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Introduction of Surface Acoustic Wave (SAW) Devices

Part 6: 2D Propagation and Waveguide

Ken-ya Hashimoto

Chiba University

k.hashimoto@ieee.org

http://www. te.chiba-u. jp/~ken

Contents

- Wavevector and Diffraction
- Waveguide
- Scalar Potential Theory

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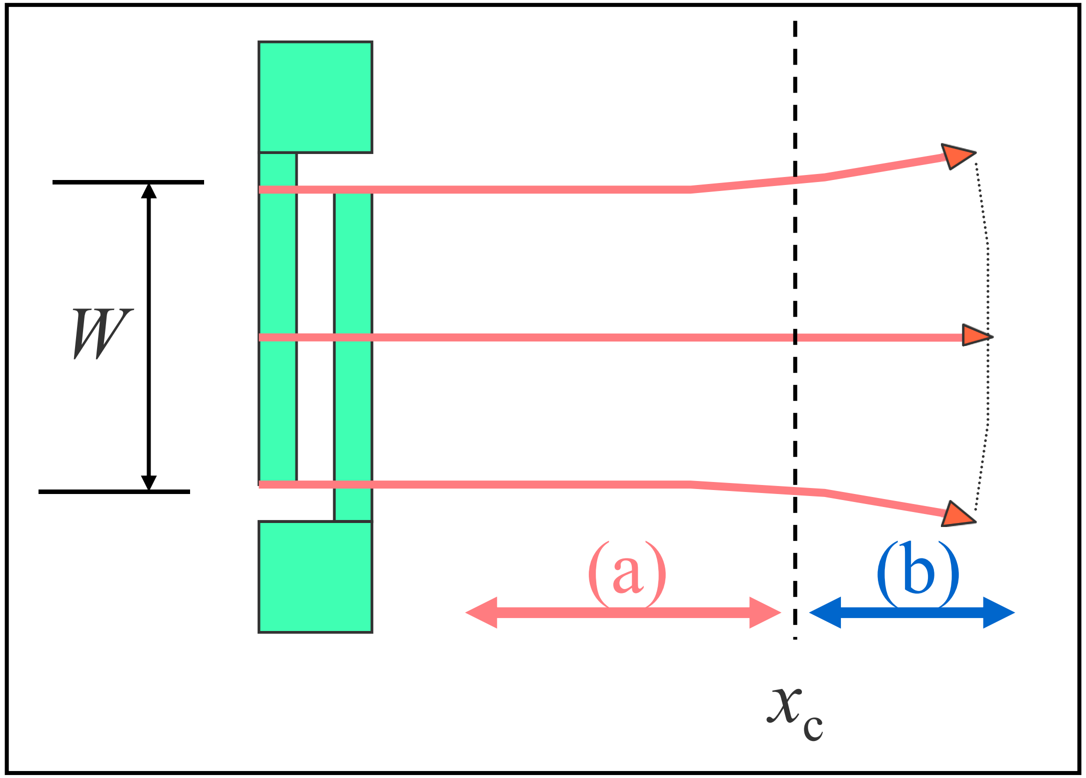
Diffraction

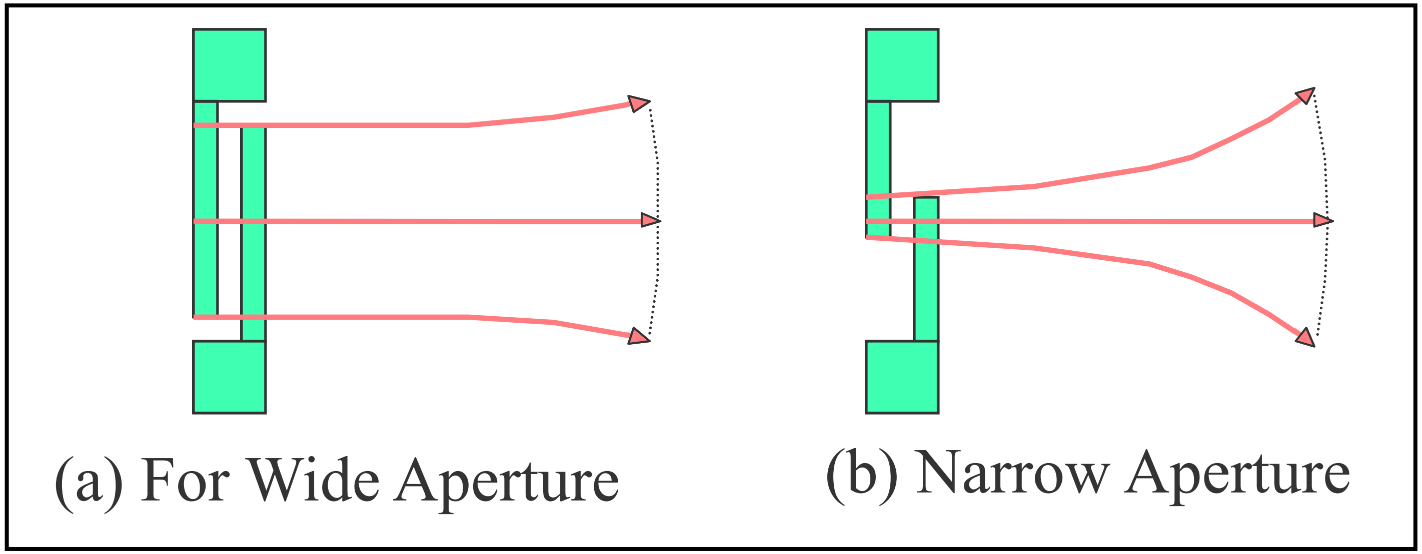


(a) Fresnel Region
(Beam Propagation)

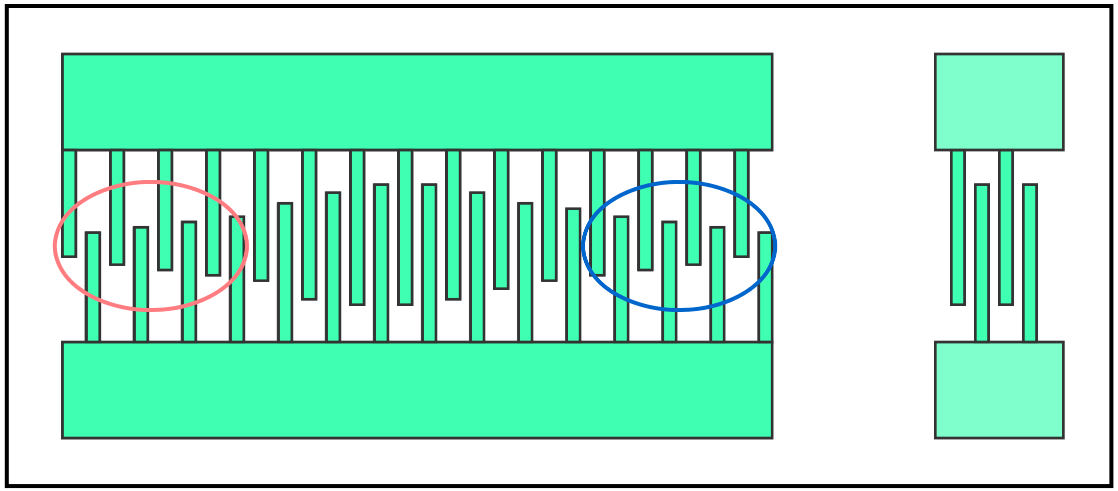
(b) Fraunhofer Region
(Cylindrical Wave
Propagation)

Critical Length : $x_c=(1+\gamma)W^2/\lambda$

γ: Parameter Determined by Anisotropy(=0 for Isotropic)



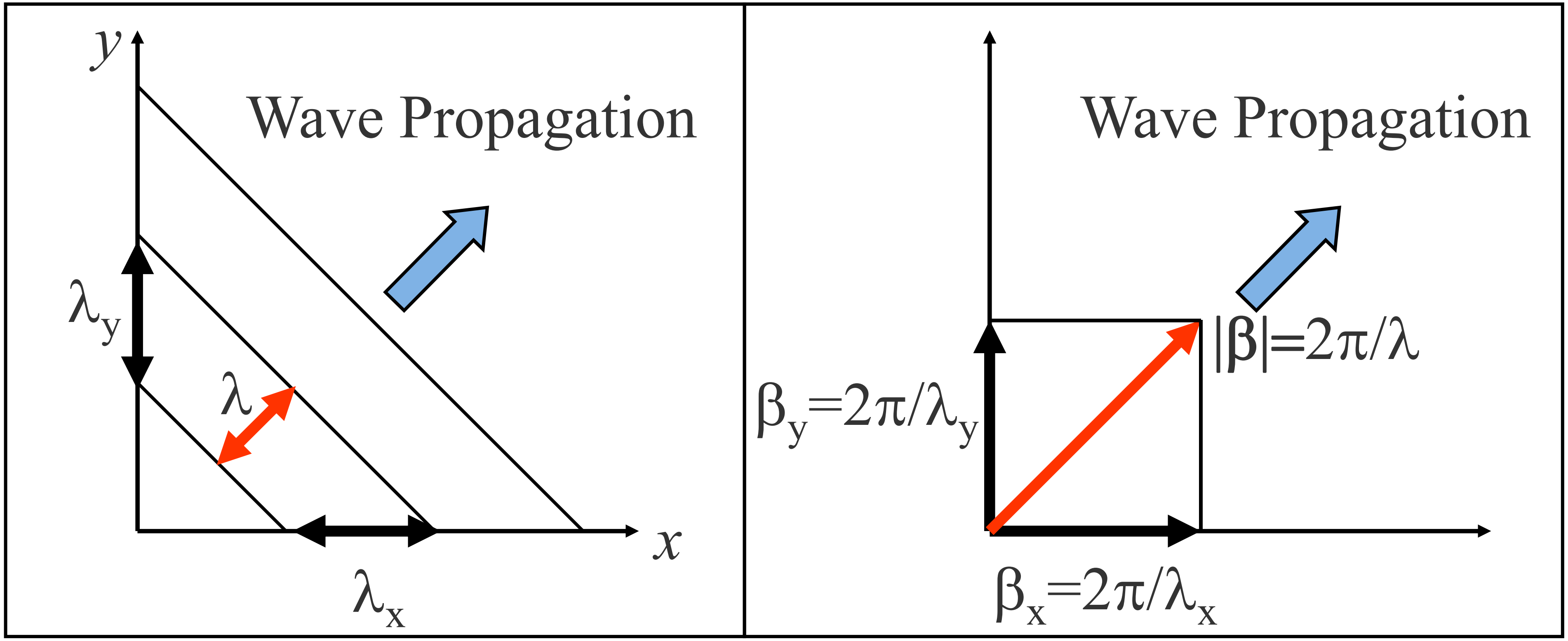
Variation with Aperture Size



For Weighted IDT

Wave Vector β

$|\beta|=2\pi/\lambda$:Phase Delay per Unit Length



$V_p\,(=f\lambda)$ does not Follow Vector Decomposition Rule!

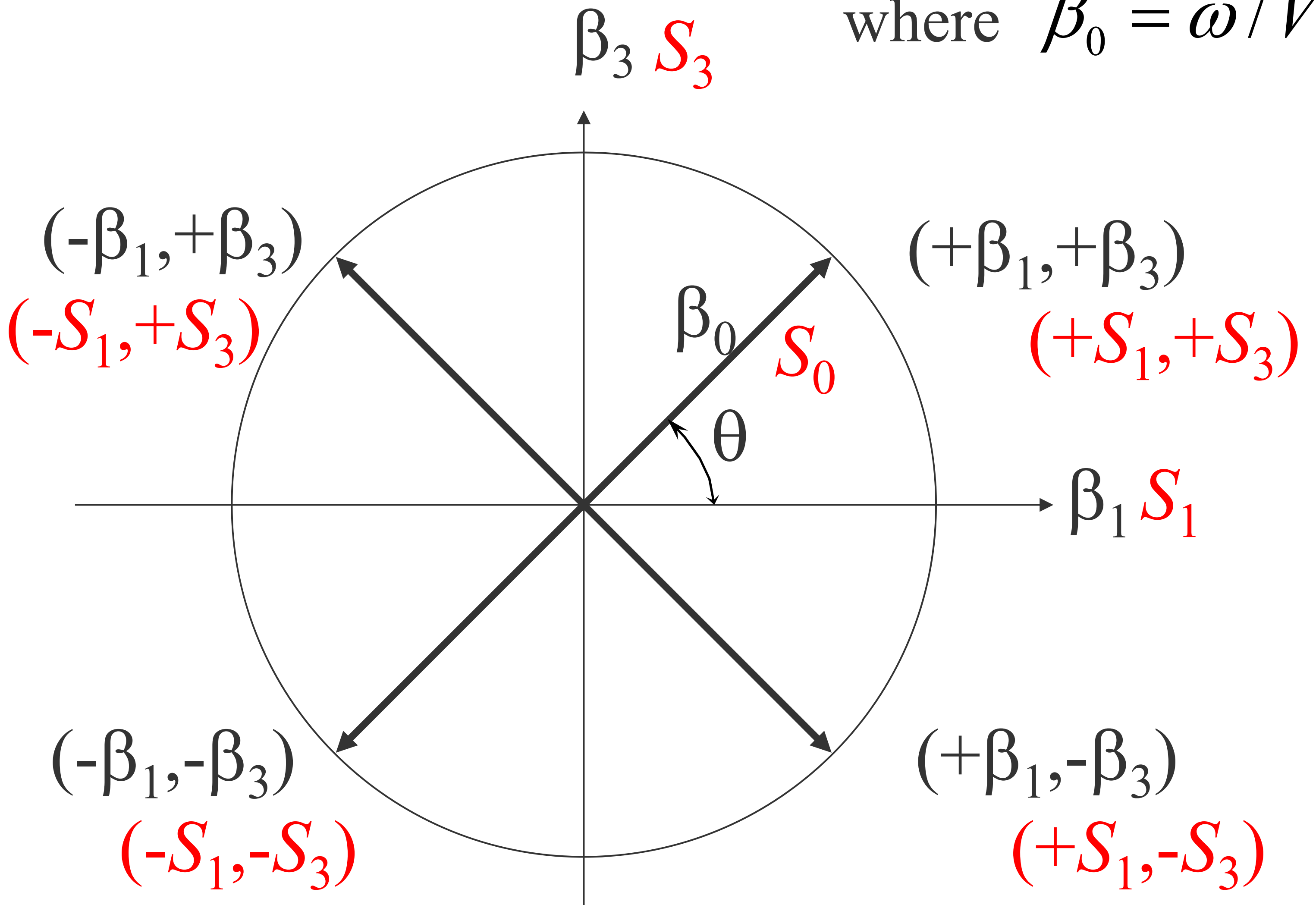
$$\exp(-j\beta\bullet\mathbf{X}) \Rightarrow \exp[-j(\beta_x x+\beta_y y+\beta_z z)]$$

2D Wave Equation

$$\rho \ddot{u} = C\nabla^2 u$$

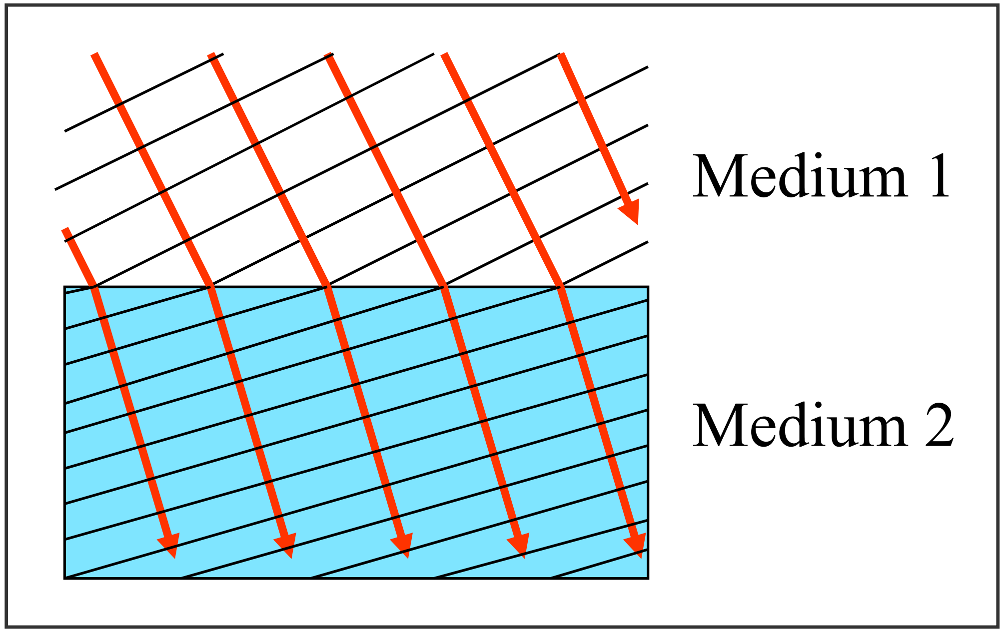
$$\phi \propto \exp[j(\omega t - \beta_x x - \beta_y y)] \Rightarrow \beta_x^2 + \beta_y^2 = \beta_0^2$$

where $\beta_0 = \omega / V$



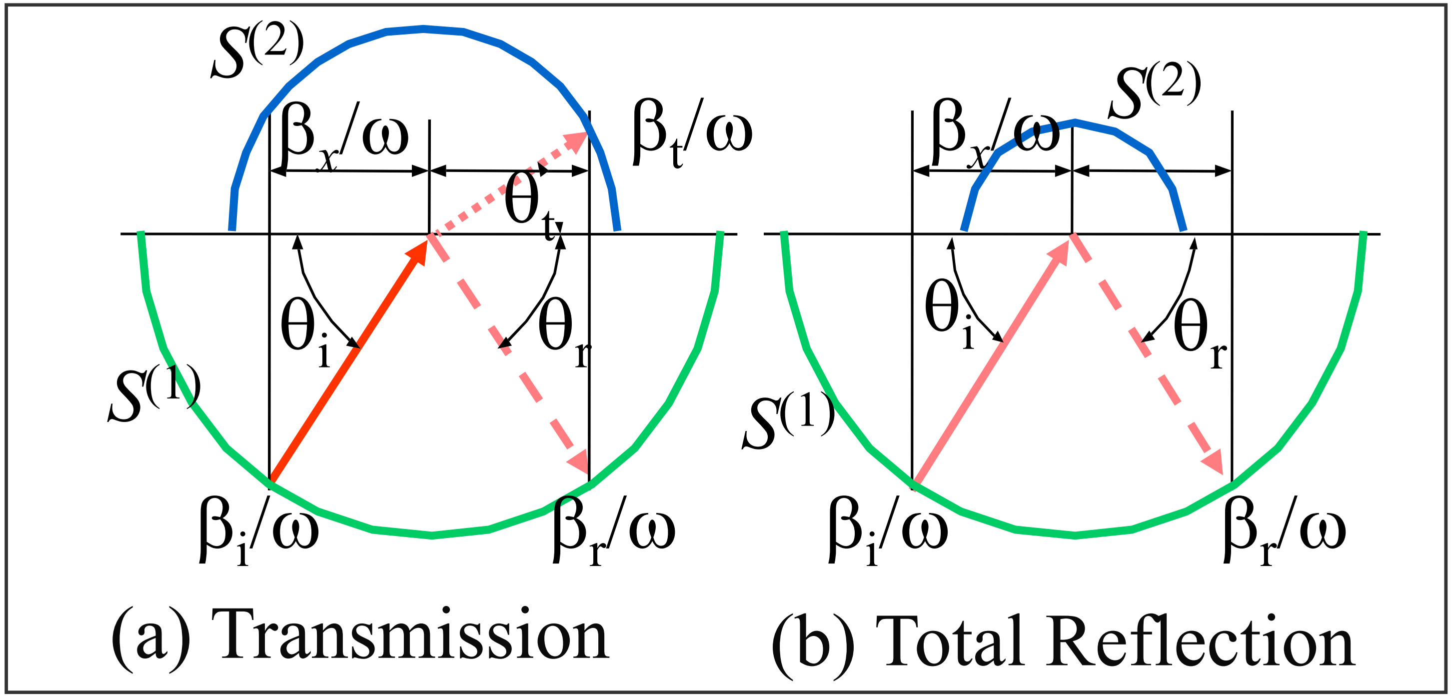
Snell's Law

Continuity of Wave Front at Boundary



Continuity of Lateral Wavelength $\Rightarrow$ **Continuity of Lateral Wavevector Component**

At Boundary Between Two media,



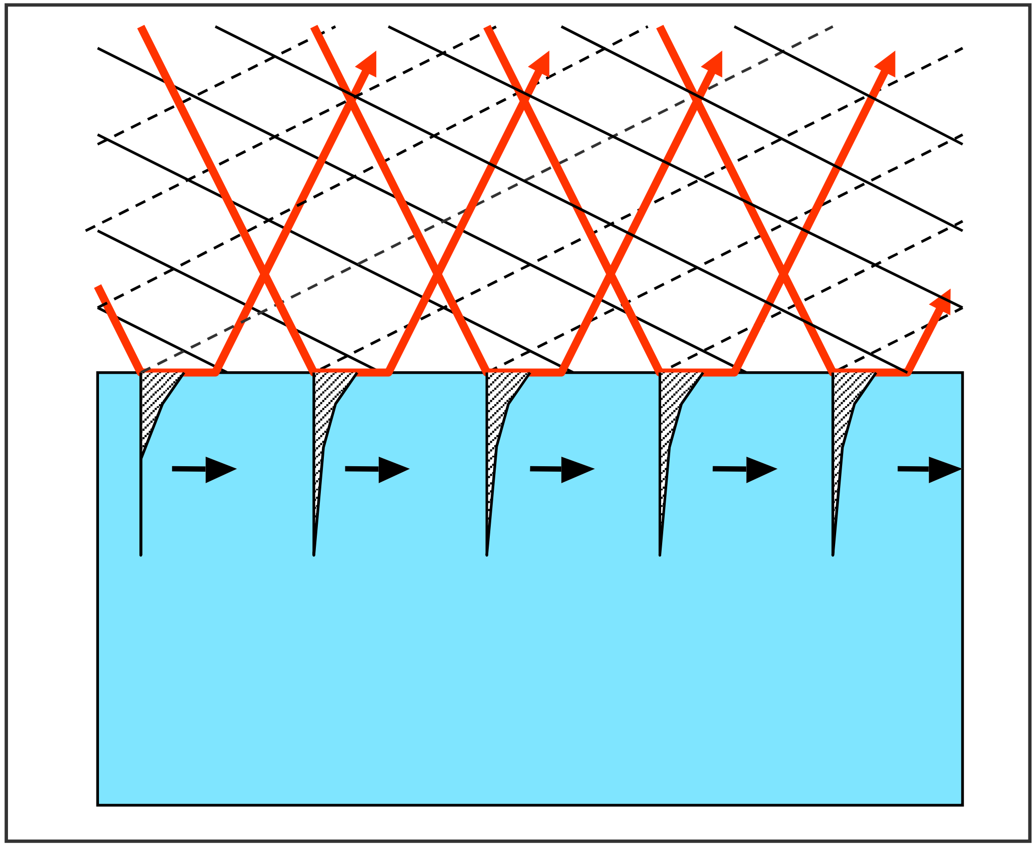
Slowness Surface (S=1/V_p) When $S^{(1)}>S^{(2)}$

$$\beta_i \sin\theta_i = \beta_r \sin\theta_r = \beta_t \sin\theta_t$$

In optics, β=nω/c, where n is refractive index and c is wave velocity in vacuum

Evanescent Field (at Total Reflection)

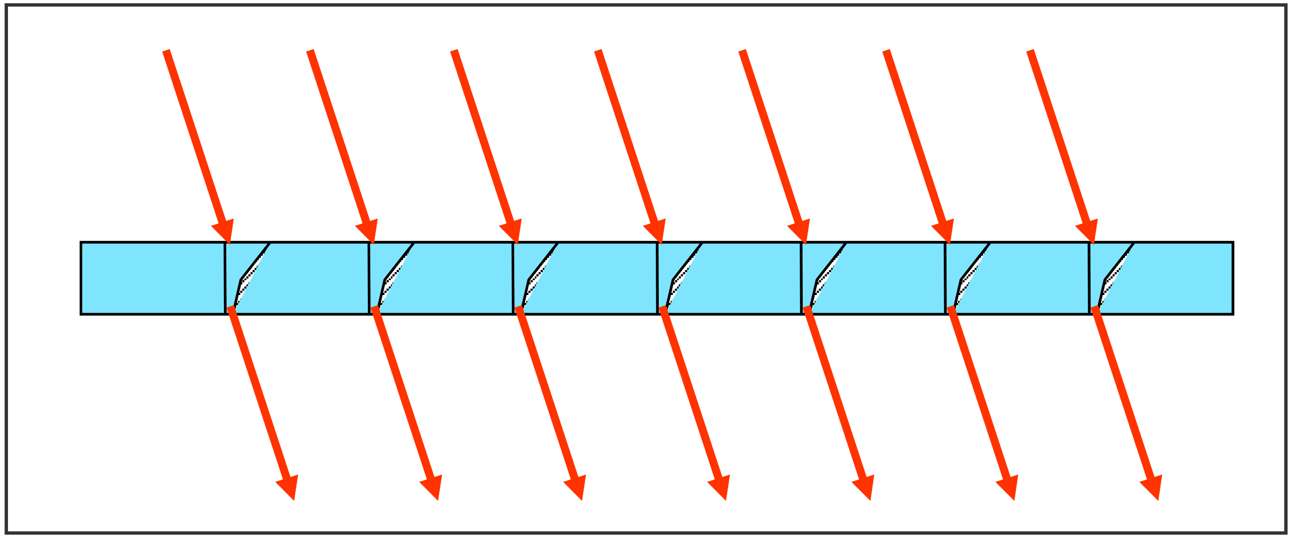
$$\beta_y^{(2)} = -j\sqrt{\beta_x^2 - \beta_0^{(2)\,2}}$$



Field Penetration

Exponential Decay
(Energy Storage)

Tunneling



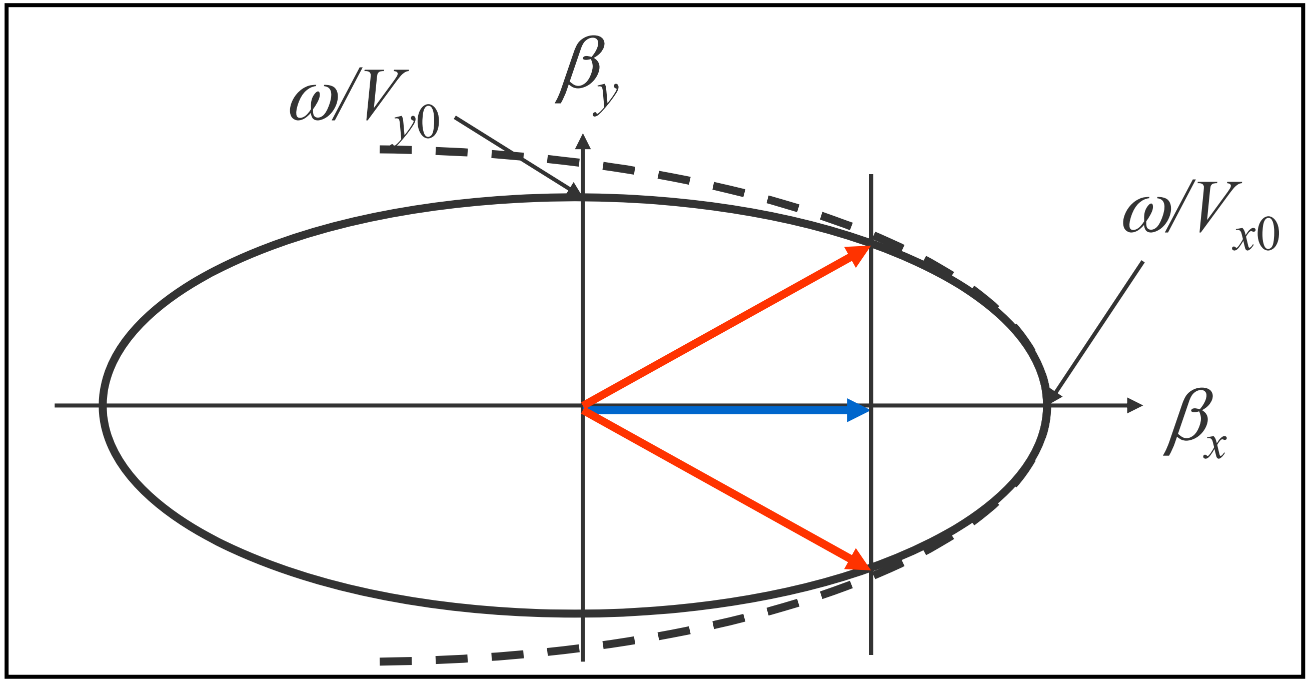
- Even for Total Reflection State, Wave Transmission Occurs when Medium is Thin
- No Phase Delay Through Transmission

Anisotropy Case

$$u \propto \exp[j(\omega t - \beta_x x - \beta_y y)]$$

$$\beta_x^2 + (V_{y0}/V_{x0})^2 \beta_y^2 = (\omega/V_{x0})^2$$

$$\beta_x \cong (\omega/V_{x0}) - 2^{-1}(\omega/V_{x0})^{-1}(V_{y0}/V_{x0})^2 \beta_y^2$$

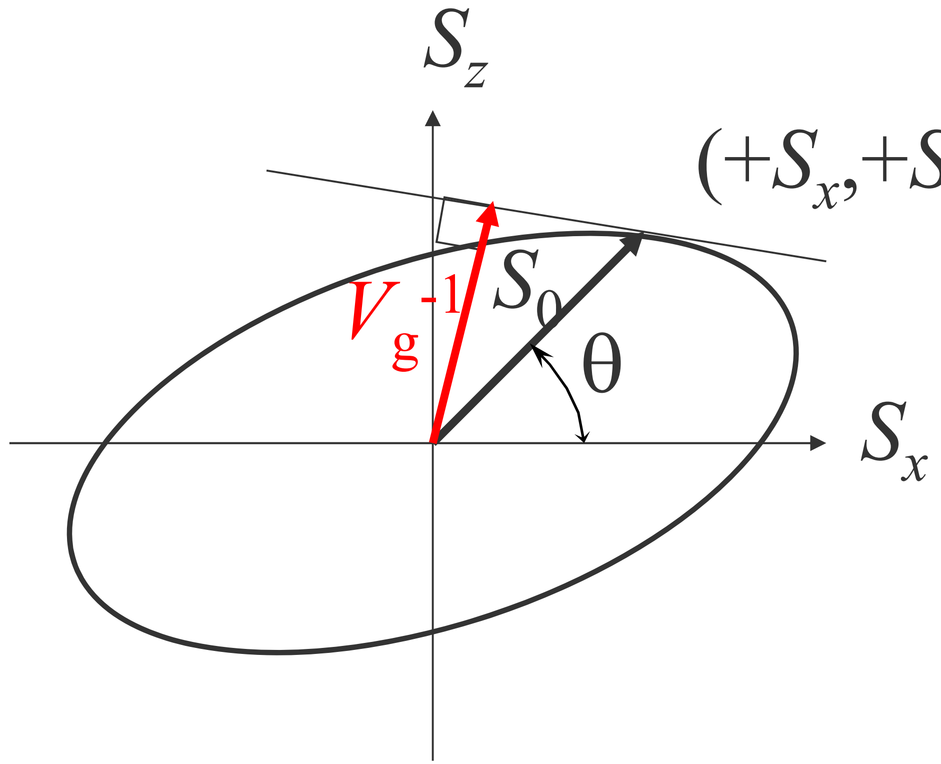


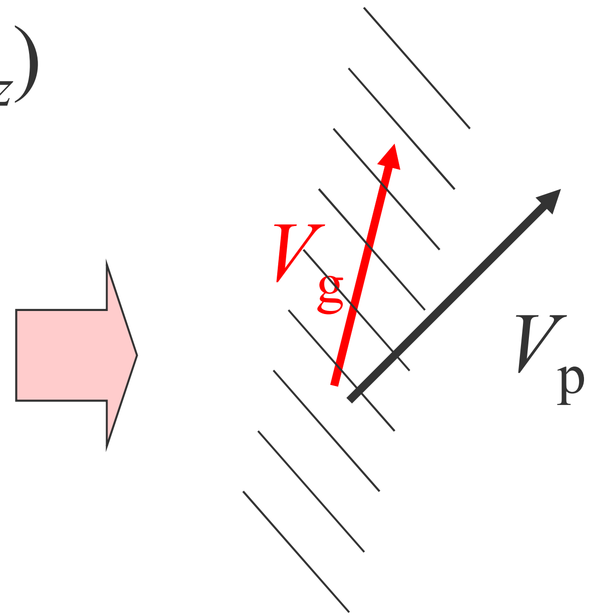
Parabolic Approximation

When Anisotropy exists,

$V_p = S_0^{-1}$: Phase Velocity

V_g: Group Velocity

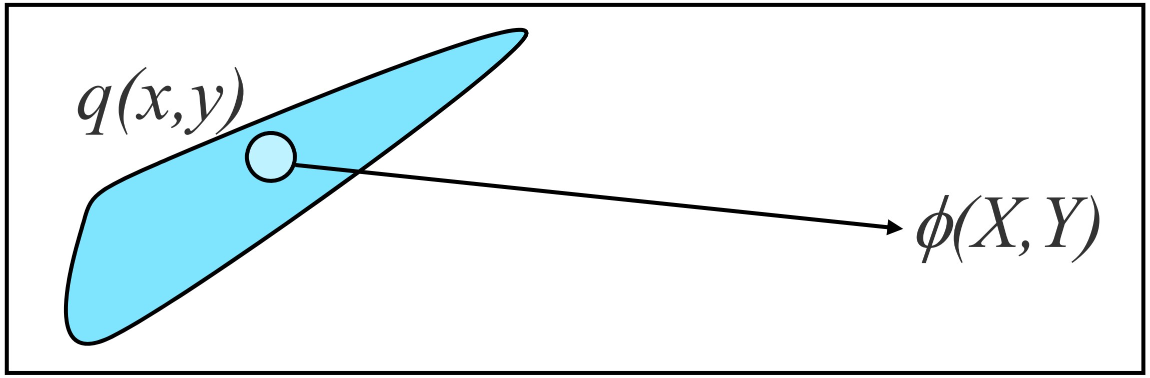




Beam Steering

cf. Birefringence

Green Function Analysis



$$\phi(X,Y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} G(X-x, Y-y)\, q(x,y)\, dx dy$$

Where *G(X,Y)* is Green Function $G(r) = \frac{F}{\sqrt{2\pi r}} \exp(-j\beta r)$

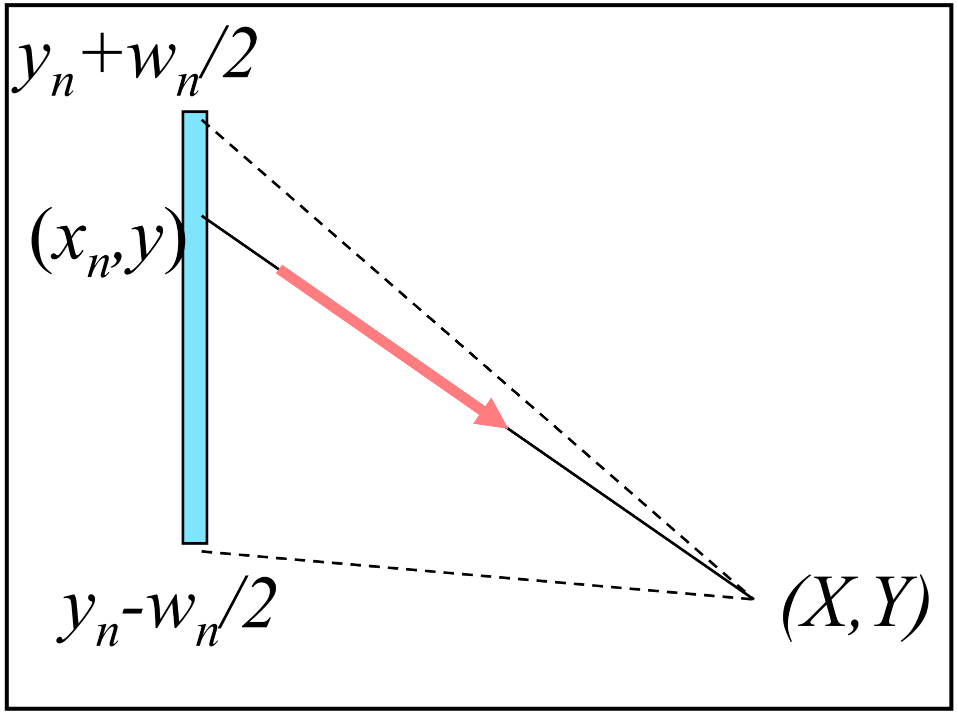
Para-Axial Approximation for X»|Y|

Approximating $\beta_x \cong \beta_{x0} - \zeta\beta_y^2$, Then

$$G(X,Y) \cong \frac{F}{\sqrt{2\pi |\zeta X|}} \exp(-j\beta |X| - jY^2/4|\zeta X|)$$

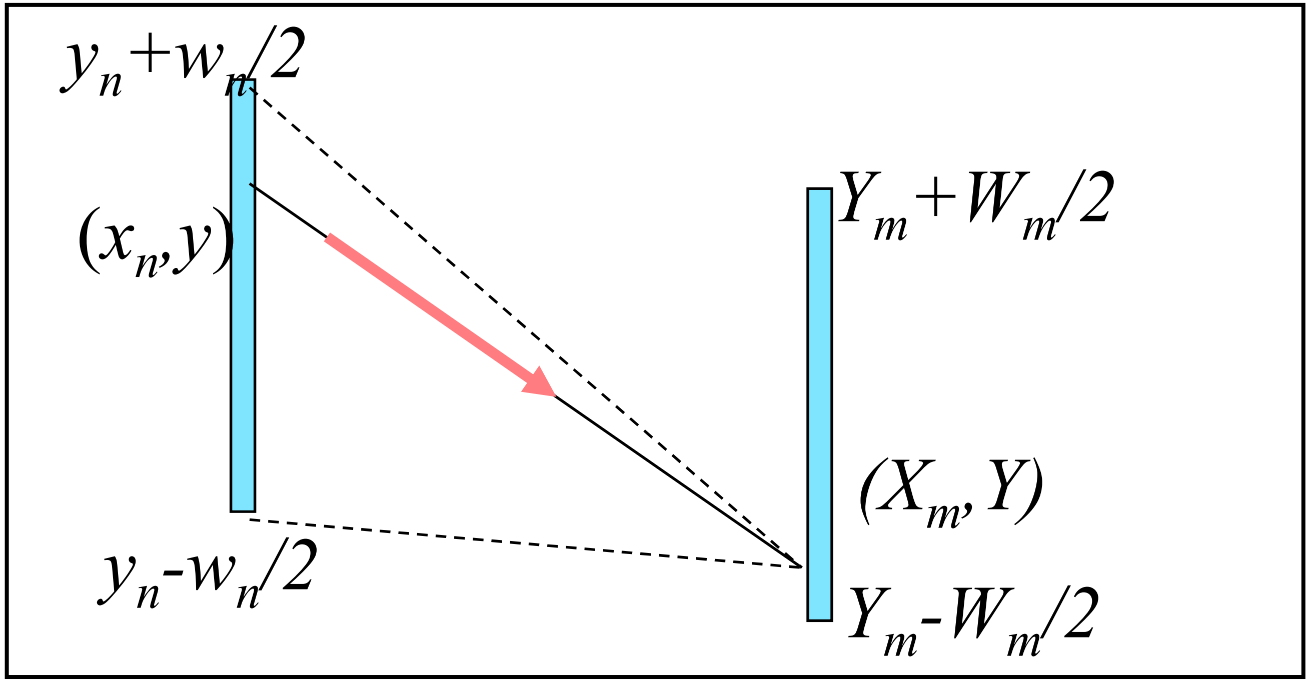
Contribution of n-th Electrode

(Width w_n, Position (x_n, y_n))



$$\phi(X,Y) = A\sum_{n=1}^{N}\int_{y_n-w_n/2}^{y_n+w_n/2} G(X-x_n, Y-y)\,dy$$

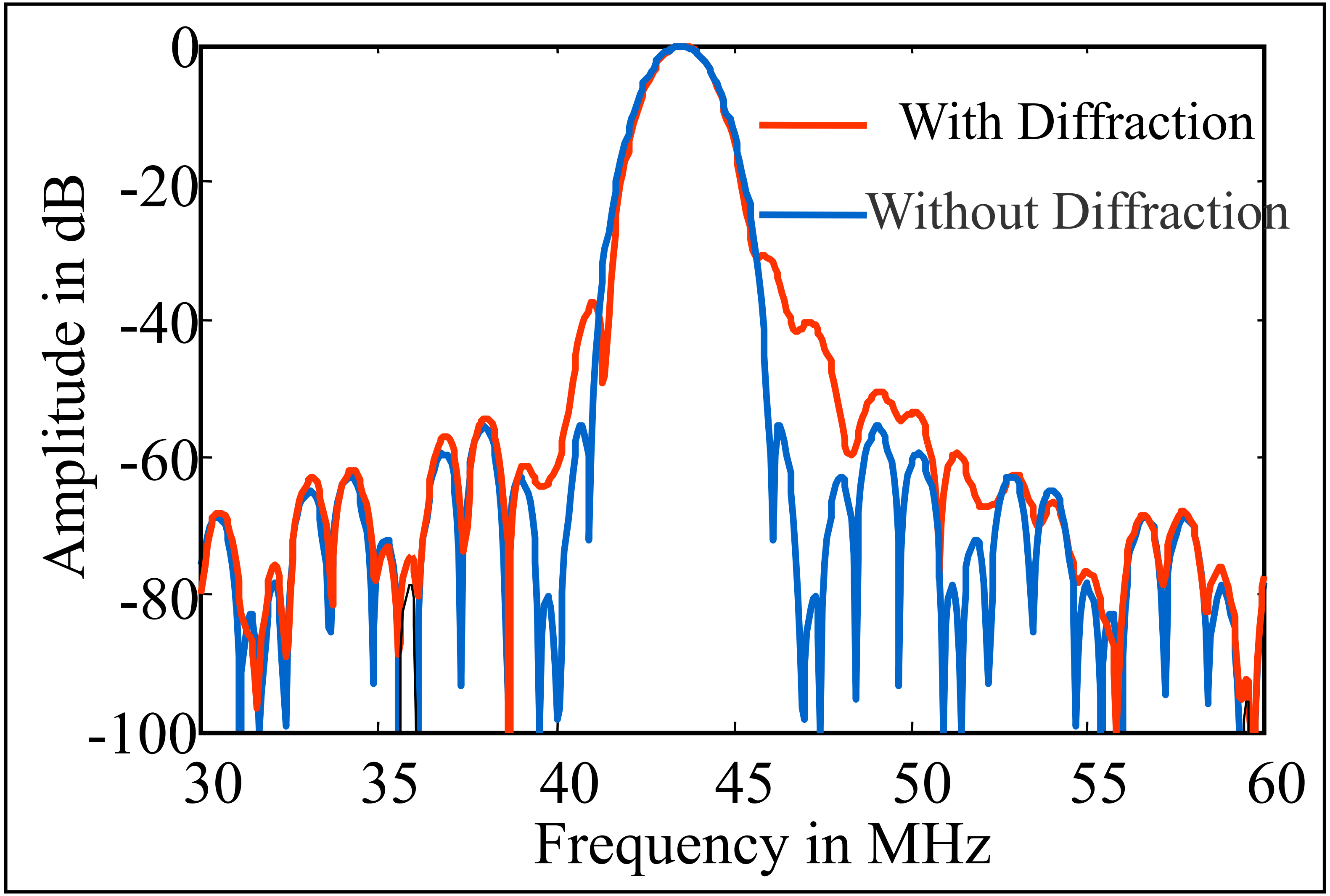
Detection by m-th Electrode
(Width W_m, Position (X_m, Y_m))



$$Q = \sum_{m=1}^{M} \int_{Y_m - W_m/2}^{Y_m + W_m/2} \phi(X_m, Y) dY$$

$$= A \sum_{m=1}^{M} \sum_{n=1}^{N} \int_{Y_m - W_m/2}^{Y_m + W_m/2} \int_{y_n - w_n/2}^{y_n + w_n/2} G(X_m - x_n, Y - y) dy dY$$

Simulation

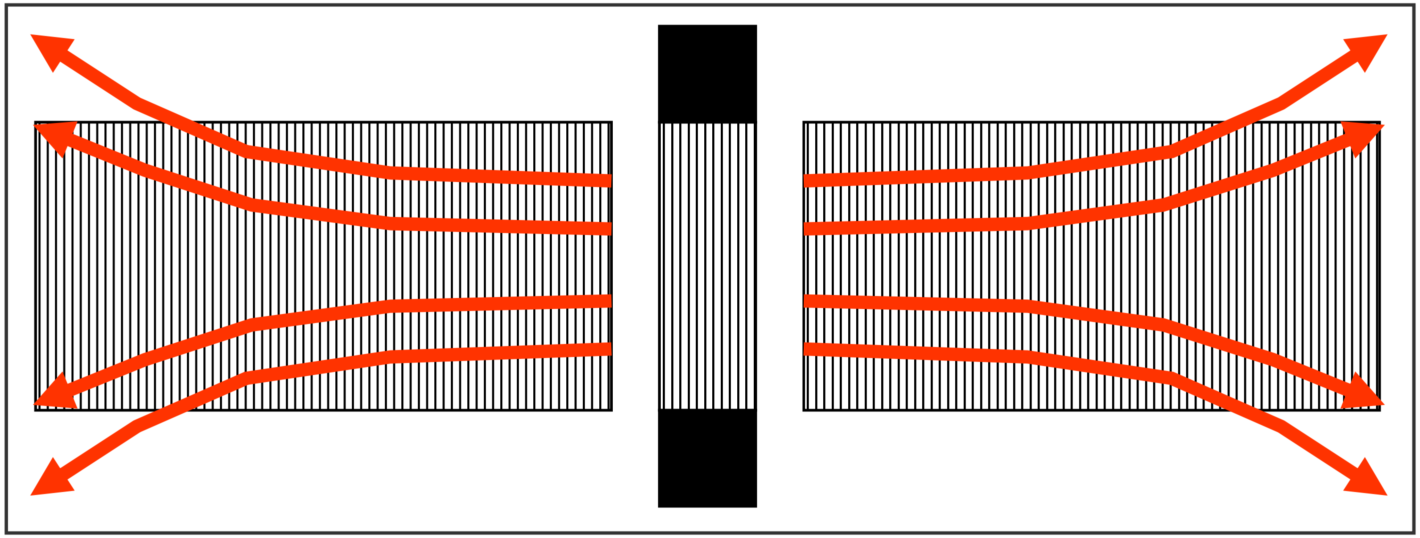


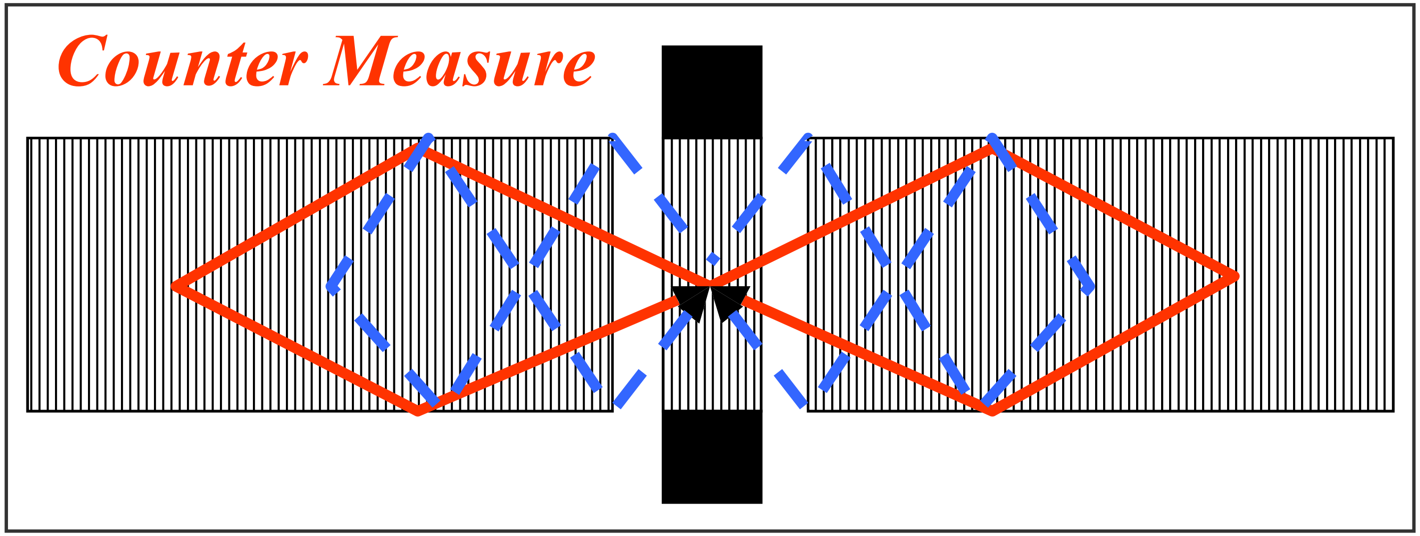
Significant at Higher Out-of-Band Rejection

Contents

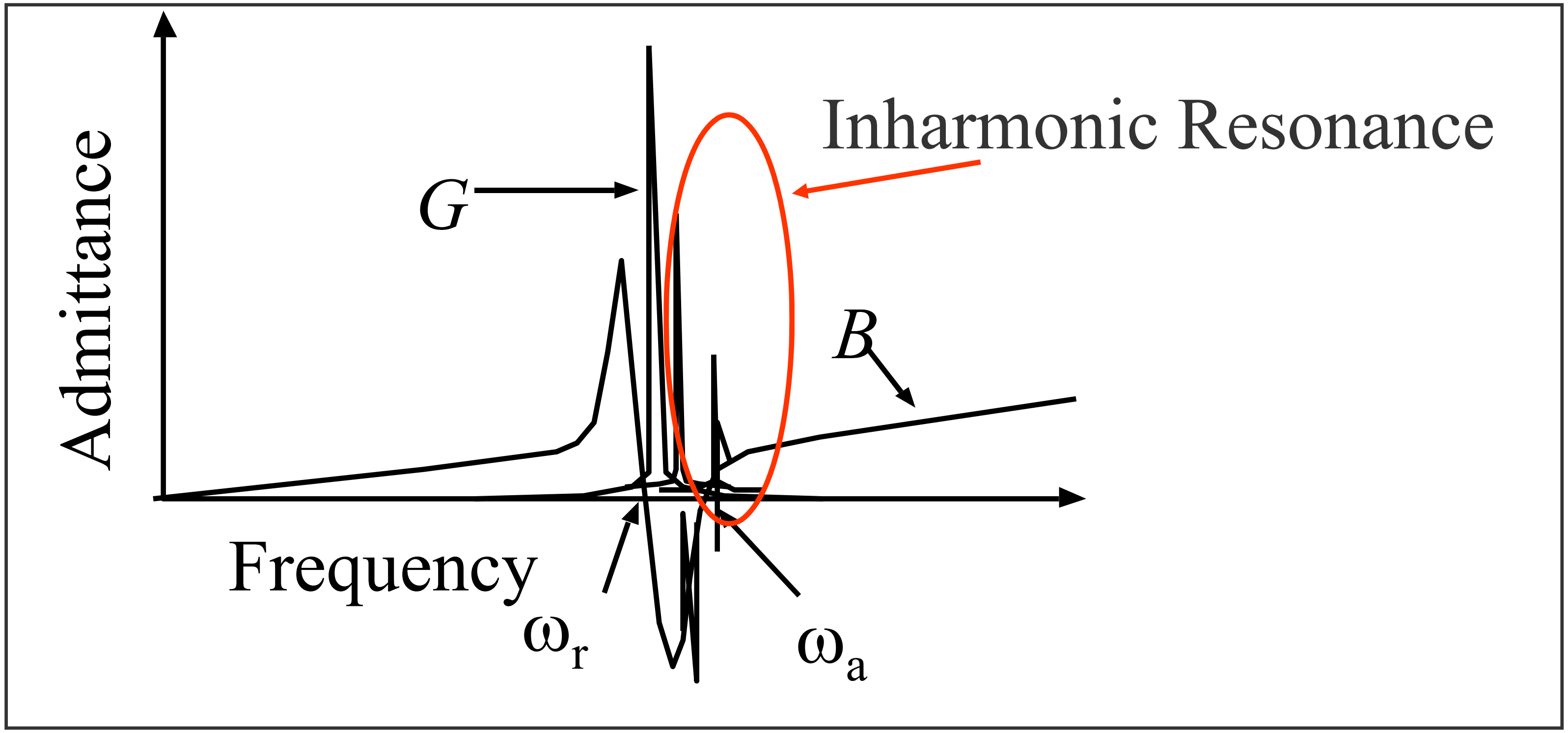
- Waveguide

Influence of Diffraction in SAW Resonators





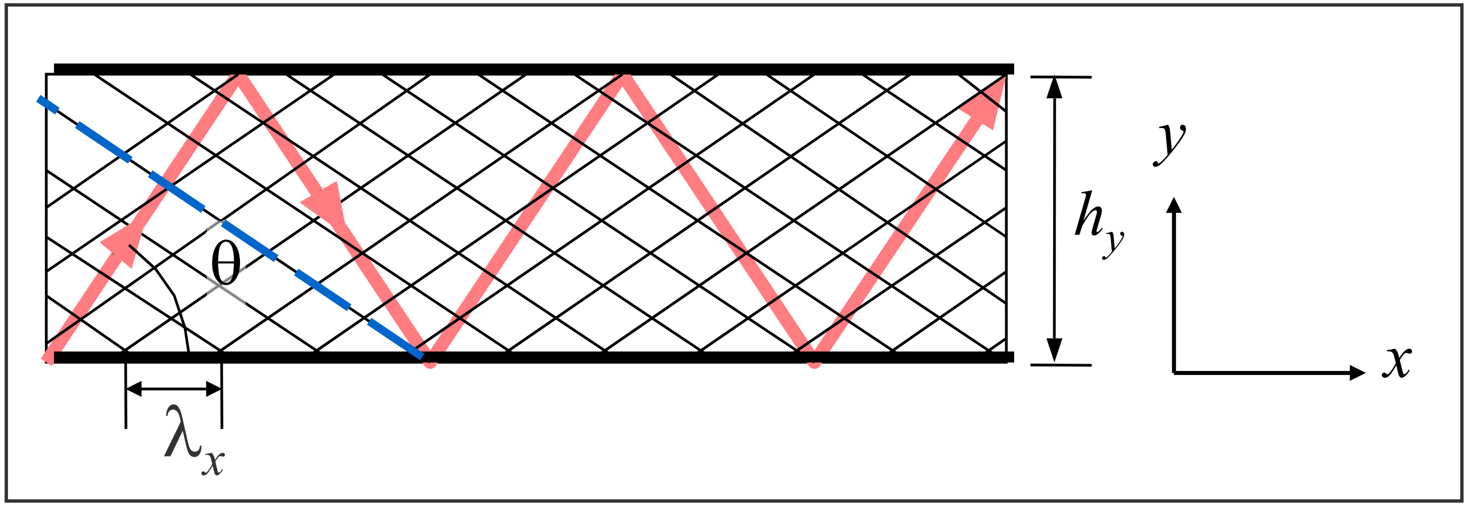
Inharmonic Resonances



Design Challenge: Suppression of Inharmonic Resonances Without Badly Affecting Main Resonance

Closed Waveguide

Wavenumber of Mode $\lambda_x = 2\pi/\beta_x$



For Phase Matching Between Incident and 2-Bounced Waves

$$-2\beta_0 h_y \csc\theta + 2\angle\Gamma = -2\beta_0 \cot\theta \times h_y \cos\theta + 2n\pi$$

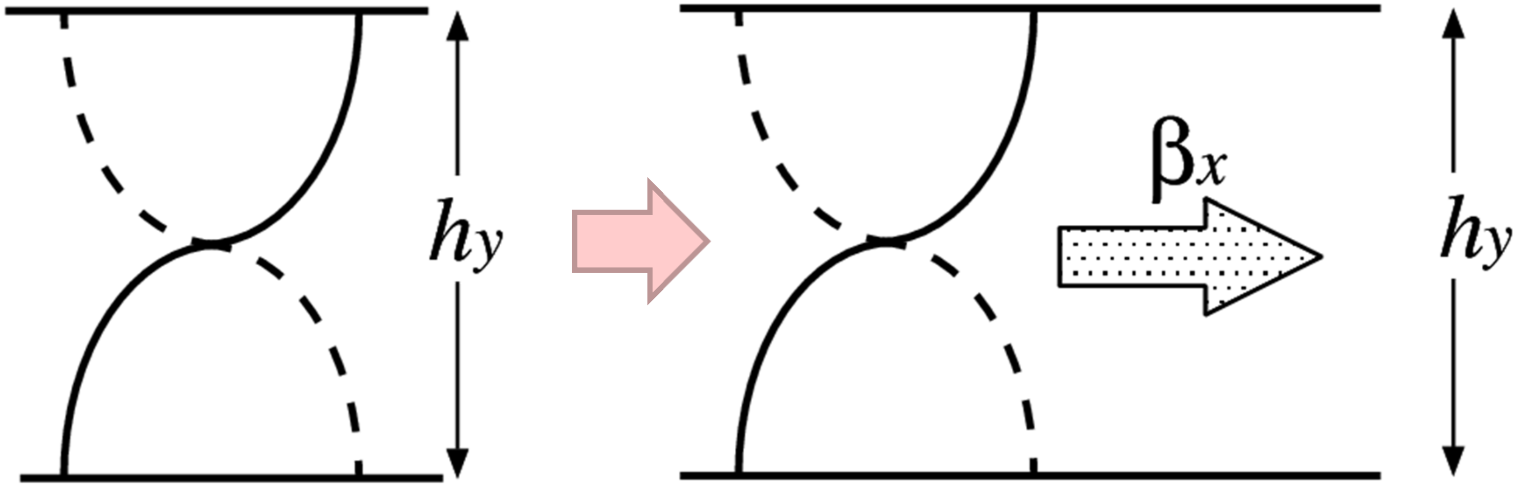
Γ: Reflection Coef. at Boundaries

⇩

Transverse Resonance Condition $-2\beta_y h_y + 2\angle\Gamma = 2n\pi$

Resonance Condition

Transverse Resonance Condition



$$\beta_y h_y = n_y \pi$$

$$\beta_y h_y = n_y \pi$$

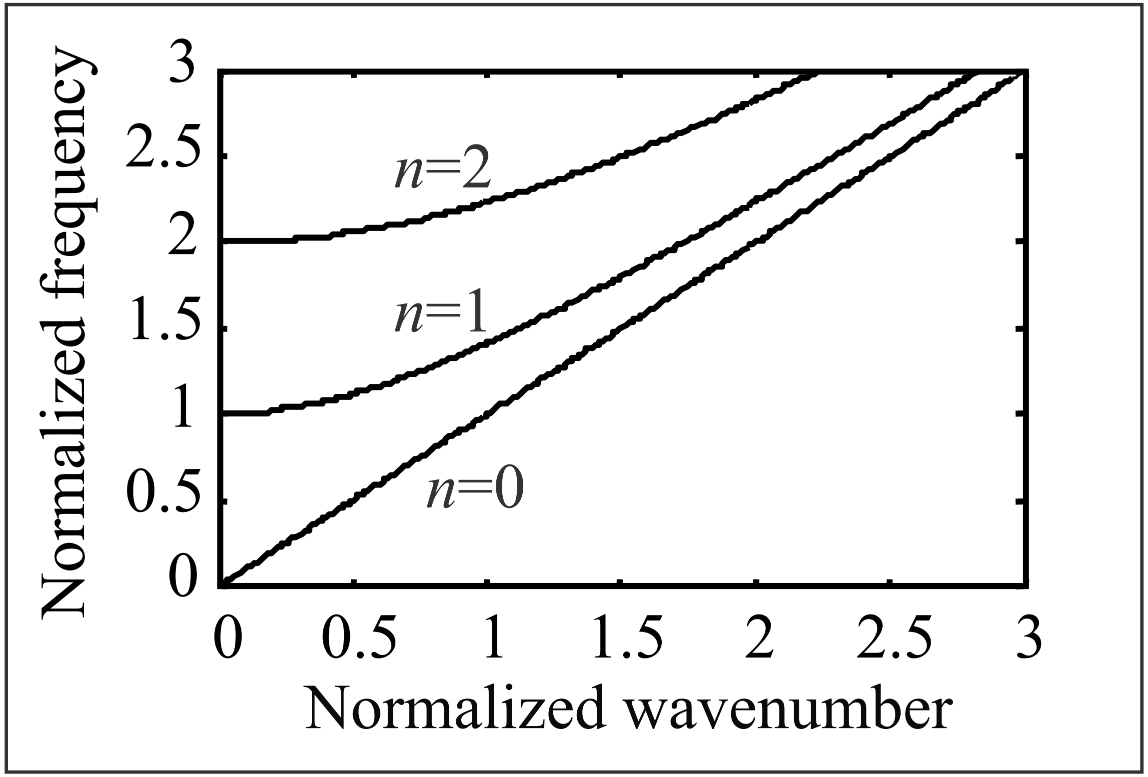
$$\left(\frac{n_y \pi}{h_y}\right)^2 = \beta_0^2$$

$$\underline{\beta_x^2} + \left(\frac{n_y \pi}{h_y}\right)^2 = \beta_0^2$$

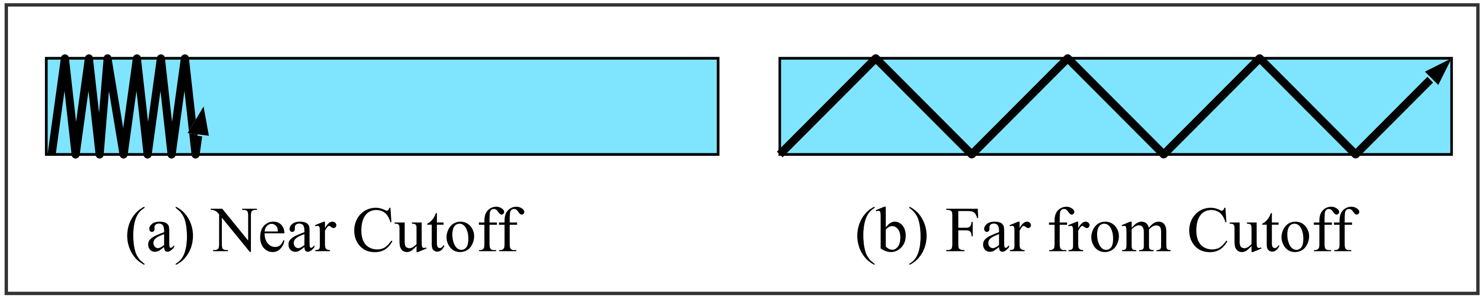
$$\beta_x = \sqrt{\beta_0^2 - \beta_y^2} \quad \text{and} \quad \beta_y h_y = n\pi$$

⇩

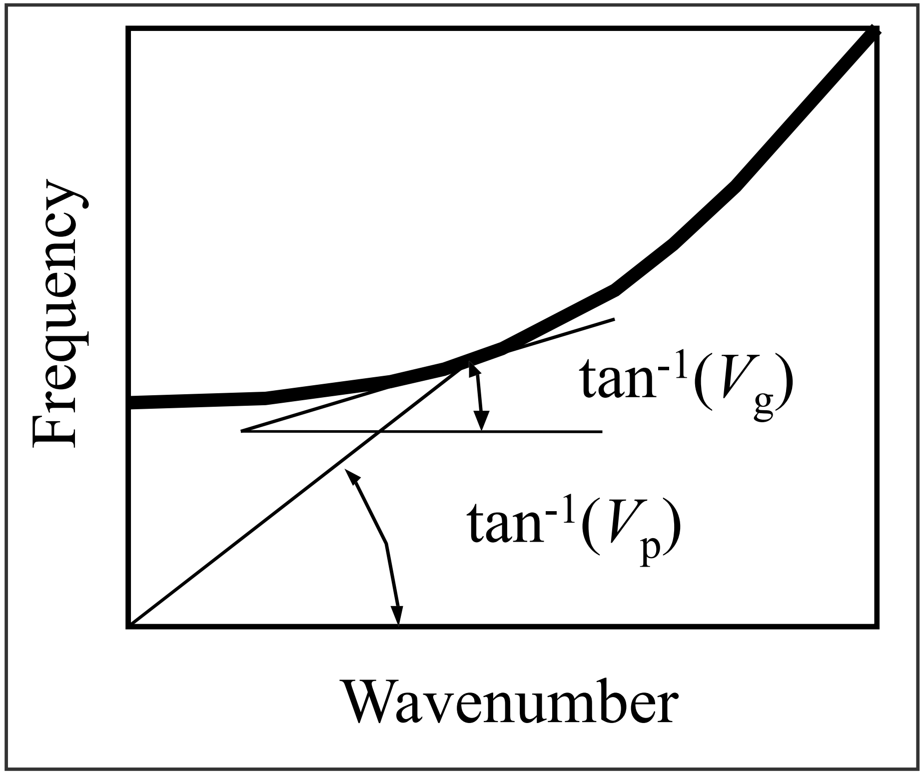
Wavenumber of Guided Mode $\beta_x = \sqrt{(\omega / V)^2 - (n\pi / h_y)^2}$



Relation Between β_x and β_0 When $\angle\Gamma=0$ or $\pm\pi$



Propagation of Waveguide Mode



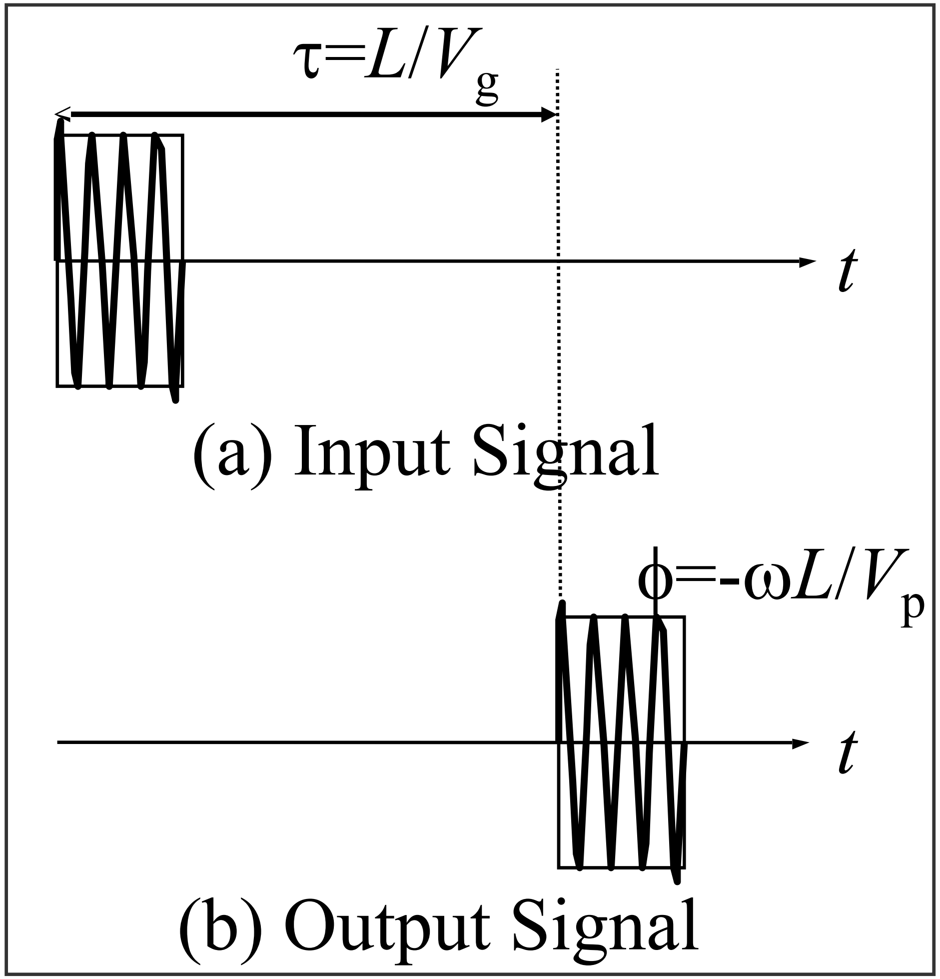
$V_p = \omega/\beta_x$: Phase Velocity

Propagation Speed of Phase Front

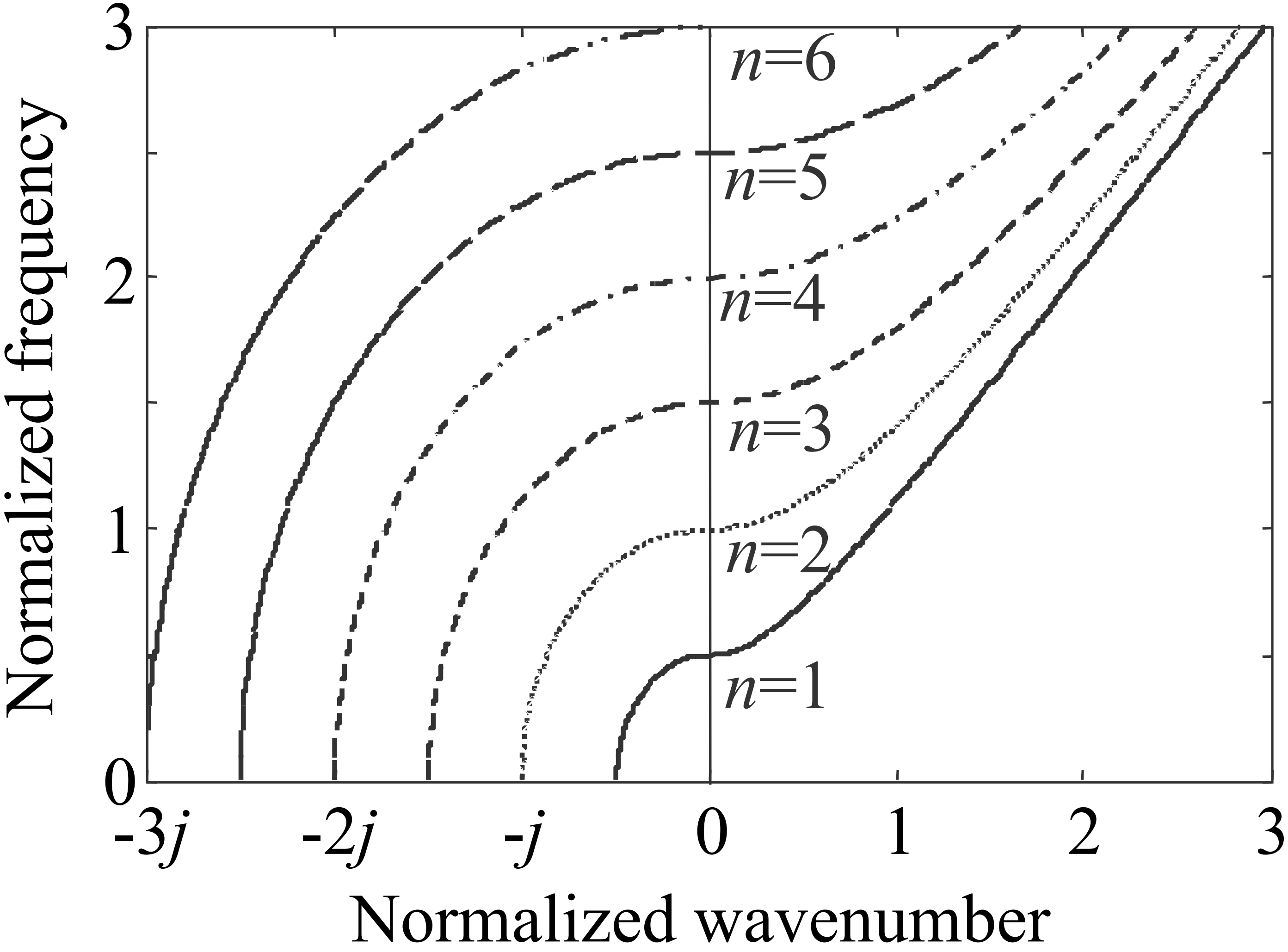
$V_g = \partial\omega/\partial\beta_x$: Group Velocity

Propagation Speed of Energy

Influence of Group and Phase Velocities on Signal Transfer

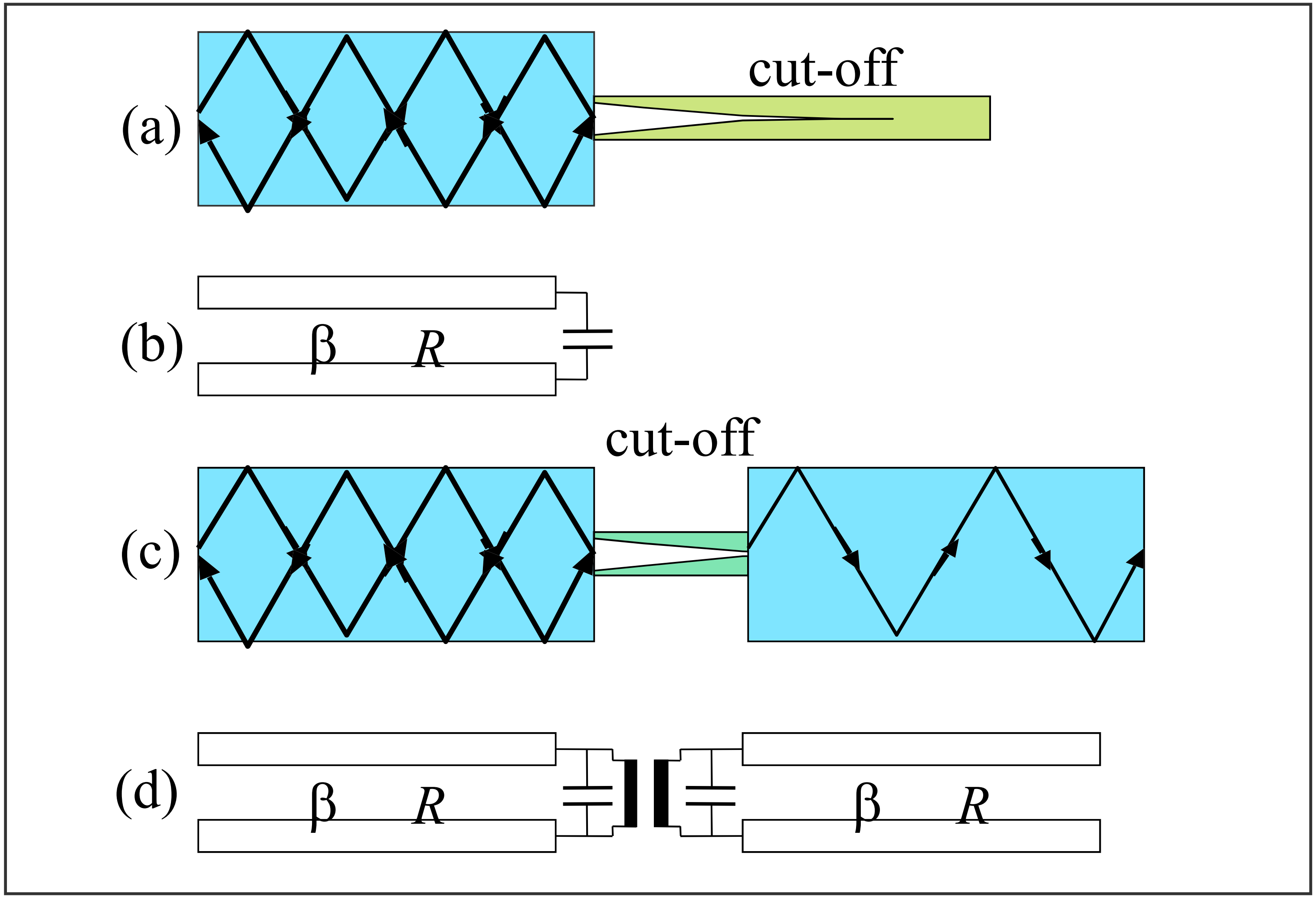


Under Cutoff Frequency



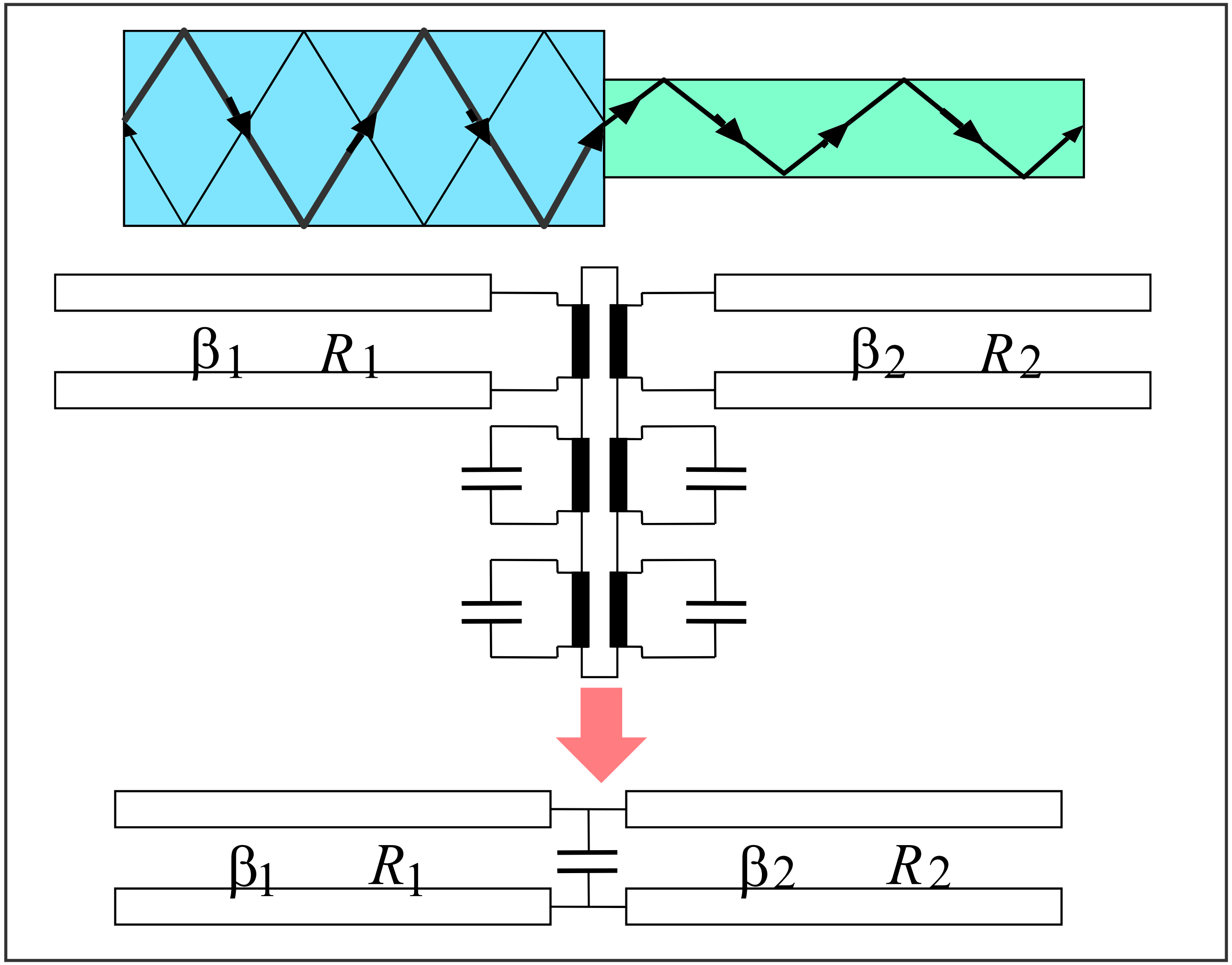
Behavior as Evanescent Field

At Cutoff



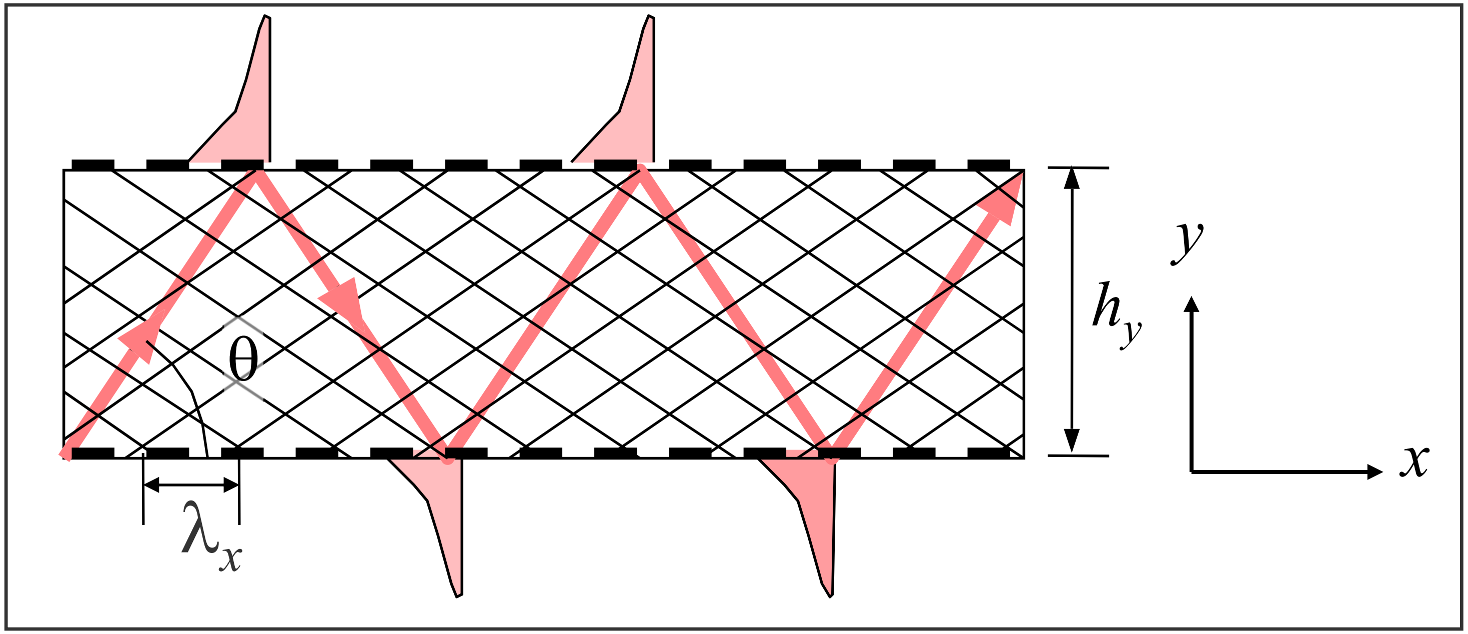
Behavior as Evanescent Field

Even if not Cutoff



Influence of Higher-Order Cutoff Modes

Open Waveguide

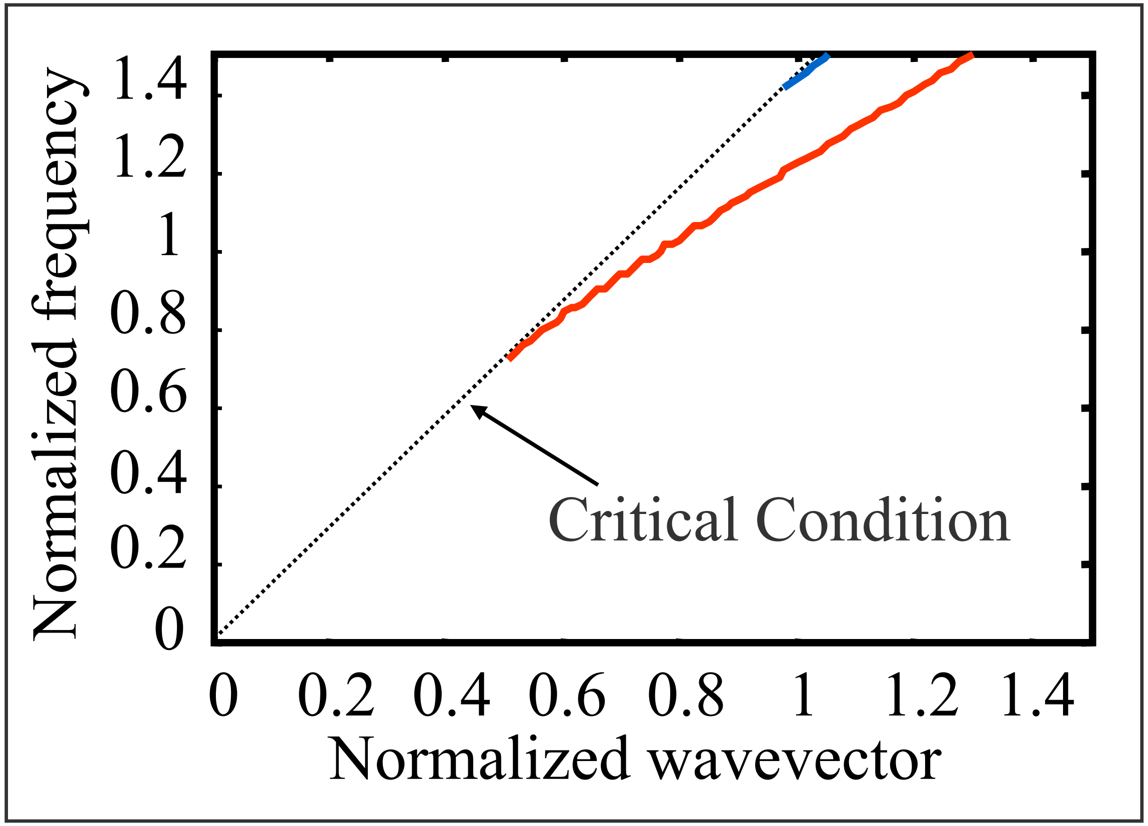


Use of Total Reflection at Surfaces ⇒
Energy Penetration to Outsides

Transverse Resonance Condition $-2\beta_y h_y + 2\angle\Gamma = 2n\pi$

$\angle\Gamma$ is Frequency (or θ) dependent

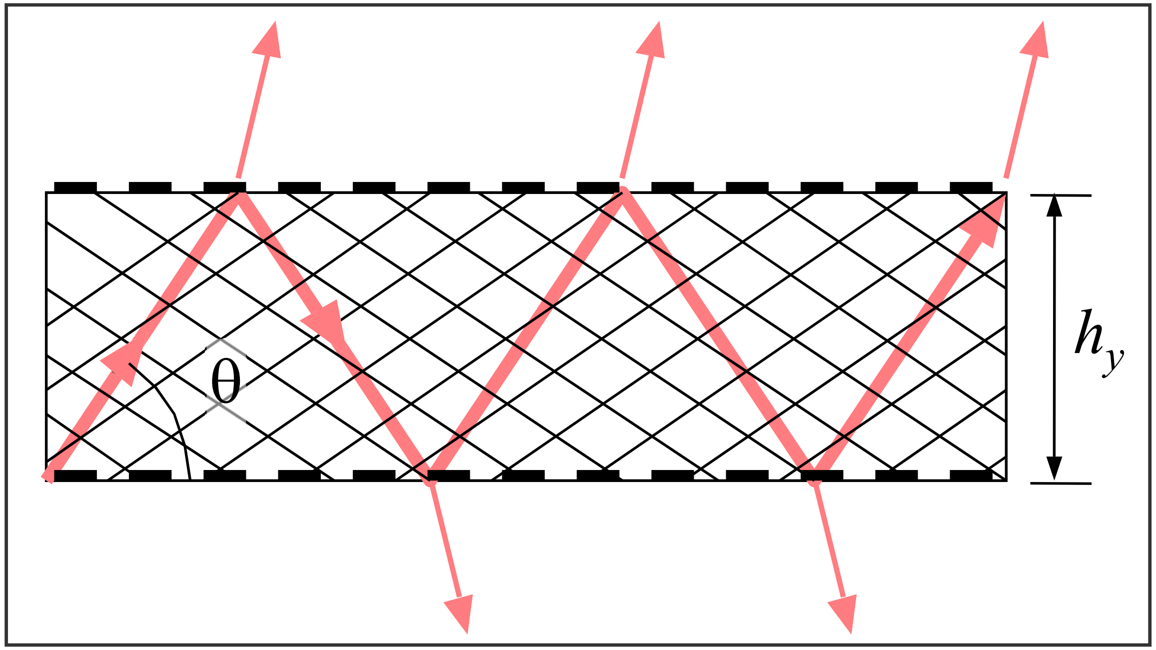
Similarity with Closed Waveguide at Total Reflection



Relation between β_x and β_0

If Total Reflection Condition is Not Satisfied?

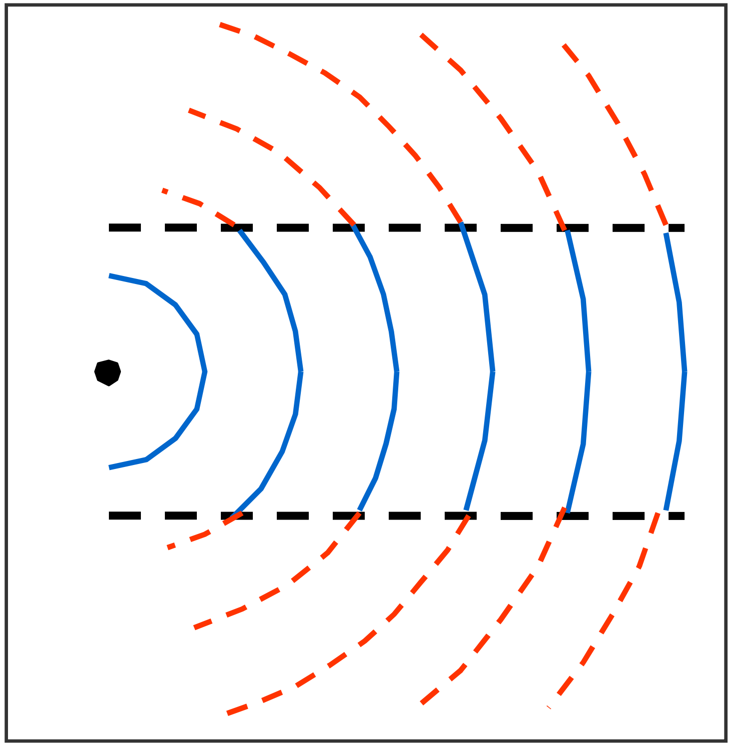
Leaky Waveguide



When Reflection Coefficient at Surfaces is Large, Pseudo Mode Propagates with Energy Leakage to Outside

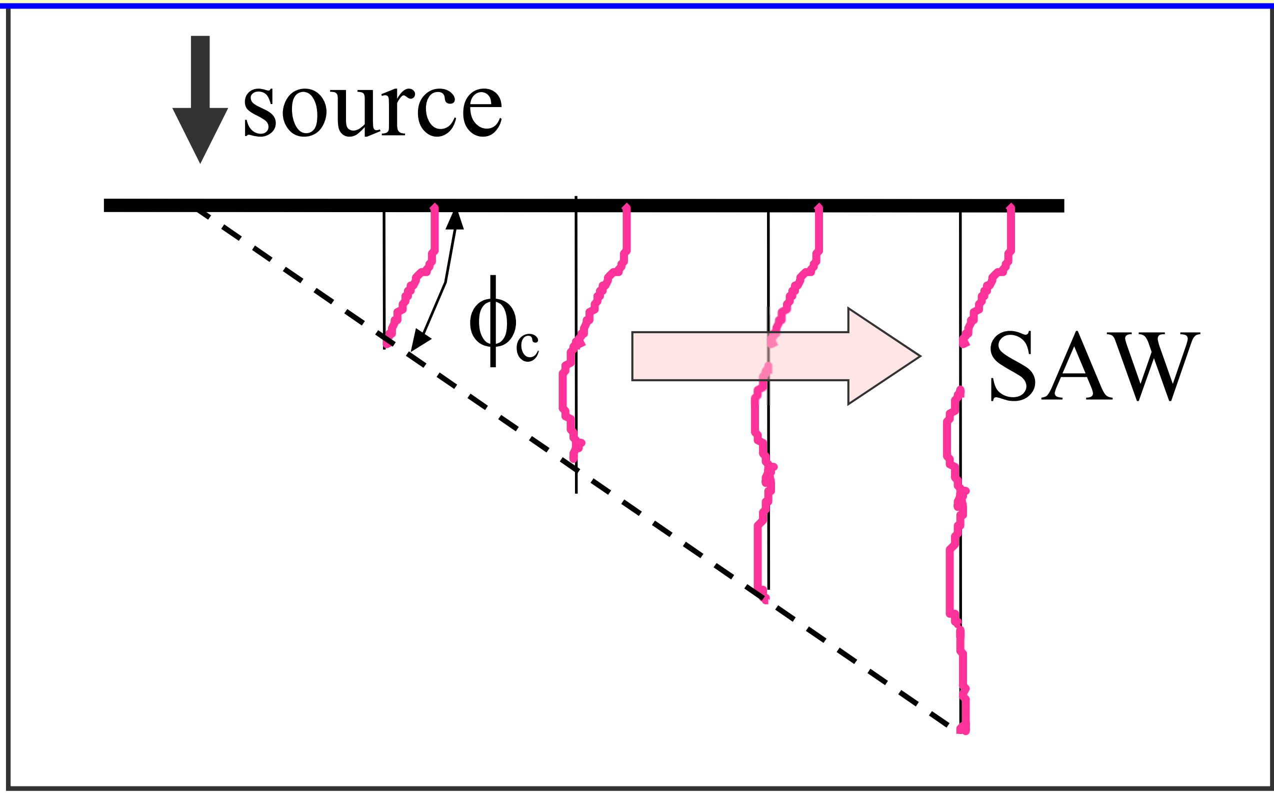
If Reflection Coefficient at Surfaces is Small?

Propagation as Free Wave（Not Guided）



Appearing When Velocities of Waveguide Mode
and Free Wave are Close (Near Cutoff)

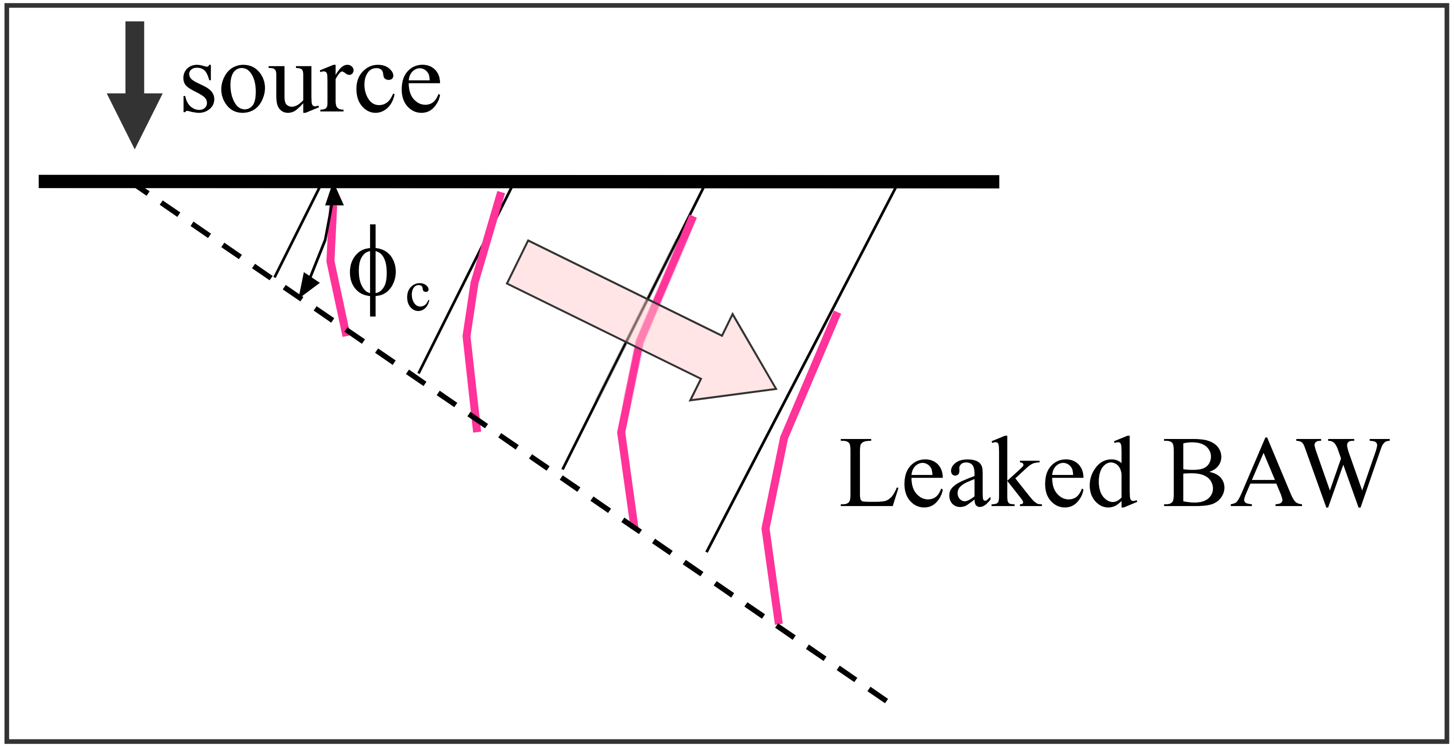
Excitation and Propagation of Non-Leaky Component



$\phi_c = \cos^{-1}(V_S/V_B)$: critical angle

V_S: SAW velocity, V_B: BAW velocity

Excitation and Propagation of Leaked-BAW Component

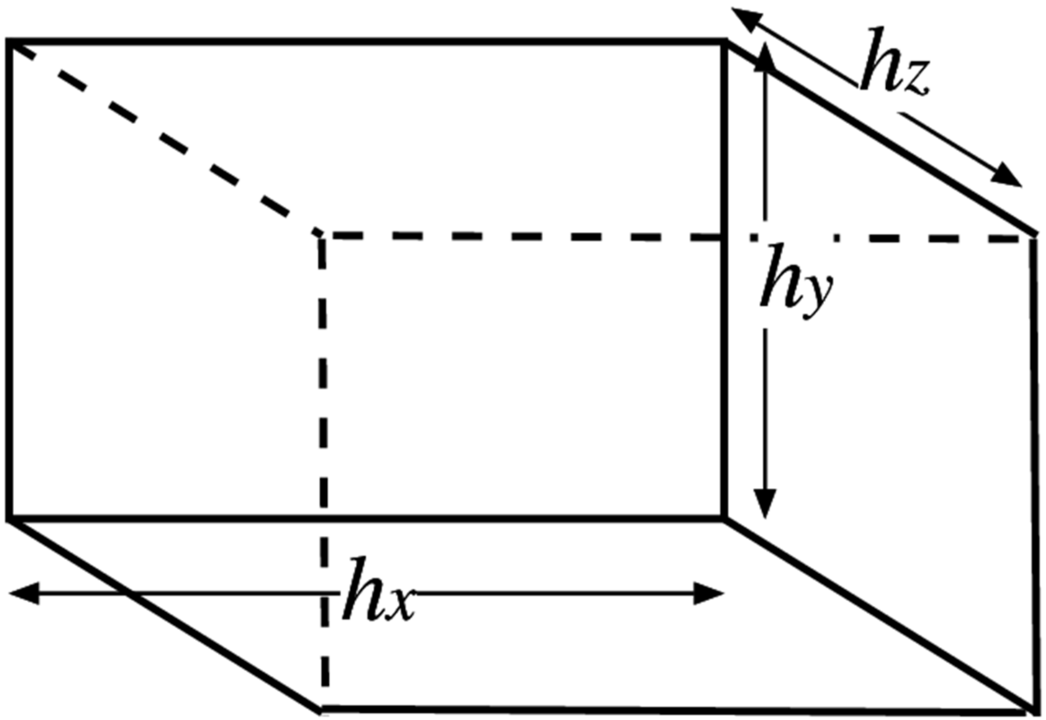


$\phi_c = \cos^{-1}(V_B/V_S)$: critical angle

V_S: SAW velocity, V_B: BAW velocity

Field Amplitude Grows Toward the Depth!

Resonance Frequency of Cuboid Cavity



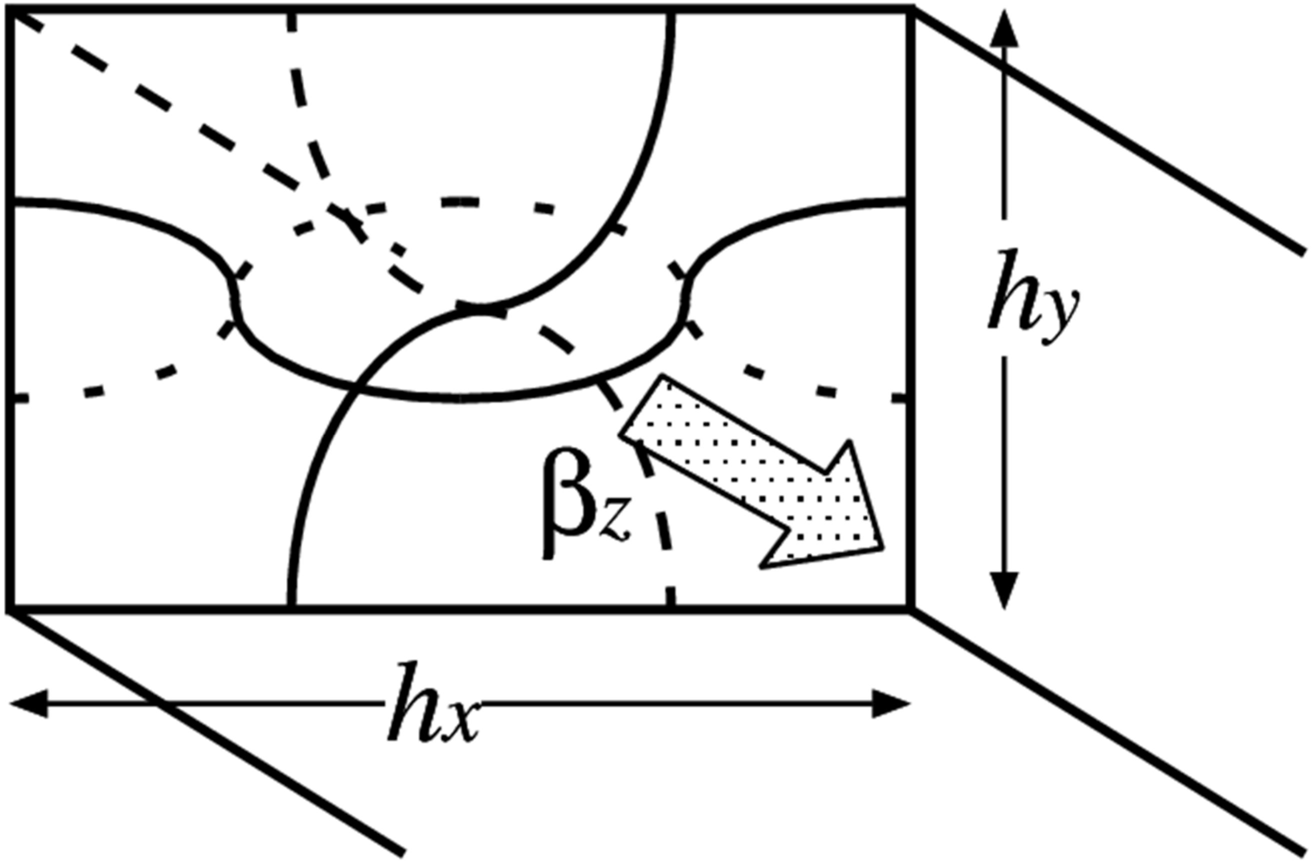
$$\beta_x h_x = n_x \pi$$

$$\beta_y h_y = n_y \pi$$

$$\beta_z h_z = n_z \pi$$

$$\Rightarrow \left(\frac{n_x \pi}{h_x}\right)^2 + \left(\frac{n_y \pi}{h_y}\right)^2 + \left(\frac{n_z \pi}{h_z}\right)^2 = \beta_0^2$$

Wavevector of Propagation Mode in Rectangular Waveguide

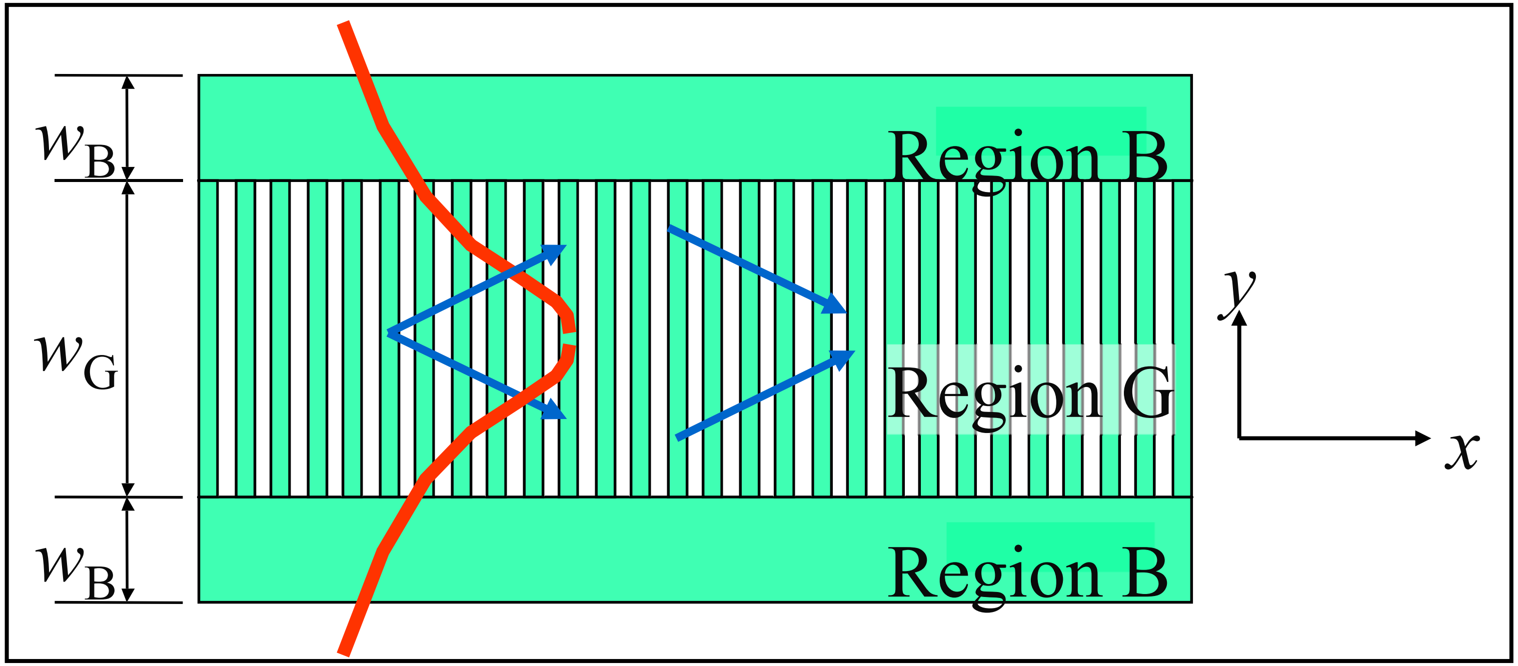


$$\beta_z = \sqrt{\beta_0^2 - \left(\frac{n_x \pi}{h_x}\right)^2 - \left(\frac{n_y \pi}{h_y}\right)^2}$$

Contents

- Scalar Potential Theory

Scalar Potential Analysis



2-D Analysis

Approximation as Uniform (Flat) IDT

Field Expression(When $w_B=\infty$ is Assumed for Simplicity)

$$\phi = \begin{cases} \phi_B^+ \exp(-\alpha_{By} y)\exp(-j\beta x) & (x \geq +w_G/2) \\ \{\phi_G^+ \exp(-j\beta_{Gy} y) + \phi_G^- \exp(+j\beta_{Gy} y)\}\exp(-j\beta x) & (|x| \leq w_G/2) \\ \phi_B^- \exp(+\alpha_{By} y)\exp(-j\beta x) & (x \leq -w_G/2) \end{cases}$$

Due to Continuity of ϕ and $\partial\phi/\partial y$ at $y=\pm w_G/2$

Symmetric Mode ($\phi_B{}^+=\phi_B{}^-$, $\phi_G{}^+=\phi_G{}^-$)

$$\phi_B^+ = 2\phi_G^+ \cos(\beta_{Gy} w_G / 2) \exp(\alpha_{By} w_G / 2)$$

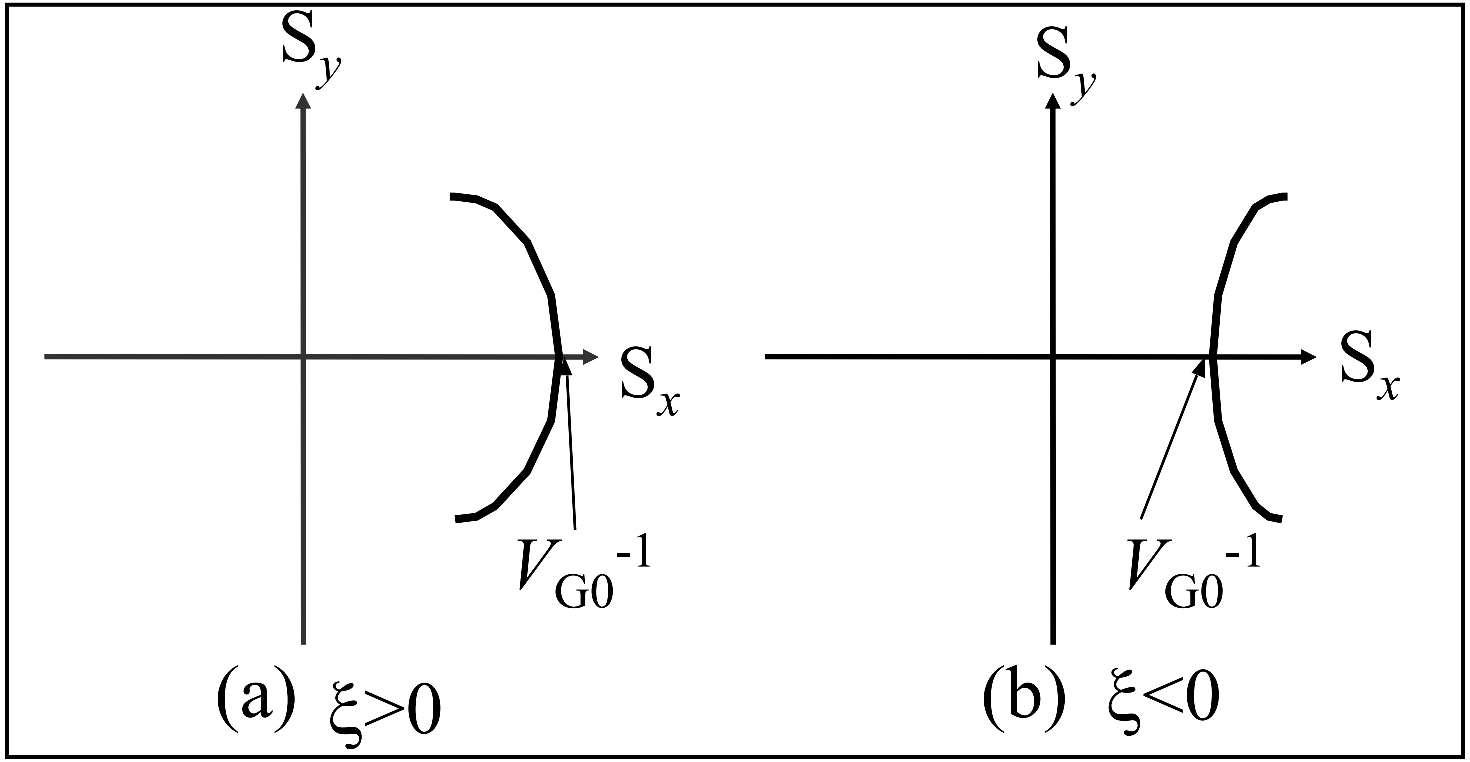
$$\alpha_{By} = \beta_{Gy} \tan(\beta_{Gy} w_G / 2)$$

Anti-Symmetric Mode ($\phi_B{}^+=-\phi_B{}^-$, $\phi_G{}^+=-\phi_G{}^-$)

$$\phi_B^+ = -2j\phi_G^+ \sin(\beta_{Gy} w_G / 2) \exp(\alpha_{By} w_G / 2)$$

$$\alpha_{By} = -\beta_{Gy} \cot(\beta_{Gy} w_G / 2)$$

Parabolic Approximation for Slowness Surface

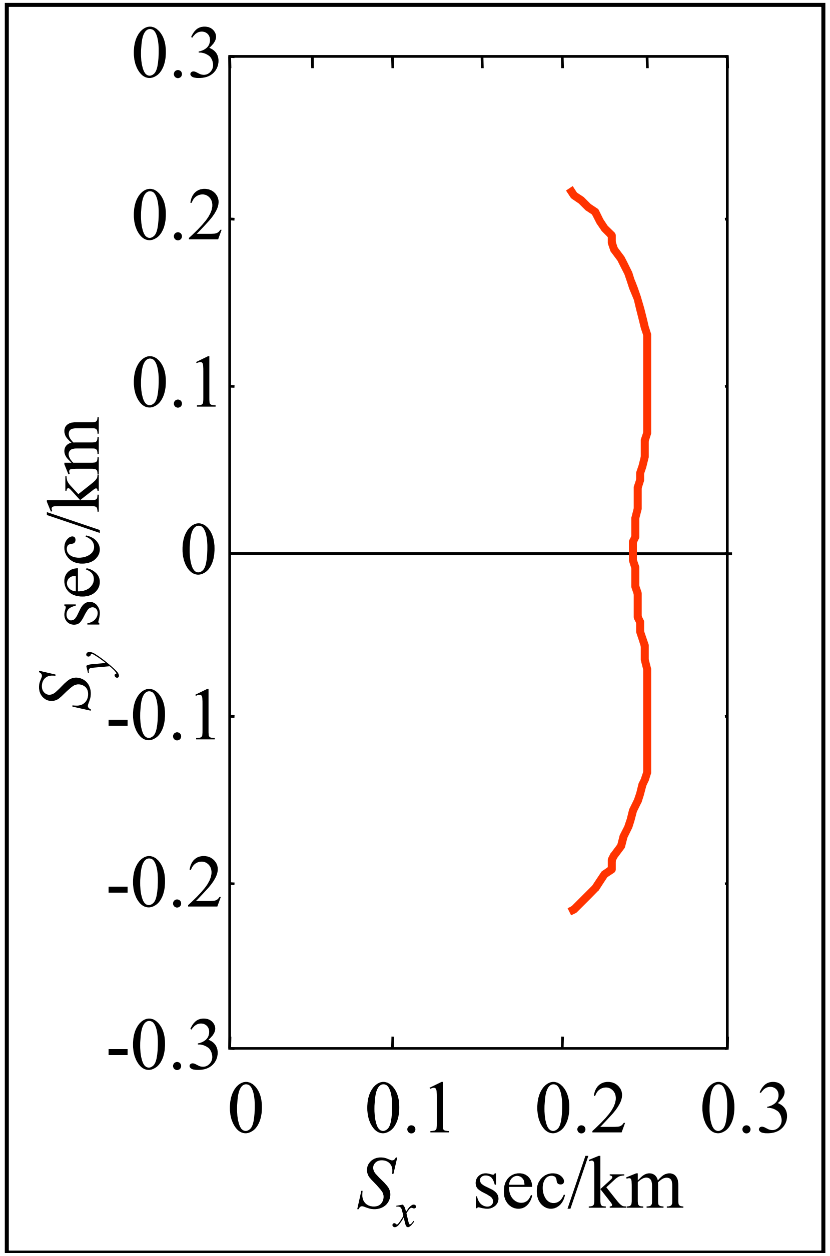


For Region G $\beta_x \cong \beta_{G0} - \xi_G \beta_{Gy}^2 / \beta_{G0}$

For Region B $\beta_x \cong \beta_{B0} + \xi_B \alpha_{By}^2 / \beta_{B0}$

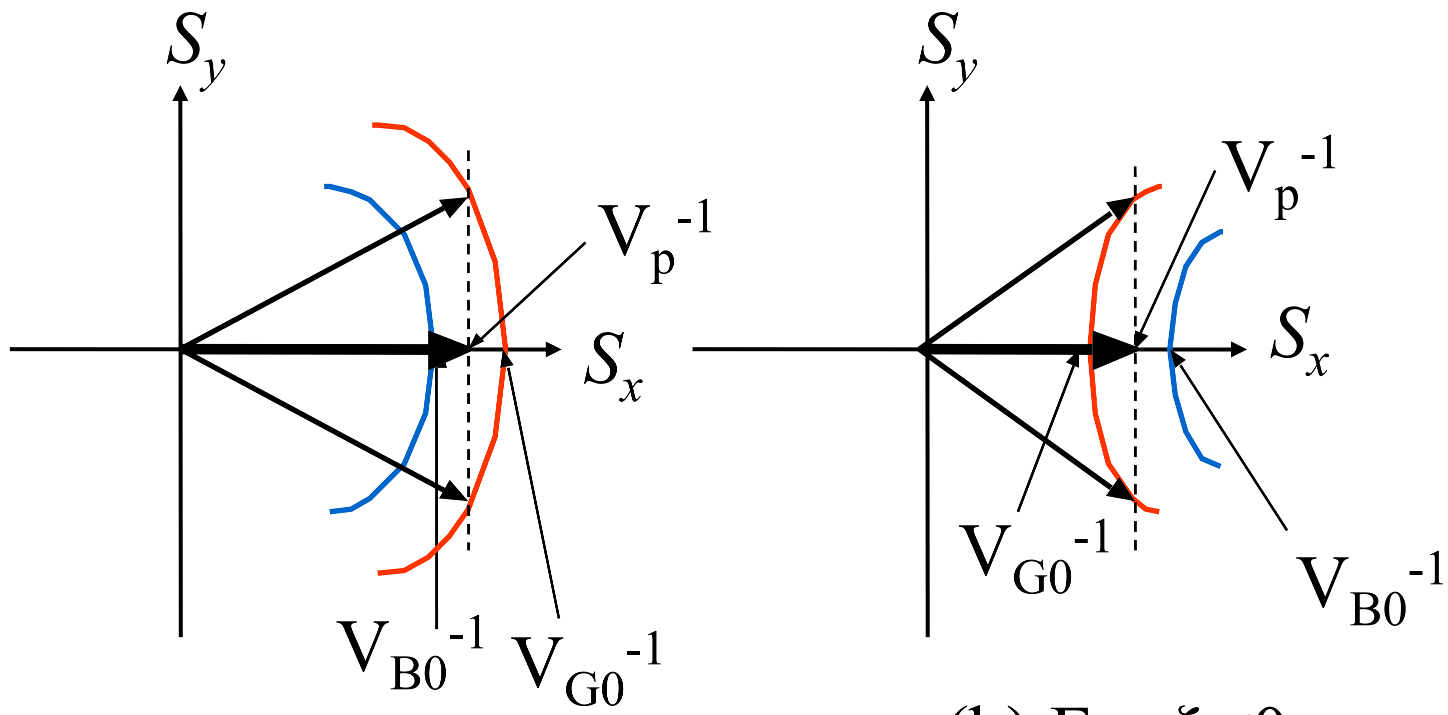
For Isotropic Case, ξ=0.5

Slowness Surface of SH-type SAW on 36-LT



Wavenumber of Grating Mode and Slowness Surface

For Energy Trapping in Waveguide $\Rightarrow$ ***Real*** $\boldsymbol{\alpha_{By}}$



(a) For $\xi>0$

$V_{G0}<V_p<V_{B0}$

Higher-order Modes Appear in Higher Frequencies

(b) For $\xi<0$

$V_{B0}<V_p<V_{G0}$

Higher-order Modes Appear in Lower Frequencies

When $|V_{B0}/V_{G0}-1| \ll 1$,

Symmetric Mode

$$\sqrt{\frac{1-\hat{V}}{1+\hat{V}}} = \tan\left[\pi \hat{w}_{G} \sqrt{\frac{1+\hat{V}}{2}}\right]$$

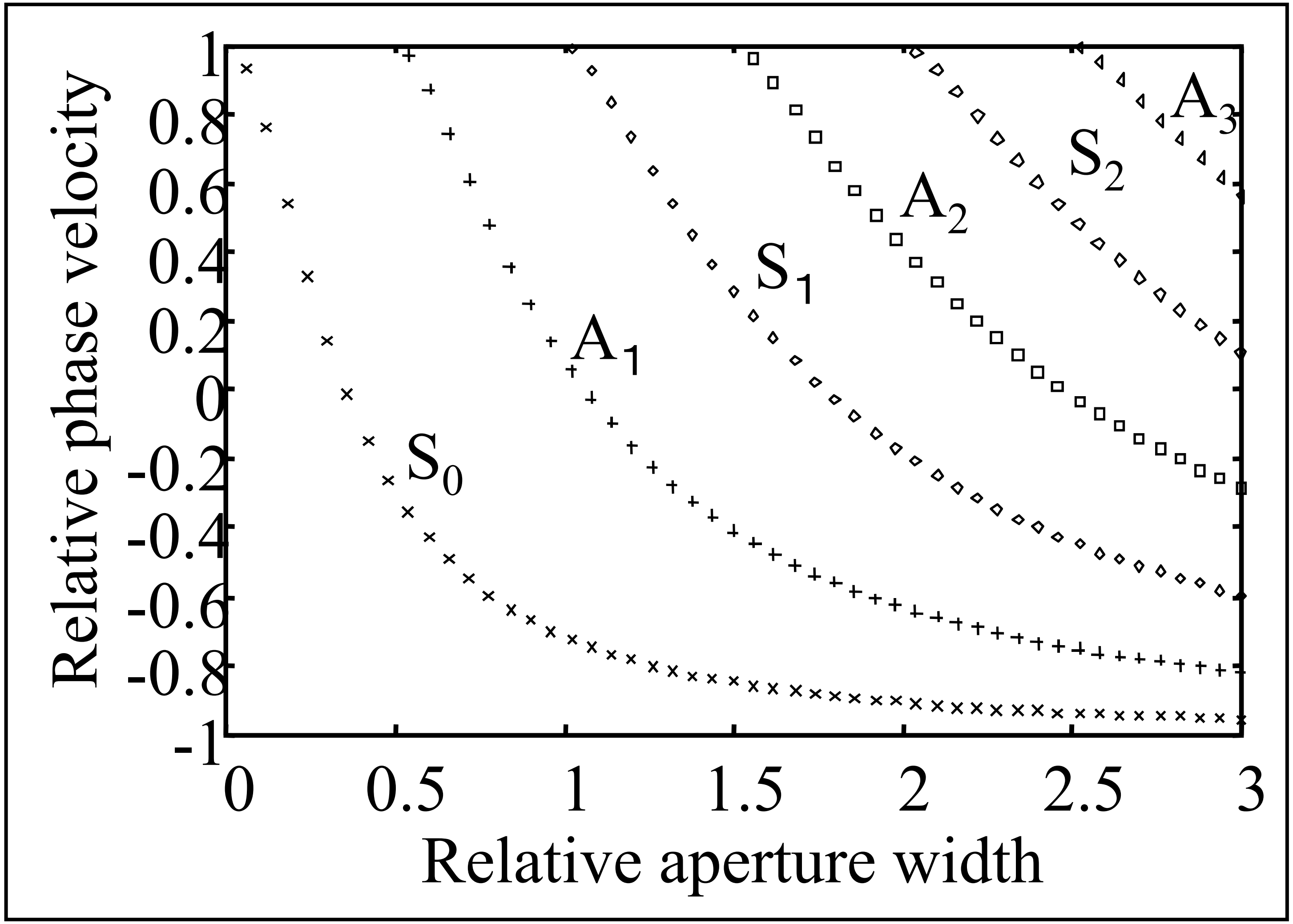
Anti-Symmetric Mode

$$\sqrt{\frac{1-\hat{V}}{1+\hat{V}}} = -\cot\left[\pi \hat{w}_{G} \sqrt{\frac{1+\hat{V}}{2}}\right]$$

Where $\hat{V} = \dfrac{2V_{p} - V_{B0} - V_{G0}}{V_{B0} - V_{G0}}$:Relative Phase Velocity

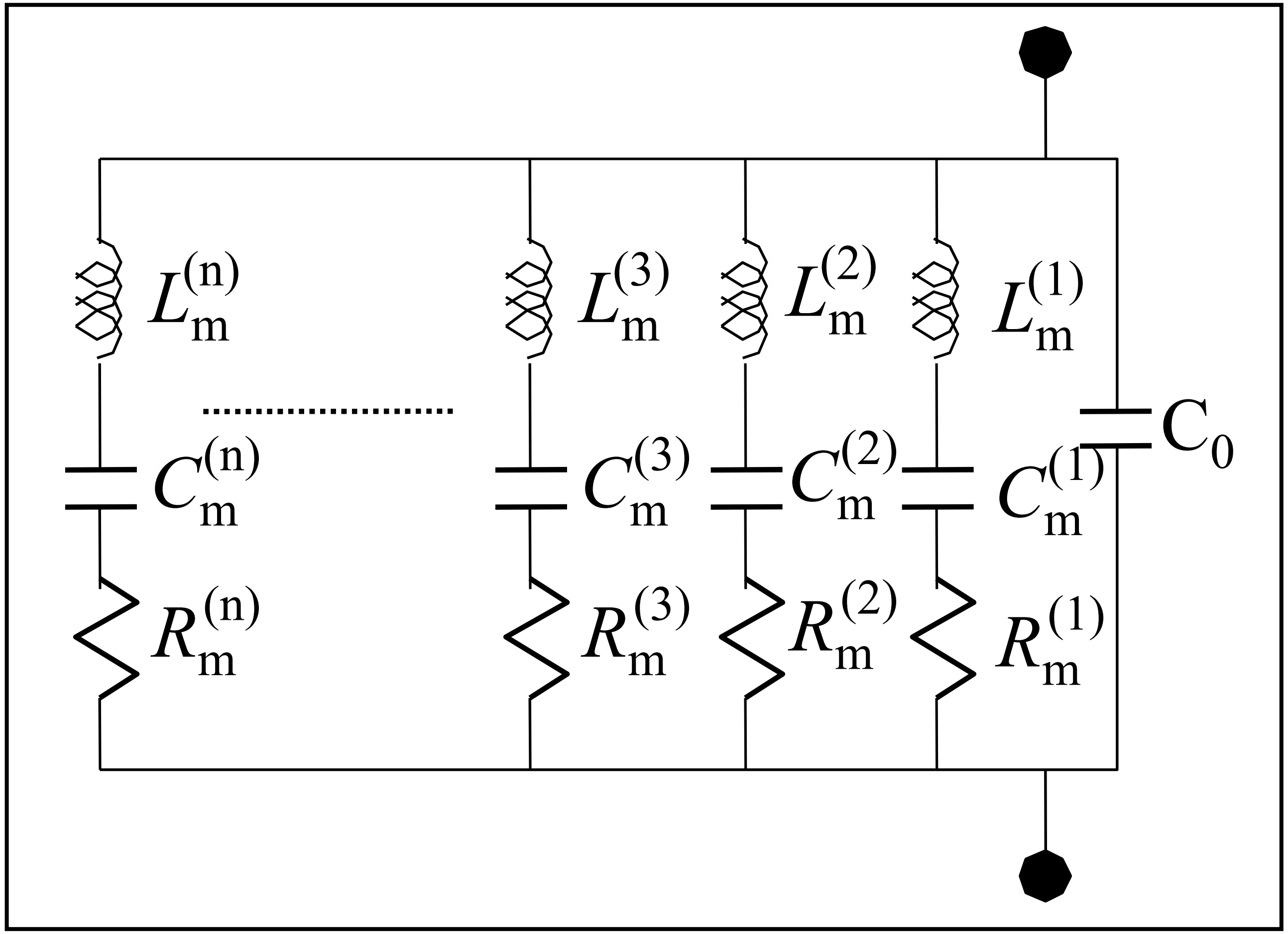
$\hat{w}_{G} = \dfrac{w_{G}}{\lambda_{p}} \sqrt{\dfrac{V_{B0} - V_{G0}}{\xi V_{G0}}}$:Relative Waveguide Width

Relative SAW Velocity vs. Relative Aperture



Velocity in Region B ⇒ Velocity in Region G

Equivalent Circuit for Multi-Mode Resonators



$$\omega_r^{(n)} = \frac{1}{\sqrt{C_m^{(n)} L_m^{(n)}}} = \frac{2\pi V_p^{(n)}}{p_I}$$

Modes Propagate without Mutual Power Interaction

$$\int_{-\infty}^{+\infty} \left|\phi_k(y) + \phi_n(y)\right|^2 dy = \int_{-\infty}^{+\infty} \left|\phi_k(y)\right|^2 dy + \int_{-\infty}^{+\infty} \left|\phi_n(y)\right|^2 dy$$

Mode Orthogonality

$$\int_{-\infty}^{+\infty} \phi_k(y)\phi_n^*(y)dy = \delta_{nk} P_k$$

$$\text{where} \quad P_k = \int_{-\infty}^{+\infty} \left|\phi_k(y)\right|^2 dy$$

Field can be Expressed as Sum of Mode Fields

Mode Completeness

$$\phi(y) = \sum_{k=1}^{\infty} A_k \phi_k(y) / \sqrt{P_k}$$

Fourier Transform $\varphi_n(x)=p^{-0.5}\exp(2n\pi jx/p)$

Orthogonality

$$\int_0^p \phi_k(x)\phi_n^*(x)dx = p^{-0.5}\int_0^p \exp[2\pi jx(n-m)/p]dx = \delta_{nk}$$

Completeness

$$\phi(x) = \sum_{k=1}^{\infty} A_k\phi_k(x) = p^{-0.5}\sum_{k=1}^{\infty} A_k \exp(2k\pi jx/p)$$

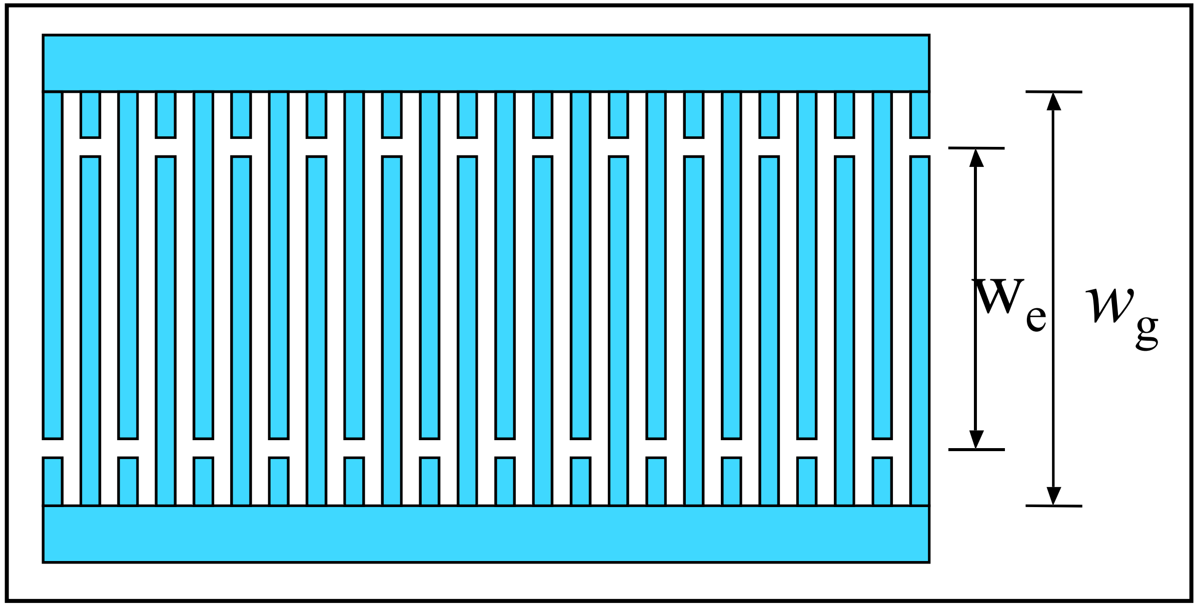
Multiplication of $\varphi_n^*(x)$ *& Integration give*

$$\int_0^p \phi(x)\phi_n^*(x)dx = \int_0^p \sum_{k=1}^{\infty} A_k\phi_k(x)\phi_n^*(x)dx = A_n$$

⇩

$$A_n = p^{-0.5}\int_0^p \phi(x)\exp(-2n\pi jx/p)dx$$

Difference of Waveguide Width w_g with Finger Overlap Width w_e



Amplitude at Excitation Source

$$\phi(y) = \begin{cases} \phi_0 & (|y| \le w_e/2) \\ 0 & (|y| > w_e/2) \end{cases}$$

Multiplying $\phi_m^*(y)$ and Integrating

$$\int_{-\infty}^{+\infty} \phi(y)\phi_m^*(y)dy = \sum_{k=1}^{\infty} A_k / \sqrt{P_k} \int_{-\infty}^{+\infty} \phi_k(y)\phi_m^*(y)dy$$

Then

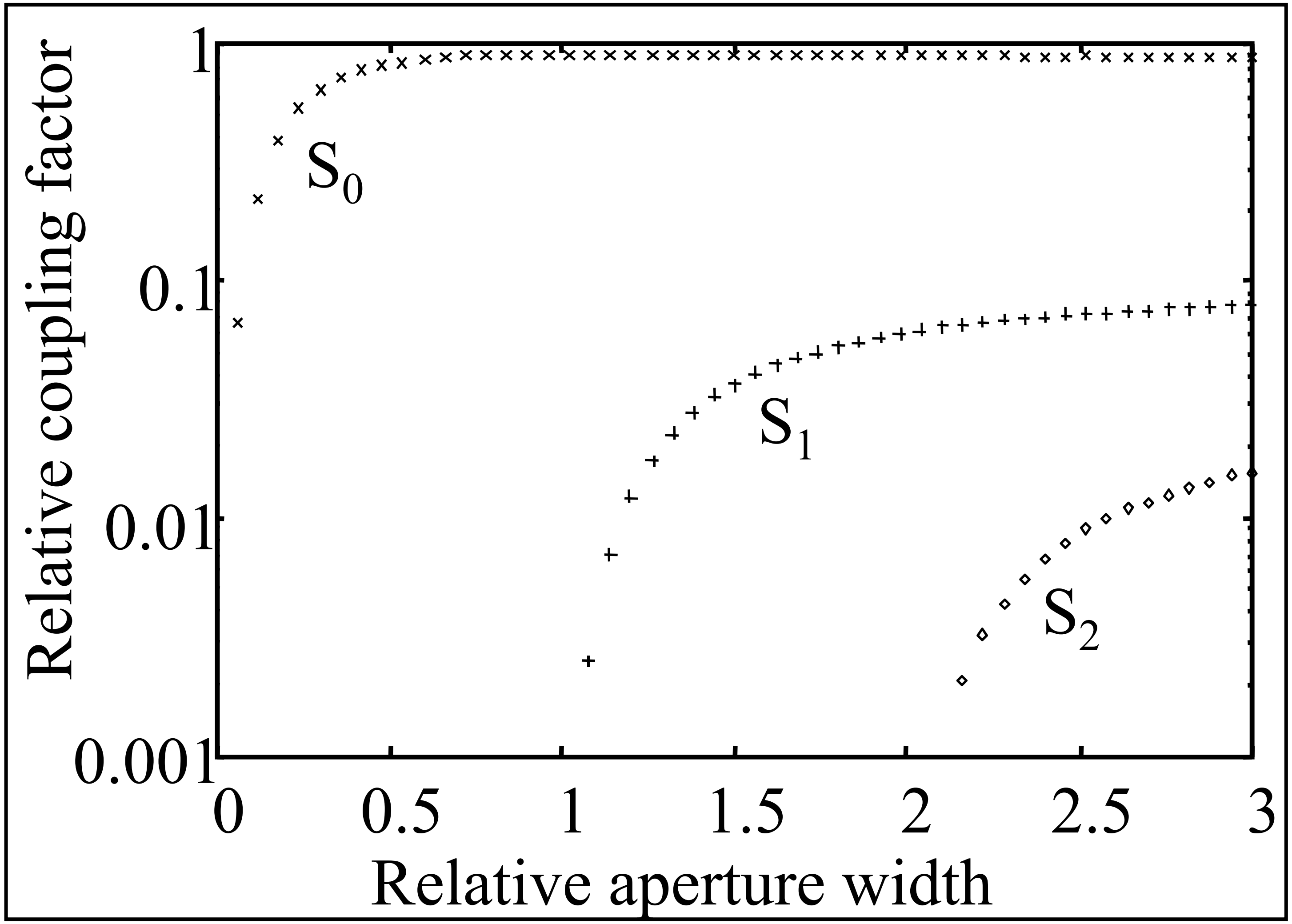
$$A_m = \frac{\phi_0}{\sqrt{P_m}} \int_{-w_e/2}^{+w_e/2} \phi_m^*(y)dy$$

1D Analysis Gives $A_0 = \phi_0 \sqrt{w_e}$

Since Motional Capacitance $\propto$ Power Excitation Efficiency,

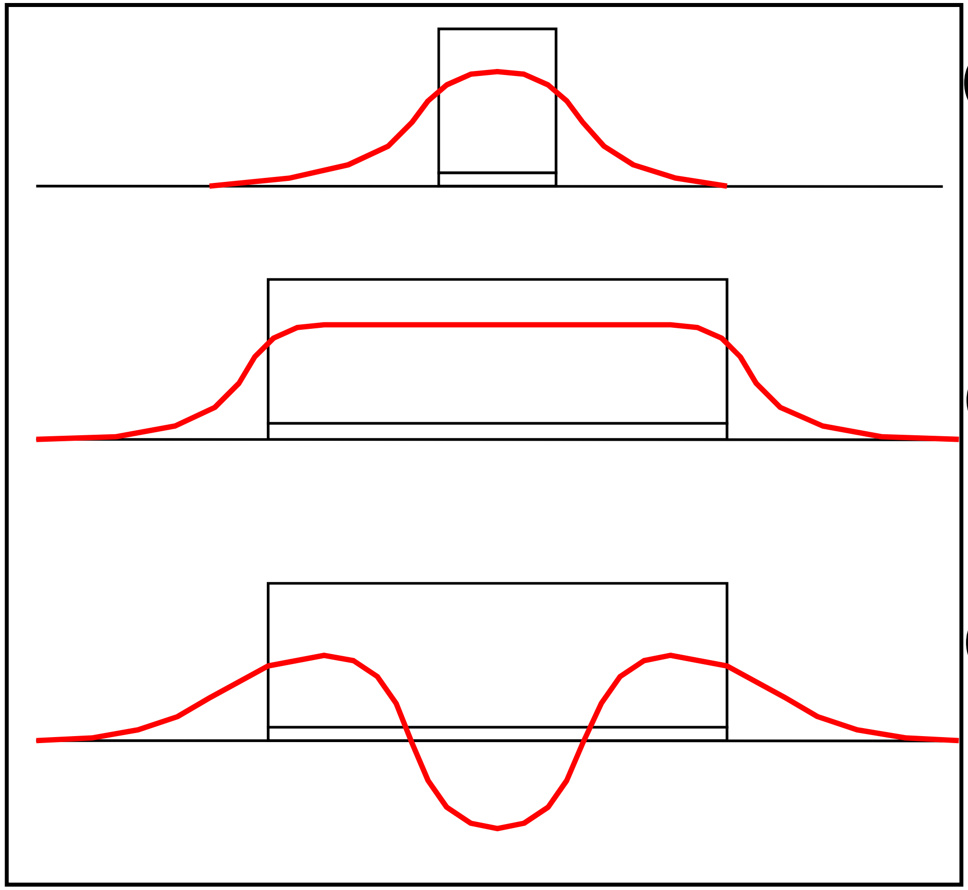
$$\frac{C_m^{(n)}}{C_m^{(0)}} = \left|\frac{A_n}{A_0}\right|^2 = \left|\int_{-w_e/2}^{+w_e/2} \phi_n(y)dy\right|^2 \left[w_e \int_{-\infty}^{+\infty} |\phi_n(y)|^2 dy \right]^{-1}$$

Effective Electromechanical Coupling Factor vs. Relative Aperture Width (When $w_e = w_g$)



Zero Excitation Efficiency for Anti-Symmetric Modes

Why Effective Coupling Factor Changes?



(a) S_0 Mode (When w is small)

Large Penetration

(b) S_0 mode (When w is large)

Small Penetration

(c) S_1 mode

Existence of Sign

Inverted Region