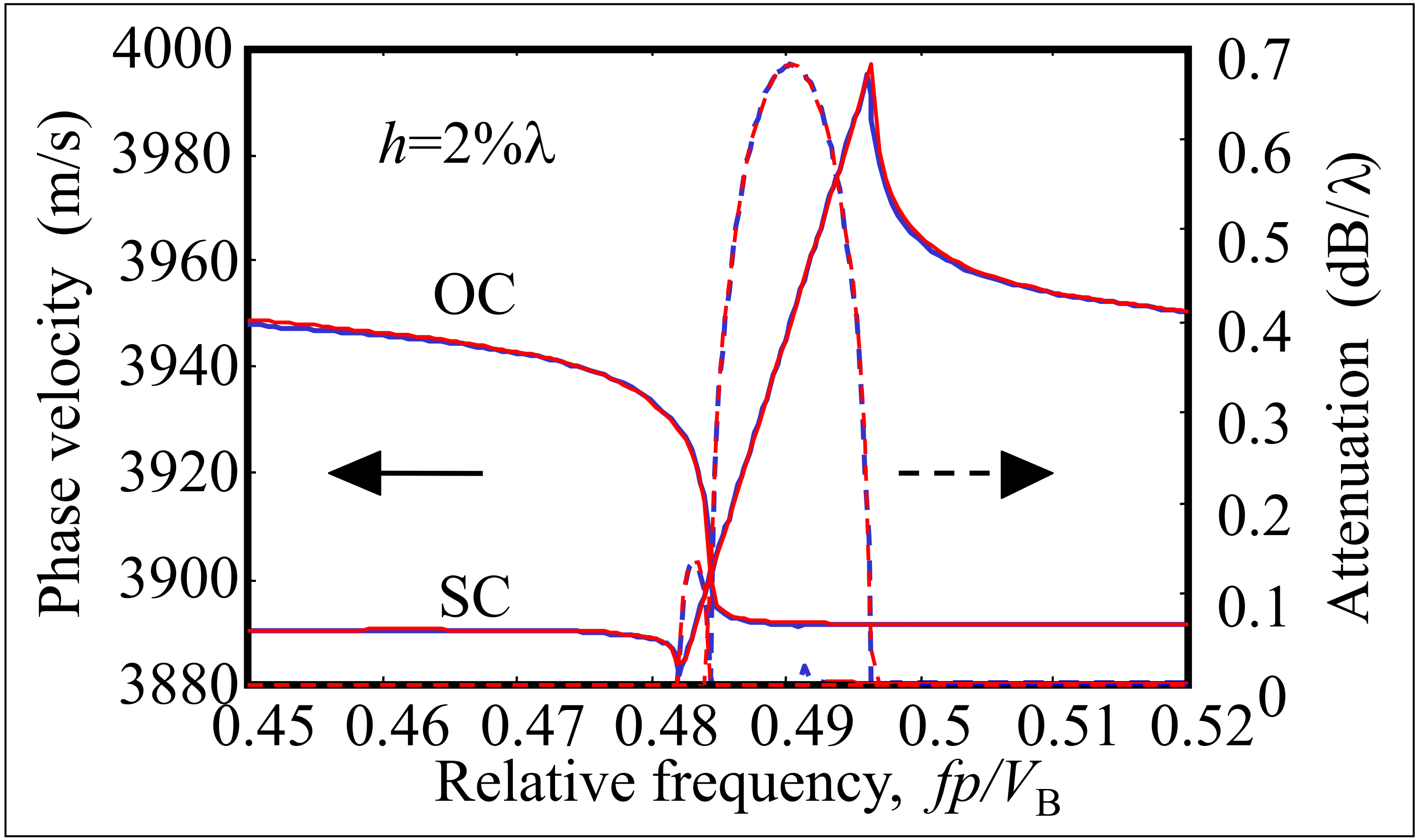
V_B=4,030.8 m/s (Slow-shear SSBW velocity)



(A) ζ with same polarity for input and output IDTs



(B) ζ with opposite polarity for input and output IDTs



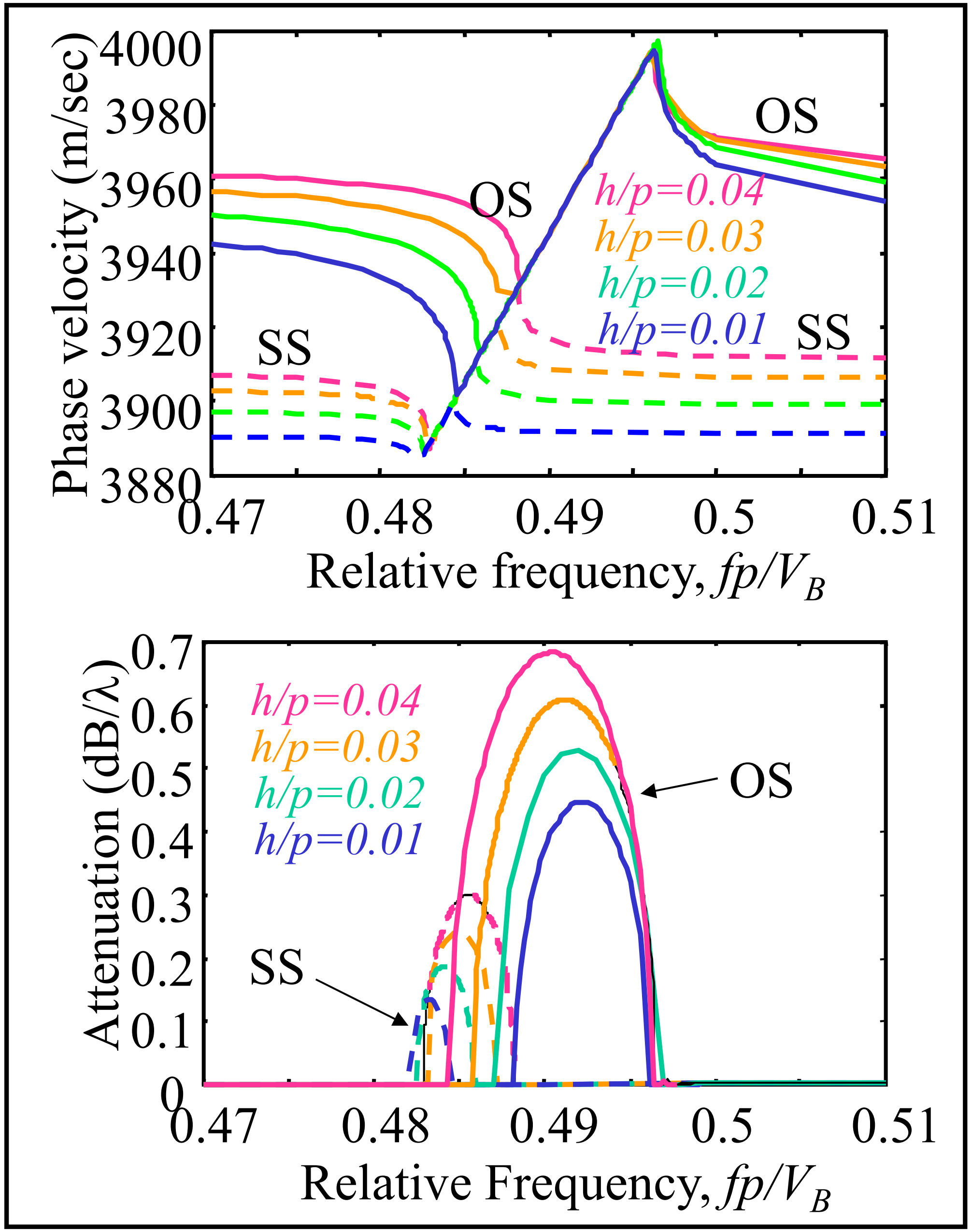
Dispersion of Rayleigh SAW on 128-LN

Blue: Analysis by ***FEMSDA***

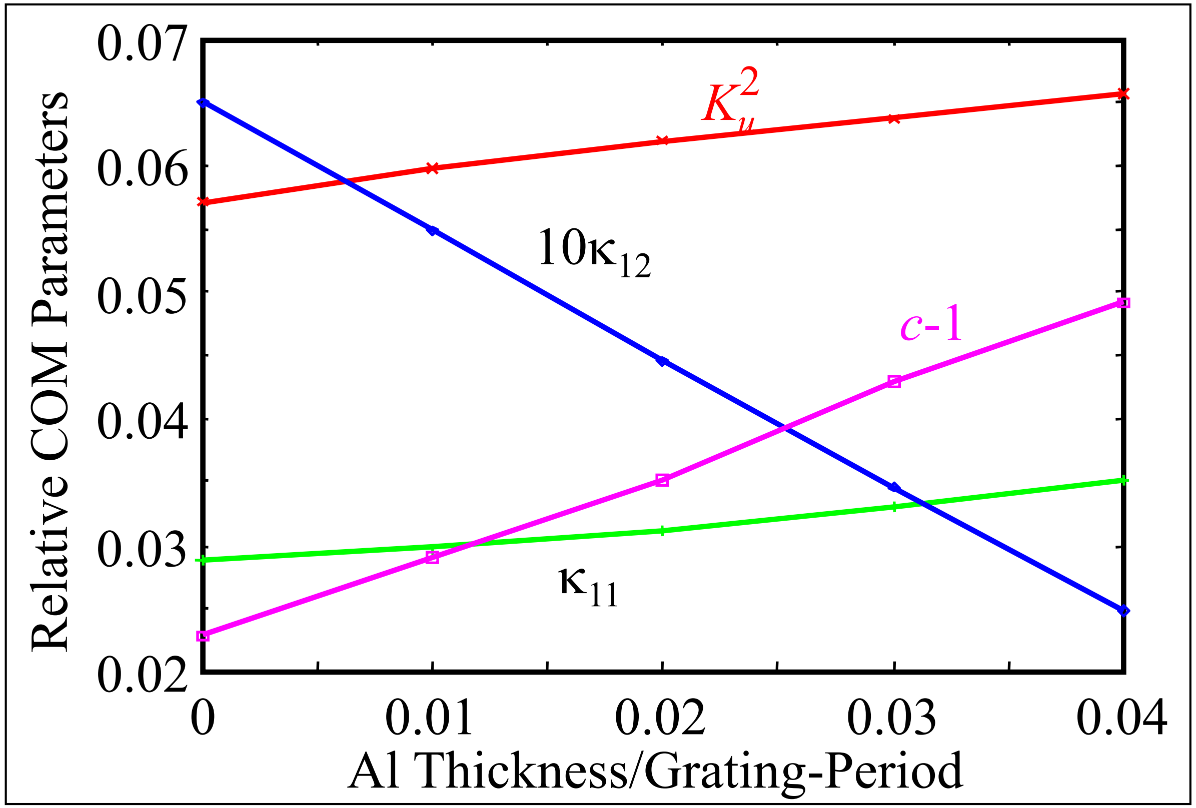
Red: Conventional COM Analysis

V_B=4,025 m/s (Slow-shear SSBW velocity)

Dispersion Relation vs. Al Thickness



Change in COM Parameters with Al Thickness



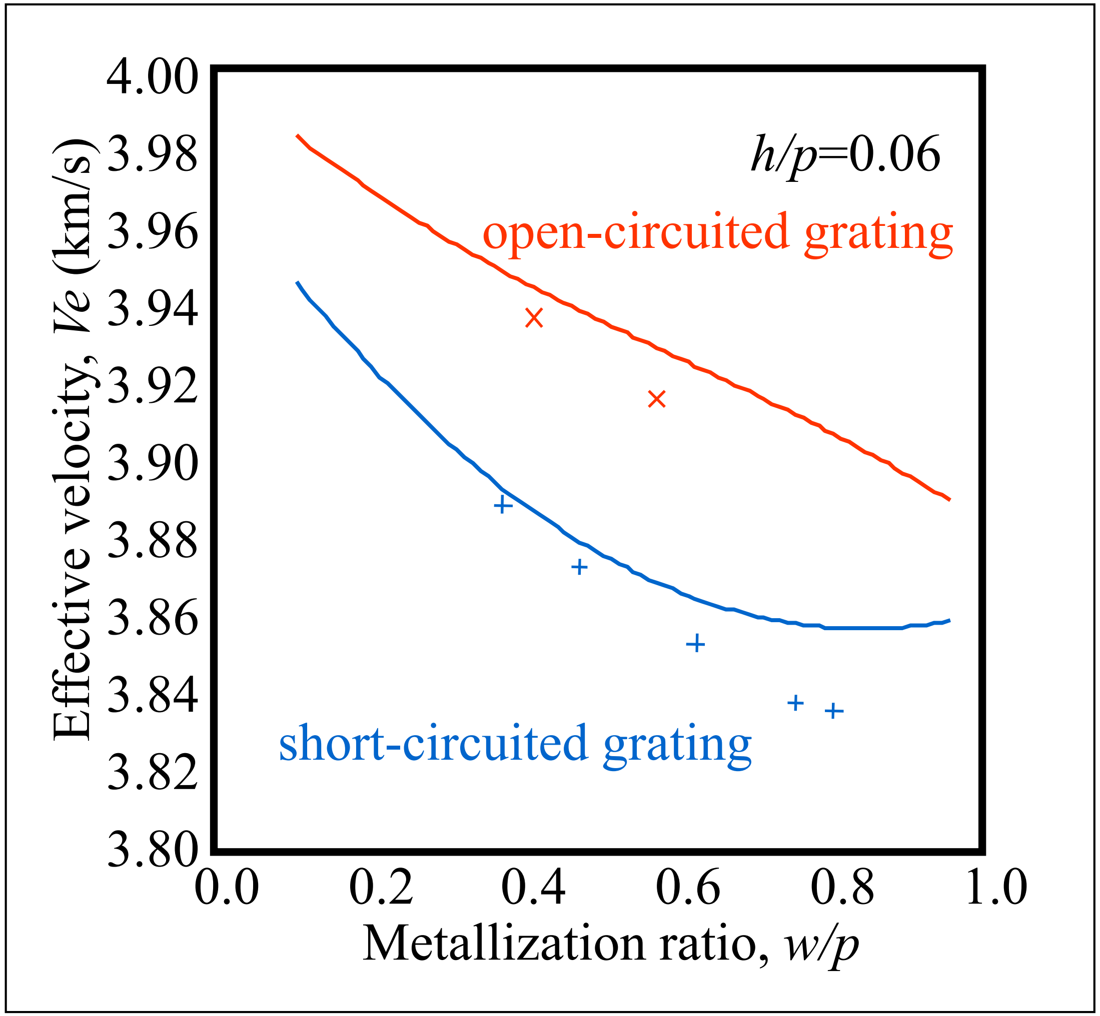
$K_u^2 = \dfrac{\pi\chi|\zeta|^2 p_I}{4\omega C}$: Electromechanical Coupling Factor for ***Perturbed Mode***

$c=V_B/V_{ref}$ V_B=4,025 m/s (Slow Shear SSBW)

Correction of Simulation Parameters

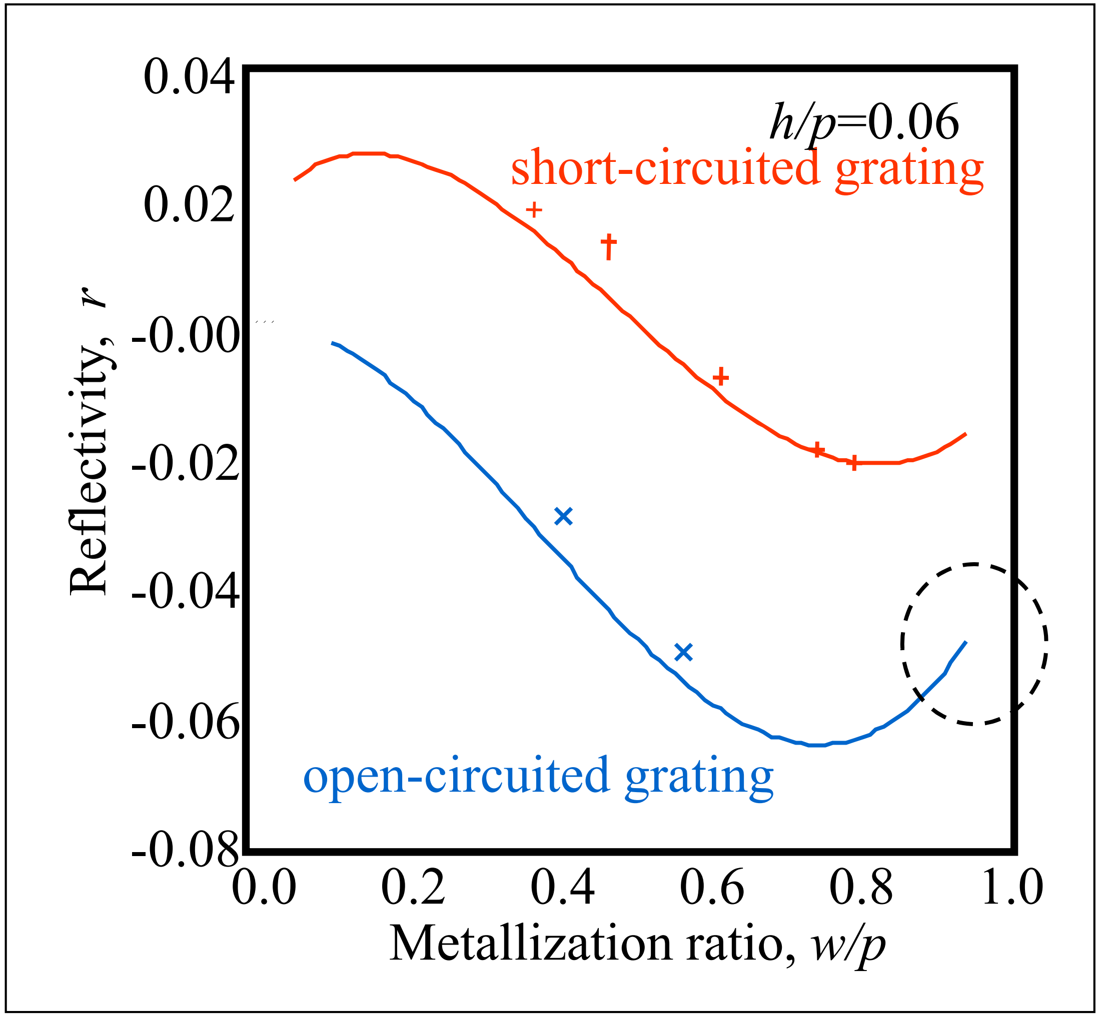
- Uncertainties in Substrate Material Constants (Supplier and Lot Dependent)
- Uncertainties in Film Material Constants (Fab. Process Dependent)
- Electrode Cross-Section (Fab. Process Dependent)

Although their Absolute Values may be Doubtful, Dependencies on Device Parameters might be held



Effective Velocity of Rayleigh-type SAW on 128-LN vs. Metallization Ratio

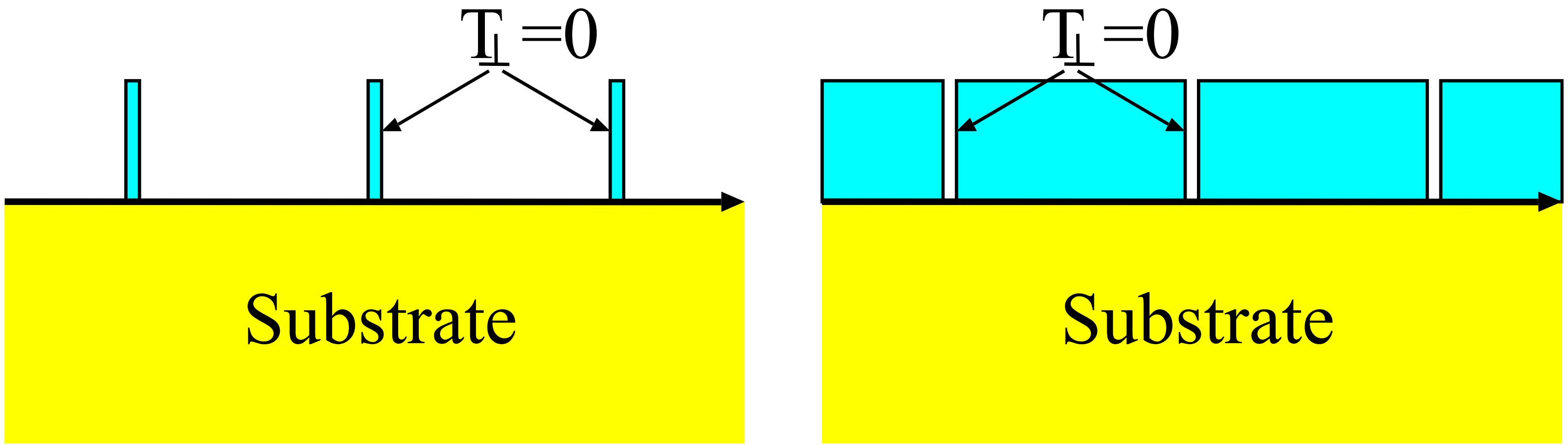
Solid Lines: FEMSDA, +×: Experiment



Reflectivity of Rayleigh-type SAW on 128-LN vs. Metallization Ratio

Solid Lines: FEMSDA, +×: Experiment

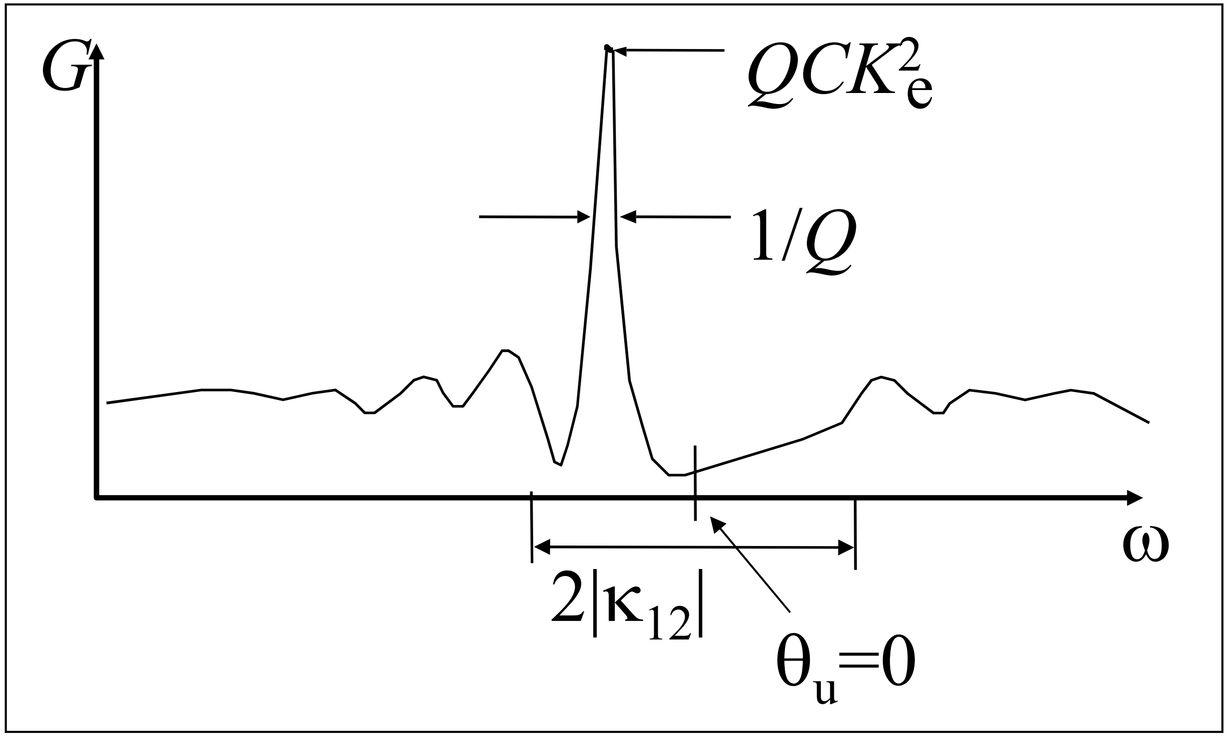
Behavior in Ultimate Situations



(a) When w/p~0 (b) When w/p~1

w/p→1 is not Equivalent to Flat Metallization!

Relation of COM Parameters with Resonance Characteristics

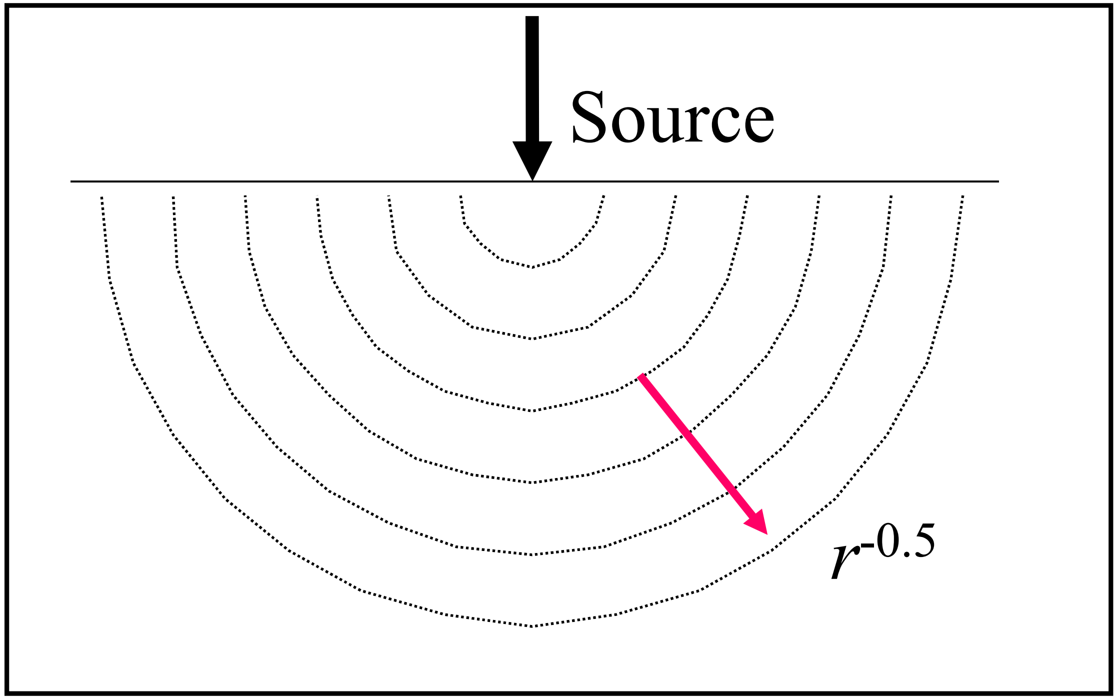


Each Parameter Independently Relates Each Property* ⇒ *Easy to Fit with Experiments

Contents

- BAWs and SH SAWs

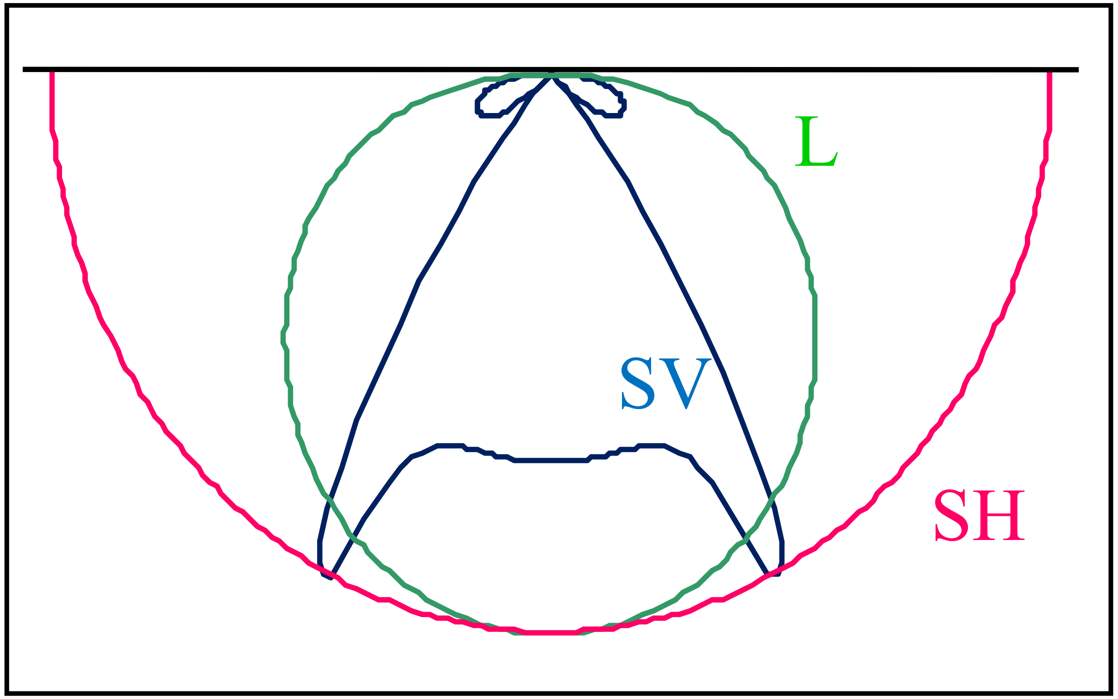
Excitation and Propagation of BAWs



Propagation of Cylindrical Wave
$(P \propto r^{-1} \Rightarrow u \propto r^{-0.5})$

Rapid Attenuation When Influence of Surface is Significant

Radiation Pattern of BAWs



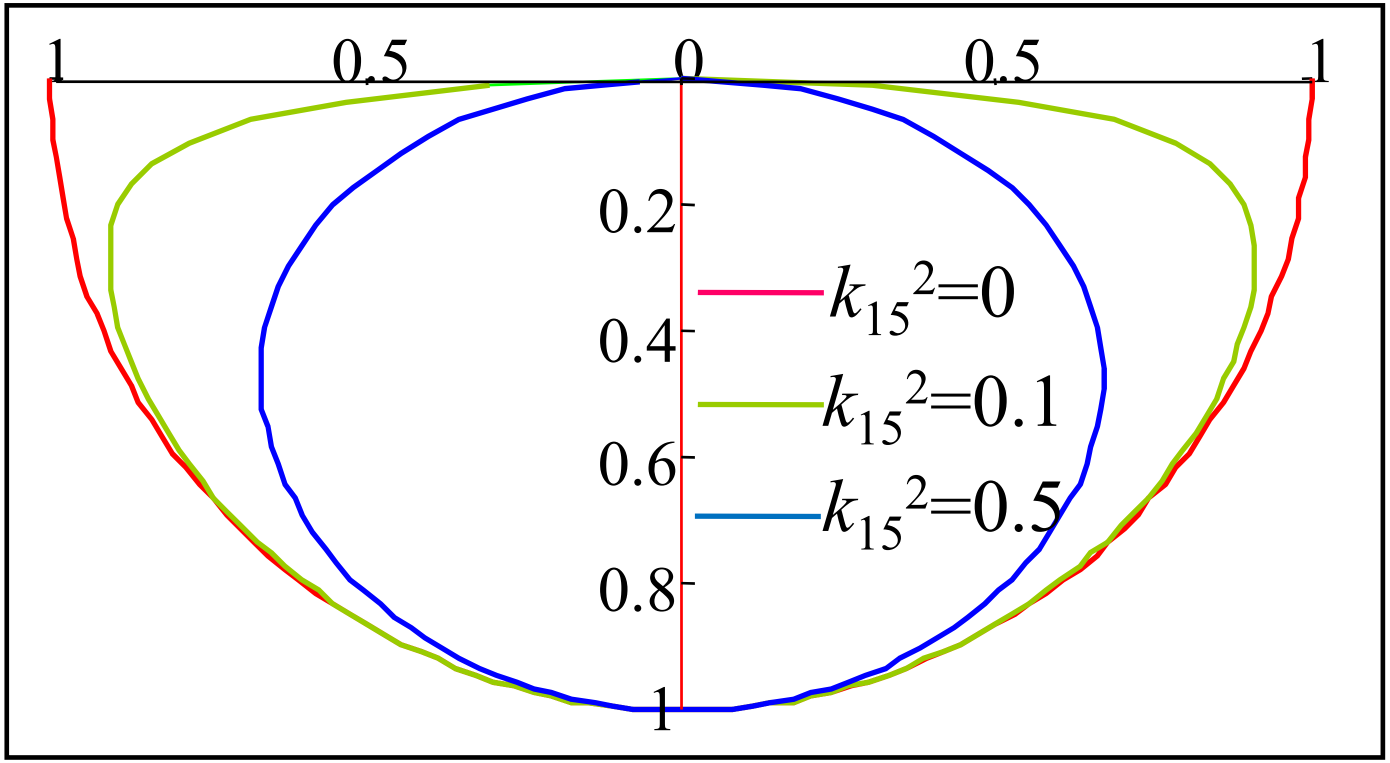
Angler Dependence of BAW Power Flow in Far Field

L & SV Do not Satisfy Surface Boundary Condition

Non-Radiative Parallel to Surface?

BAW Radiated to Surface Changes into SAW

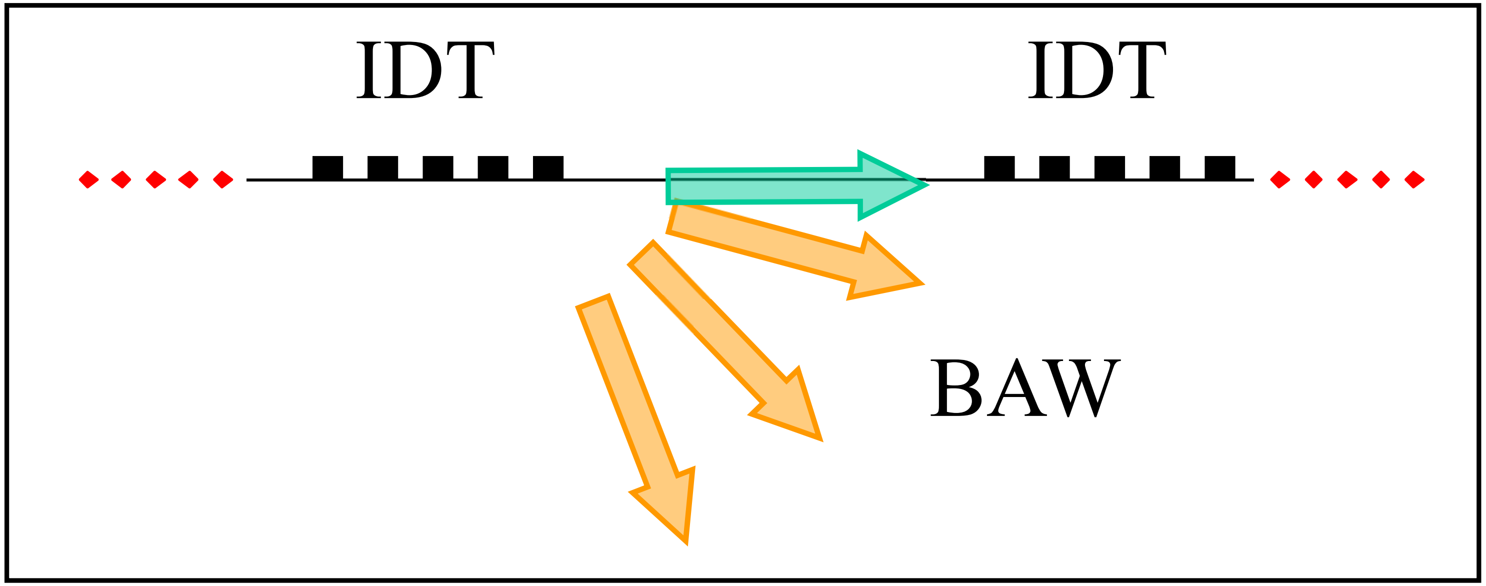
Coupling of SH & Φ Components



SH+φ ⇒ SH-Type SAW

Efficient SAW Radiation ⇔
Suppression of SSBW Radiation

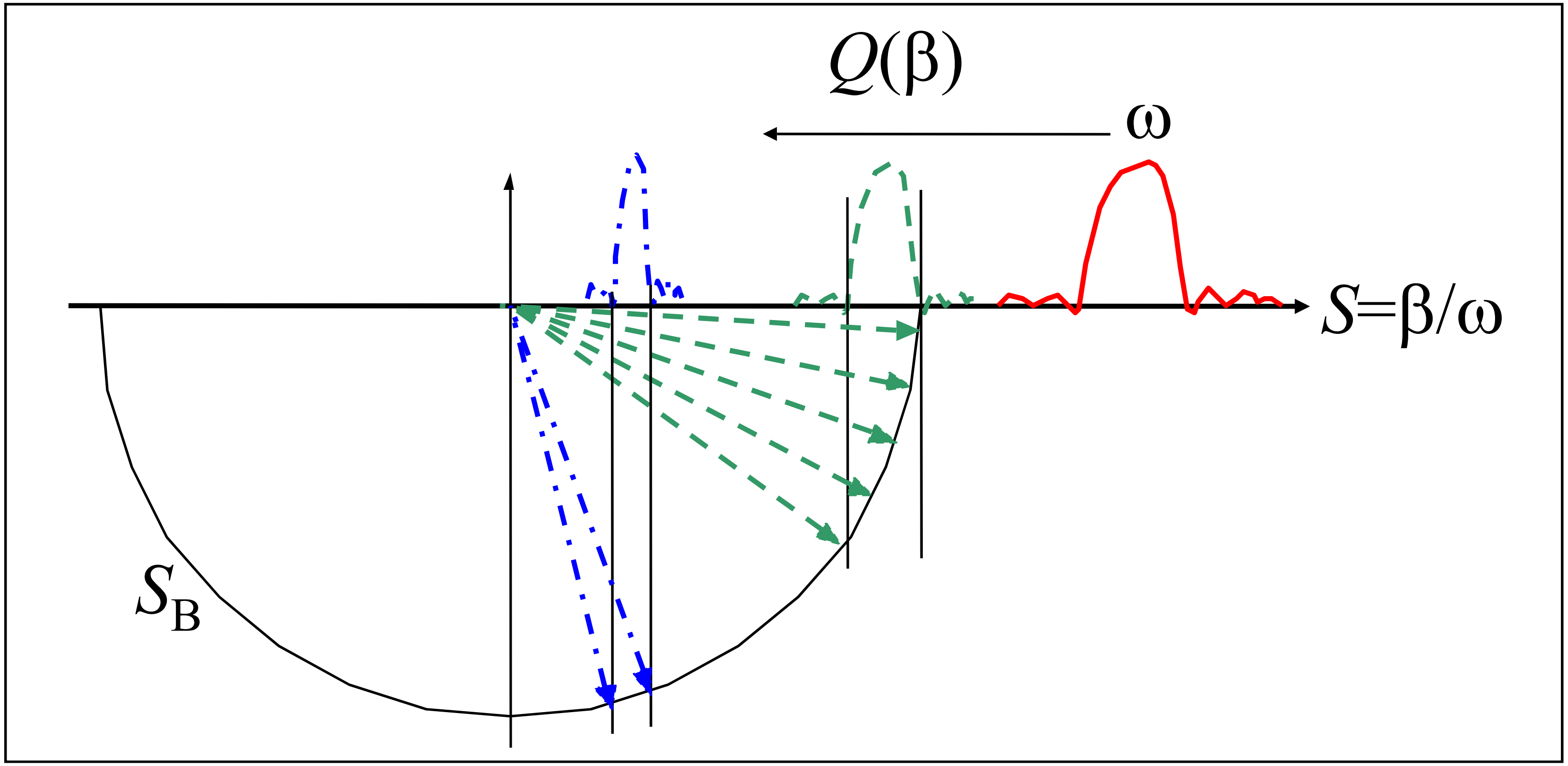
SSBW: Surface Skimming Bulk Wave BAW Propagating on Surface



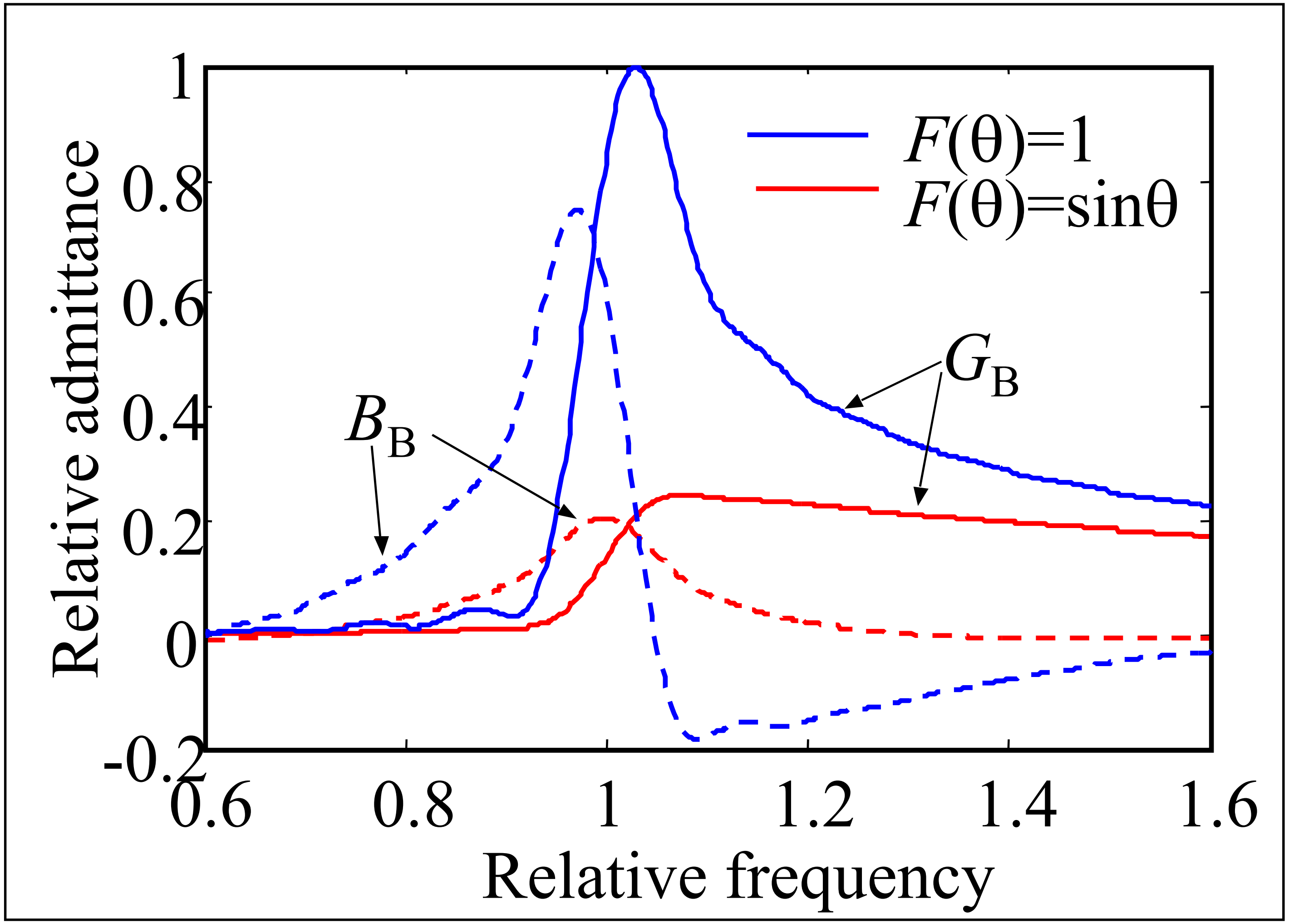
Excitation and Propagation: Very Sensitive to Surface Condition

Characterized by Velocity Difference Between SAW and BAW

Frequency Response of BAW Radiation

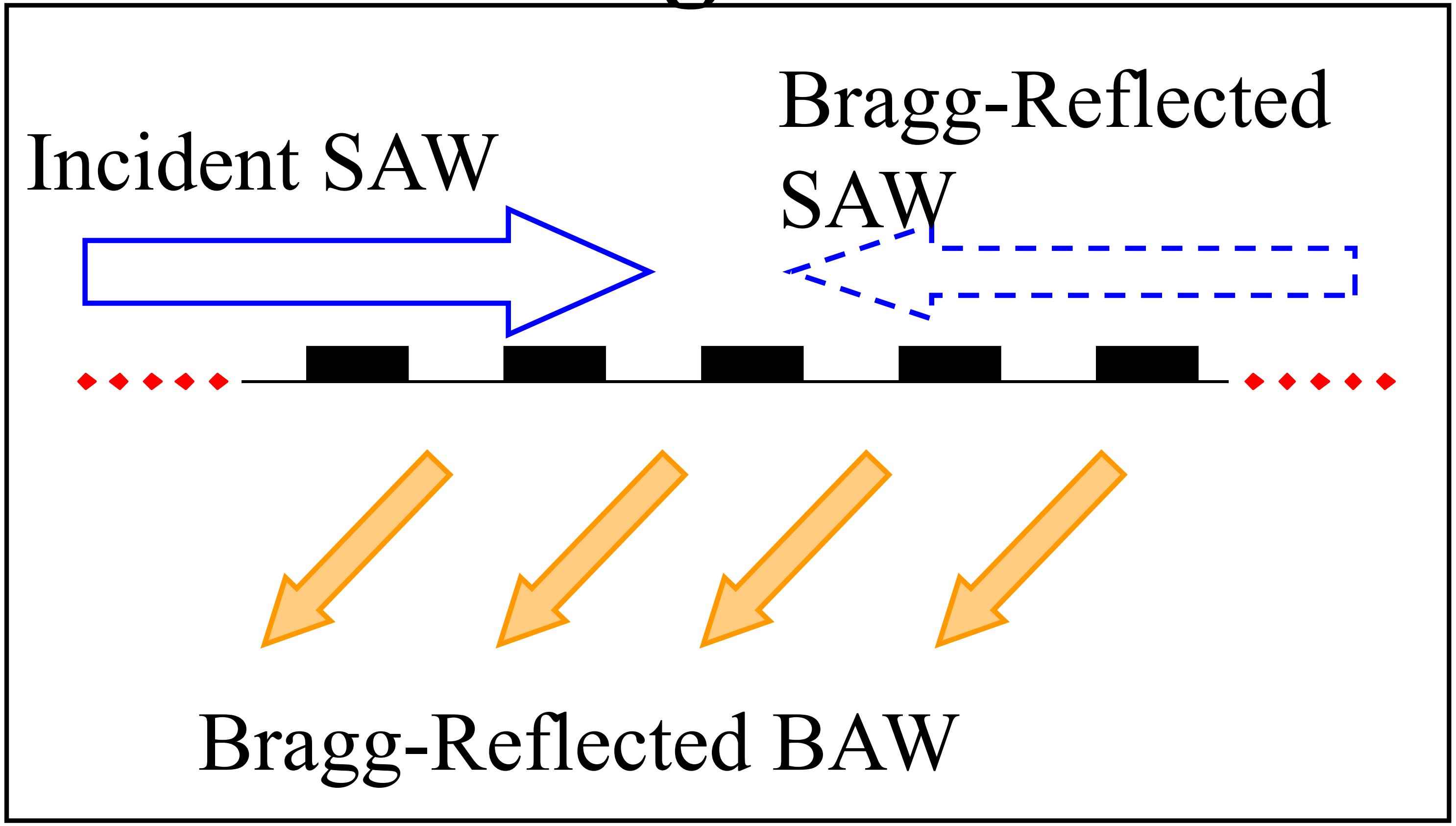


- Cutoff Nature
- Radiation Peak just above the Cutoff Freq.



$F(\theta)$: θ Dependence of BAW Radiation

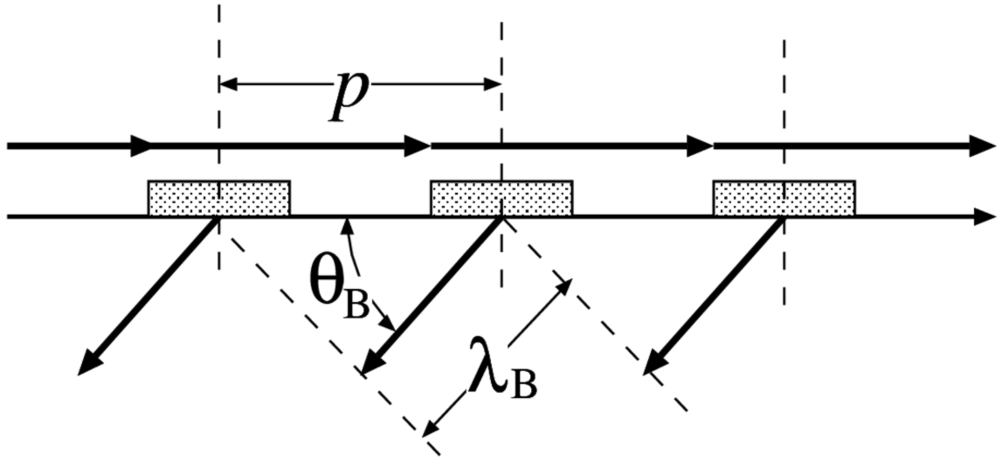
Back-Scattering to BAW



For SH-type SAW,

Cutoff for BAW Back-Scattering ≈ SAW Resonance Frequency

SAW Bragg Reflection to Bulk Waves



Phase Matching Condition

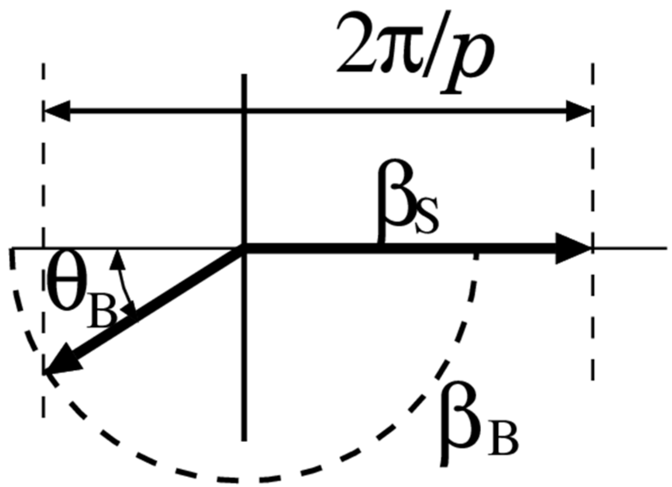
$$-\beta_S p - \beta_B p \cos\theta_B = -2n\pi$$

⇩

$$\beta_S - 2n\pi / p = -\beta_B \cos\theta_B$$

⇩

$$f = \frac{np^{-1}}{S_S + S_B \cos\theta_B}$$



When $S_S > S_B$

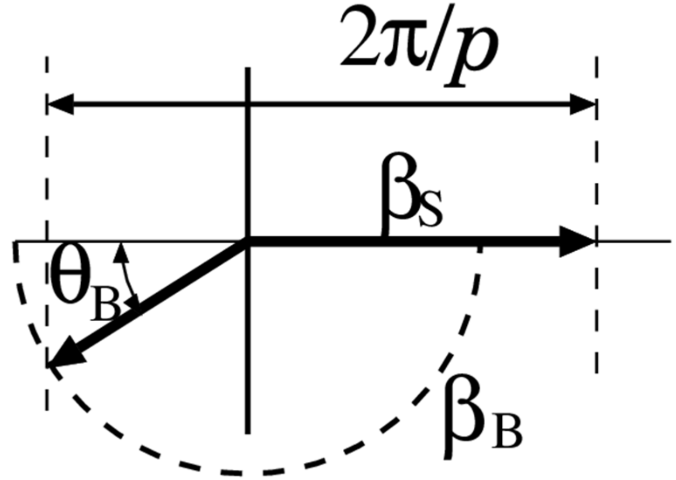
Cutoff Freq. f_{BC} for BAW Back-Scattering

$$f_{Bc} = \frac{n}{(S_S + S_B)p} > \underline{\frac{n}{2 S_S p}}$$

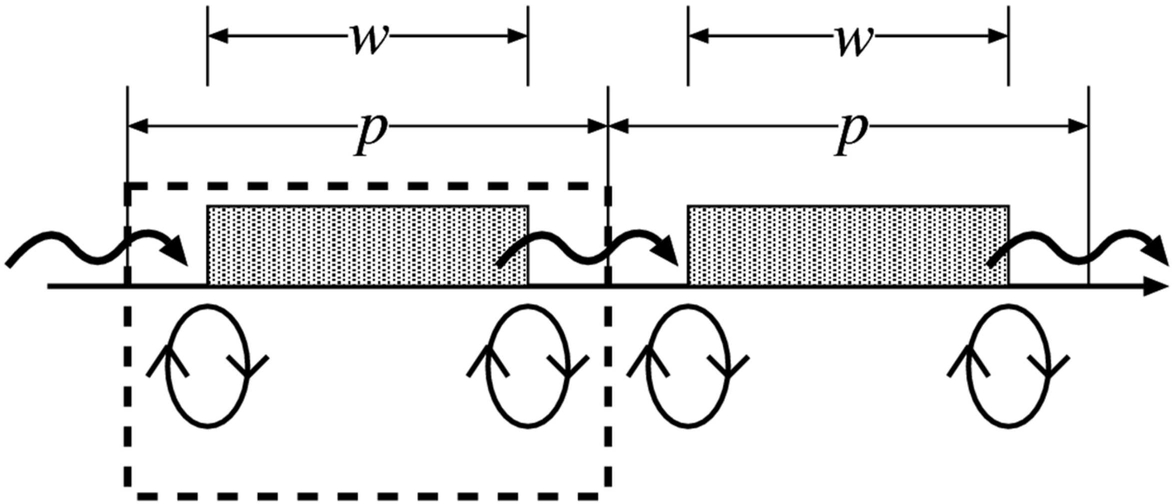
SAW Bragg Freq.

BAW Bean Scan with f

$$\cos\theta_B = \frac{f_{Bc}}{f}\left(S_S S_B^{-1} + 1\right) - S_S S_B^{-1}$$

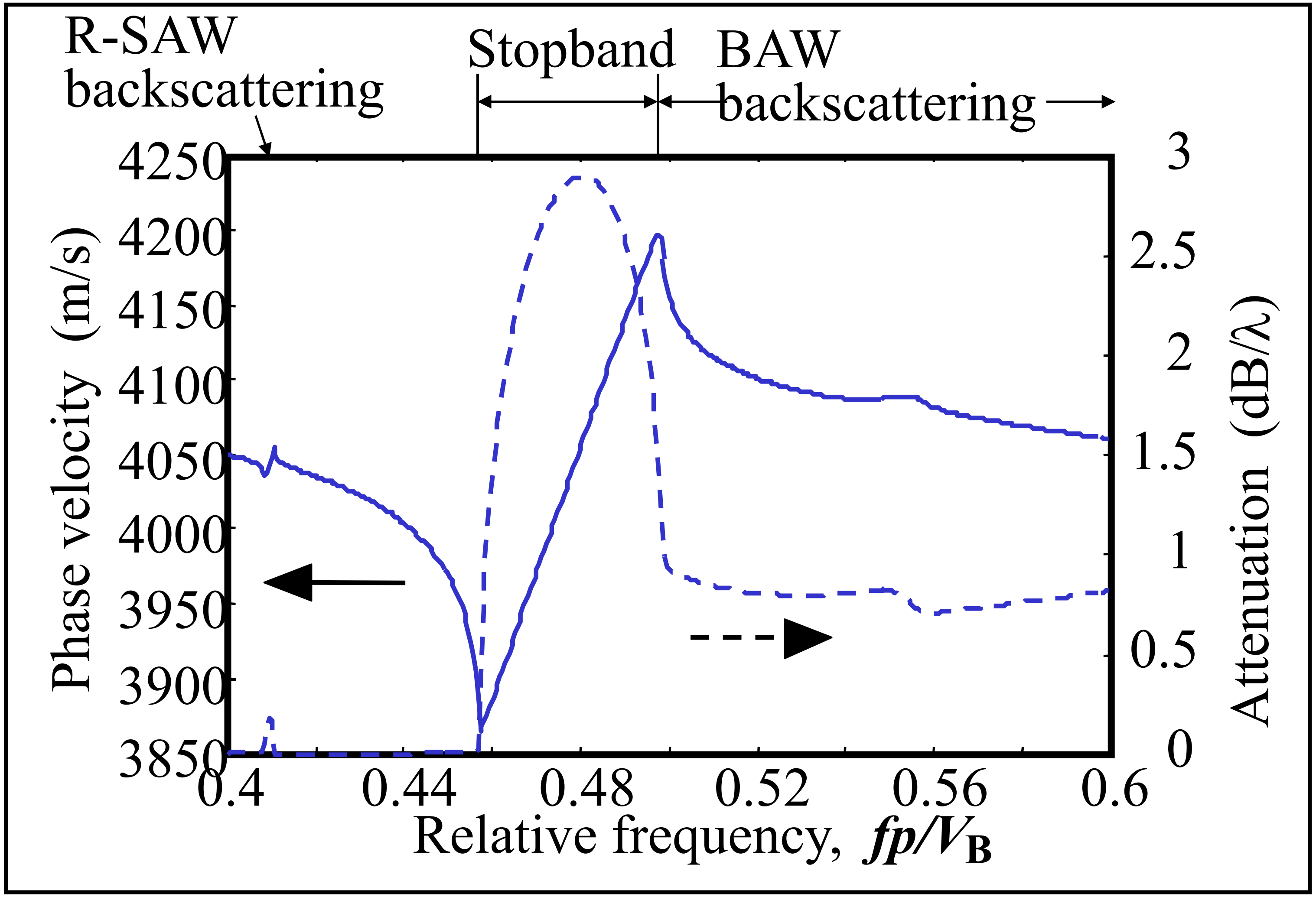


At Frequency Below f_{Bc}



Evanescent Field via Non-Radiated BAW

Energy Storage (SAW Velocity Reduction) Effect



Dispersion in phase velocity and attenuation of ***SH-type SAW*** on the SC grating (h/λ=0.1) on 42-LT. Blue lines: calculated by FEMSDA

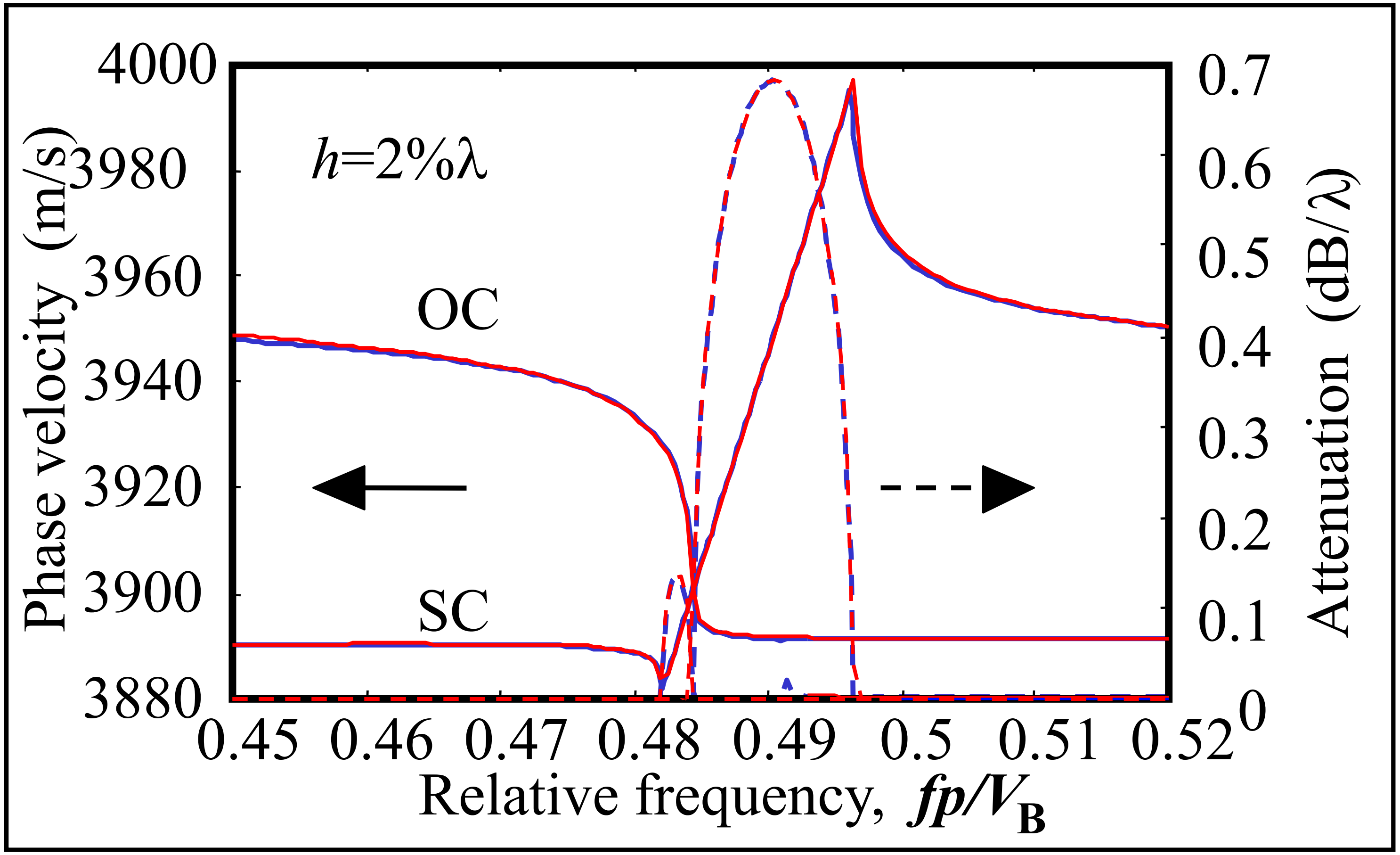
SAW Wavenumber Derived by Conventional COM Theory

$$\beta_s = \pi / p + \sqrt{\theta_u^2 - \kappa^2}$$

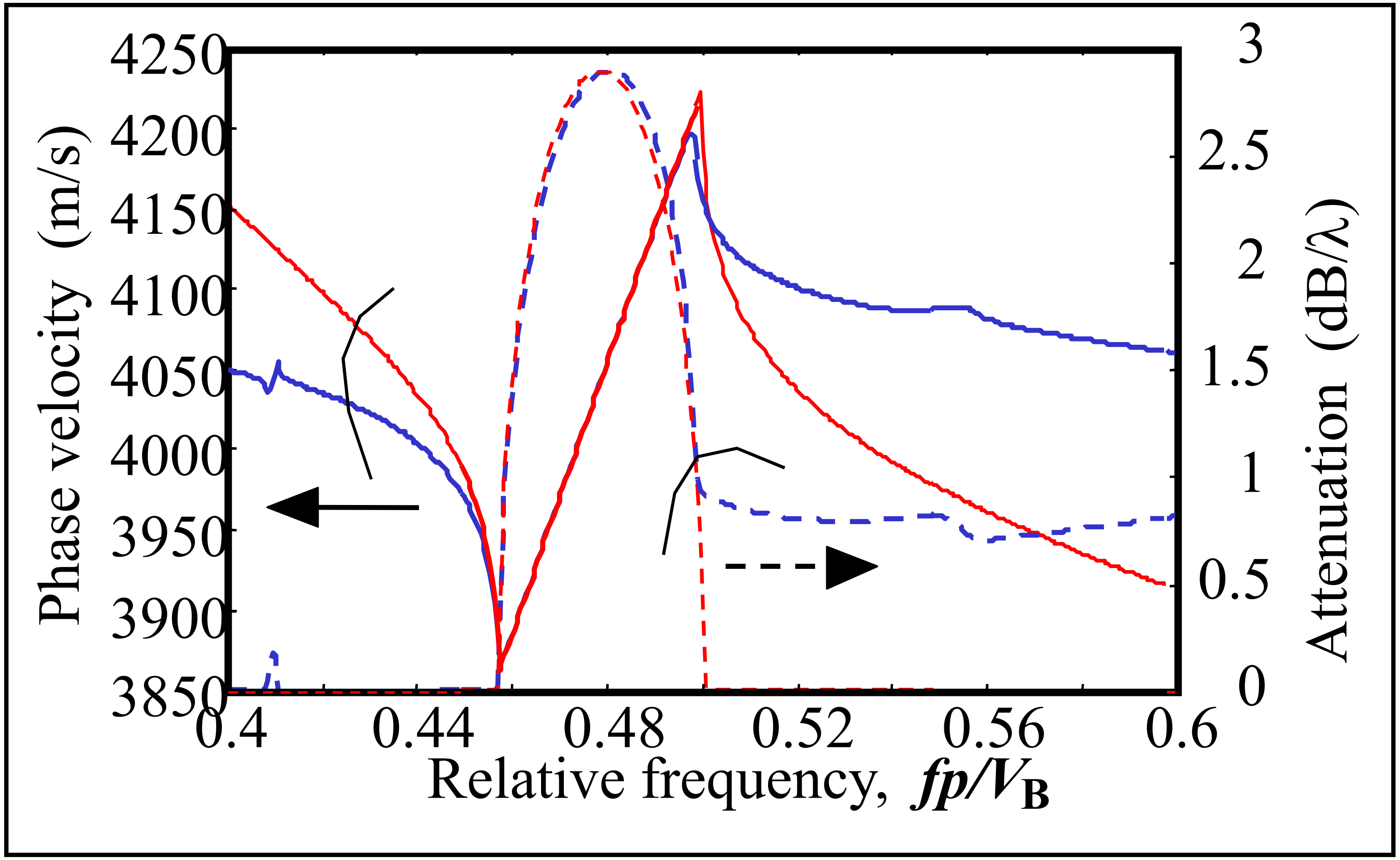
where θ_u is detuning factor
(linearly dependent on frequency)

κ: mutual coupling factor (const.)

p: grating period



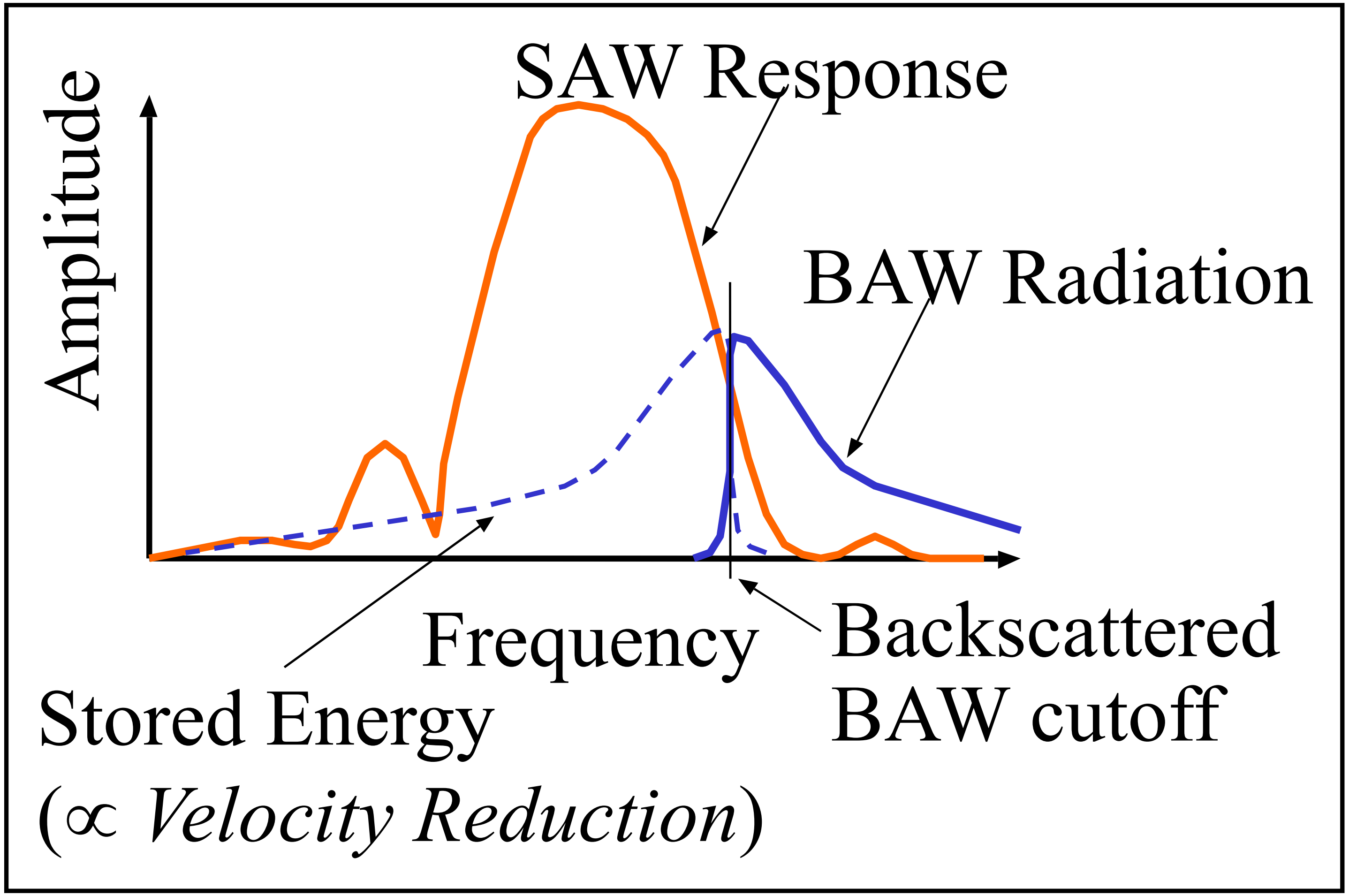
Dispersion in phase velocity and attenuation of ***Rayleigh-type SAW*** on gratings on 128-LN.
Blue lines: calculated by FEMSDA, and
red lines: calculated by conventional COM.



Dispersion in phase velocity & attenuation of ***SH-type SAW*** on SC grating on 42-LT.

Blue lines: calculated by FEMSDA, and

red lines: calculated by conventional COM.



Origin of dispersion near stopband

Characterisation by Plessky's Model

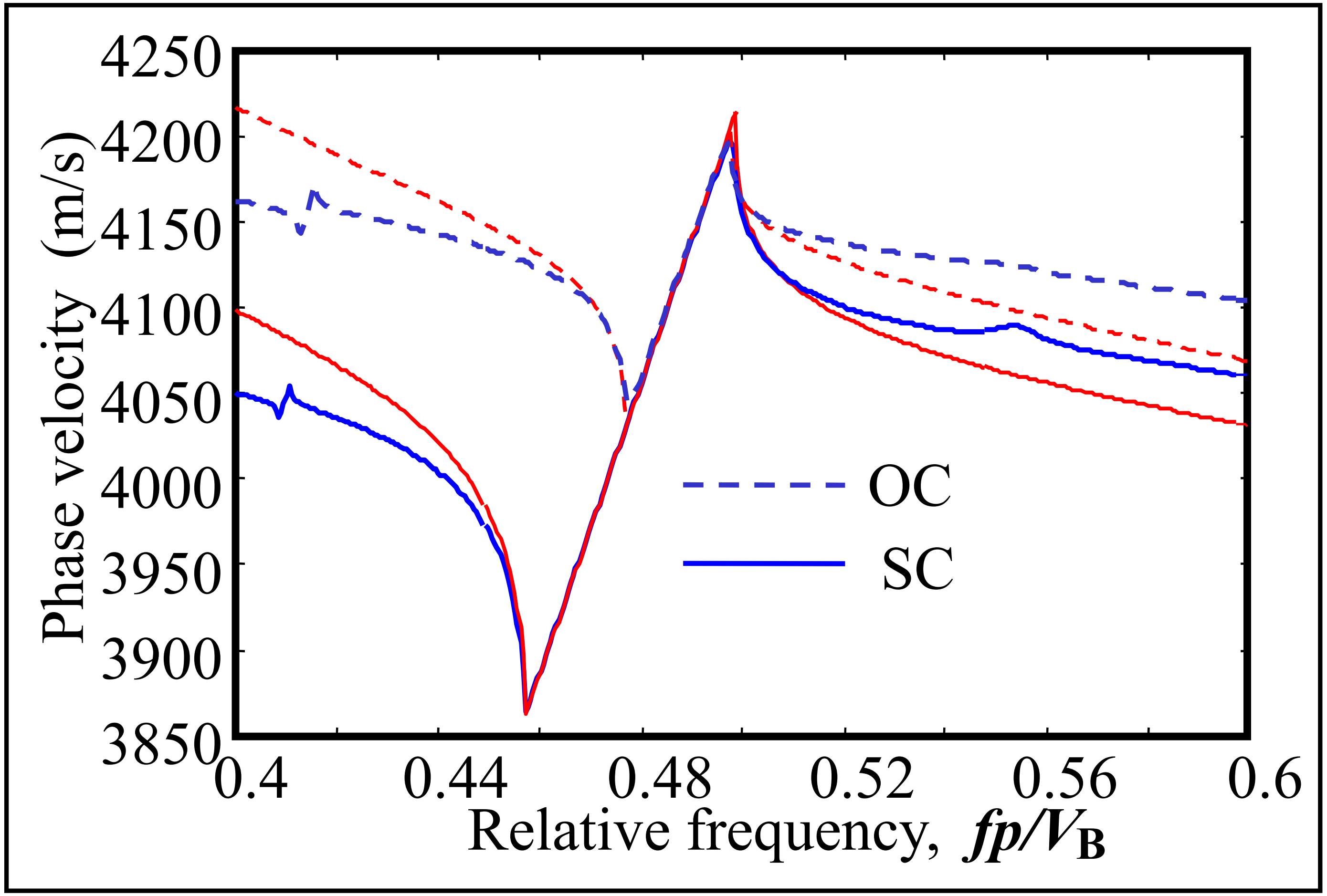
- Dispersion Relation

$$\beta_s = \pi / p + \sqrt{\Delta^2 - (\varepsilon_s^2 / 2 + \eta_s \sqrt{\Delta_{Bs} - \Delta})^2}$$

where $\Delta = c(\omega / V_B - \pi / p)$: normalised frequency

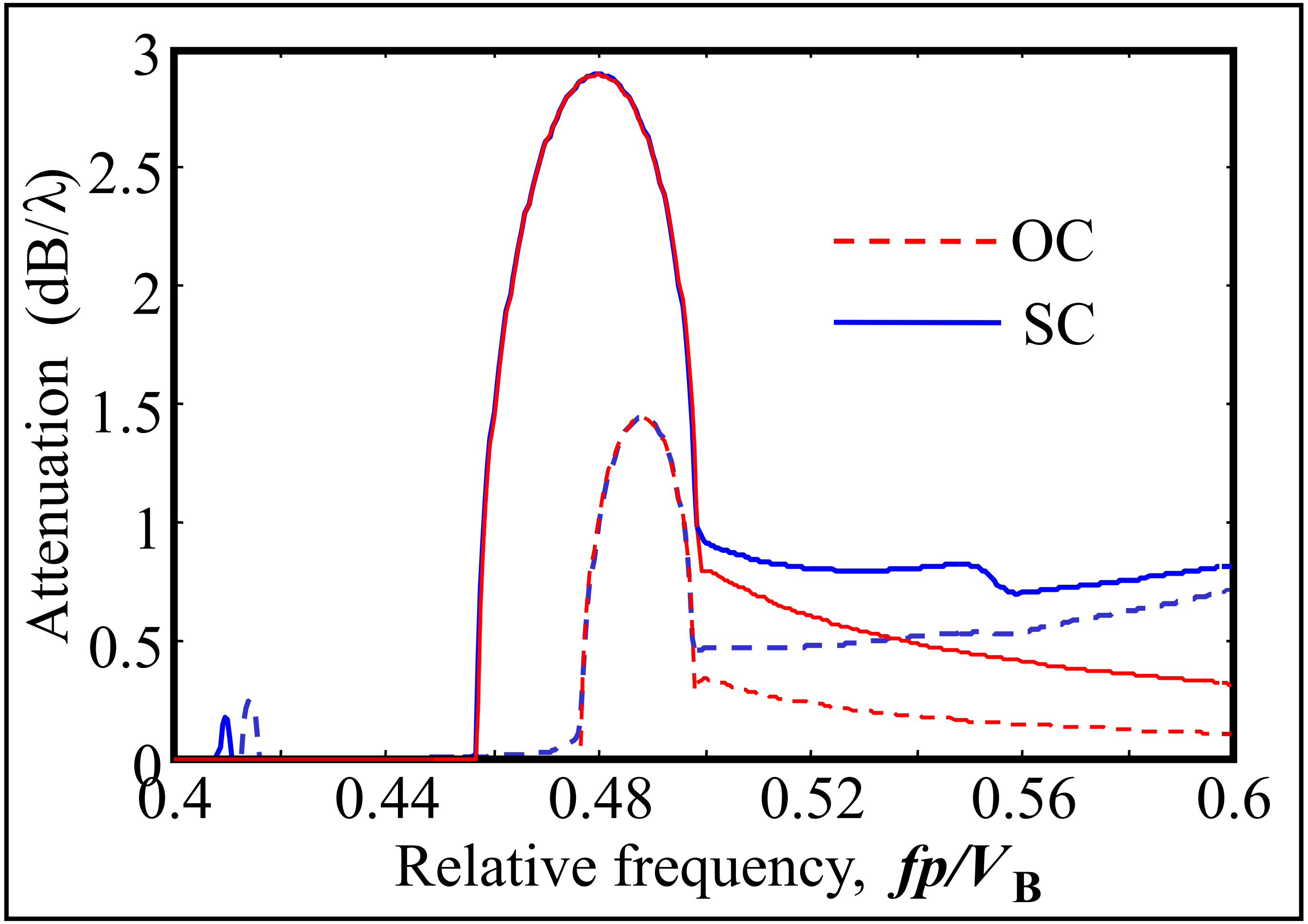
$\Delta_{Bs} = \varepsilon_s^2 / 2 - \eta_s^2 / 4$: normalised BAW-cutoff frequency

ε_s, η_s, c : parameters for SC grating (constant)



Phase velocity of ***SH-type SAW*** for grating structure on 42-LT.

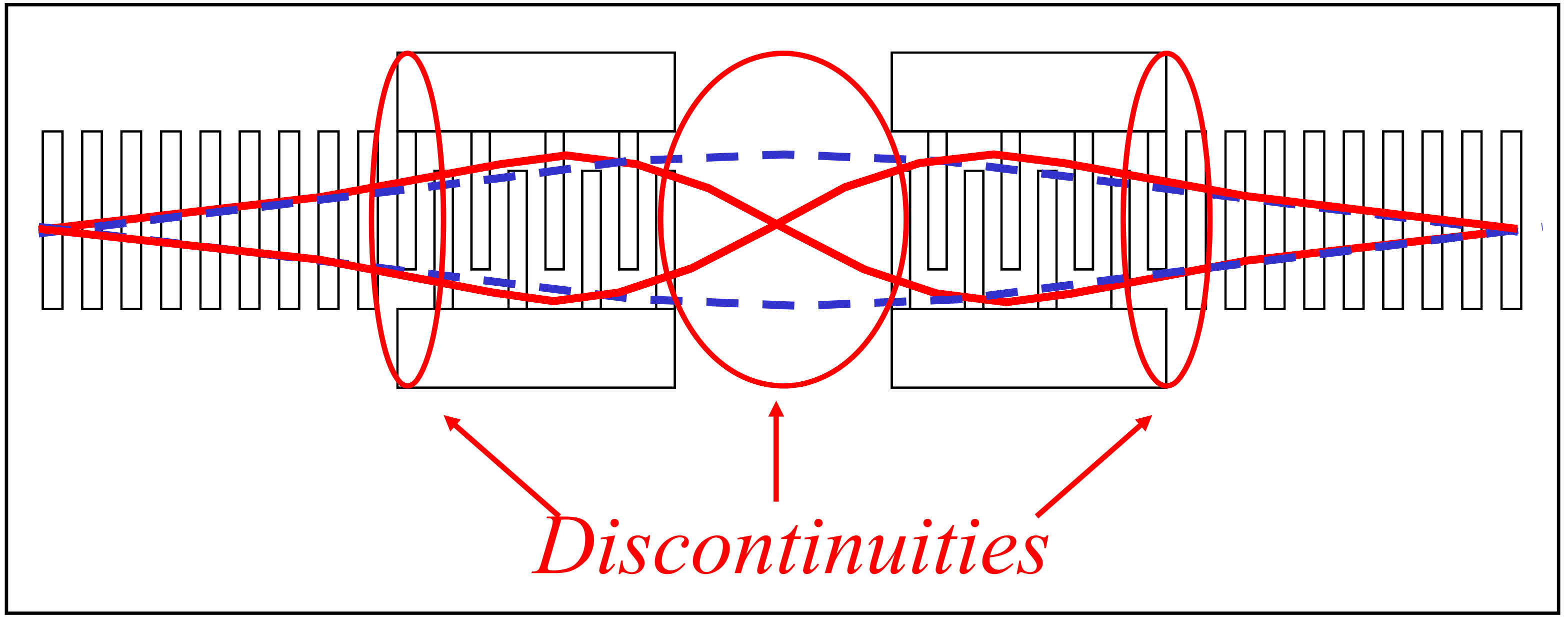
Blue lines: calculated by FEMSDA, and red lines: calculated by Plessky's model.



Attenuation of ***SH-type SAW*** for grating structure on 42-LT.

Blue lines: calculated by FEMSDA, and red lines: calculated by Plessky's model.

What will Happen When Periodicity Breaks?



BAW Radiation + Additional Phase Shift
(Frequency Dependent)